

## WS-3 Types of vector

### Task

① Given vector is  $\vec{A} = 6\hat{i} + 8\hat{j} + 10\hat{k}$

unit vector parallel to  $\vec{A} = \frac{\vec{A}}{|\vec{A}|}$

$$\Rightarrow \hat{A} = \frac{6\hat{i} + 8\hat{j} + 10\hat{k}}{\sqrt{6^2 + 8^2 + 10^2}}$$

$$\Rightarrow \hat{A} = \frac{6\hat{i} + 8\hat{j} + 10\hat{k}}{\sqrt{36 + 64 + 100}}$$

$$\Rightarrow \hat{A} = \frac{2(3\hat{i} + 4\hat{j} + 5\hat{k})}{\sqrt{200}}$$

$$\Rightarrow \hat{A} = \frac{2(3\hat{i} + 4\hat{j} + 5\hat{k})}{10\sqrt{2}}$$

$$\therefore \hat{A} = \frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$$

② Let the Given position vector  $\vec{R} = -6\hat{i} - 4\hat{j} - 12\hat{k}$

$\therefore$  unit vector parallel to  $\vec{R} = \frac{\vec{R}}{|\vec{R}|}$

$$\Rightarrow \vec{R} = \frac{-6\hat{i} - 4\hat{j} - 12\hat{k}}{\sqrt{(-6)^2 + (-4)^2 + (-12)^2}}$$

$$\Rightarrow \vec{R} = \frac{-2(3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{36 + 16 + 144}} = \frac{-2(3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{196}}$$

$$\vec{R} = \pm \frac{2(3\hat{i} + 2\hat{j} + 6\hat{k})}{14} \quad (\sqrt{196} = \pm 14)$$

$$\vec{R} = \pm \frac{1}{7} (3\hat{i} + 2\hat{j} + 6\hat{k})$$

③ If  $\vec{A} = 3\hat{i} + 4\hat{j}$  and  $\vec{B} = 7\hat{i} + 24\hat{j}$

let  $\vec{C}$  be the vector whose magnitude is same as that of  $\vec{B}$   $C = |\vec{B}|$

and also  $\vec{C}$  is parallel to  $\vec{A}$  (ie)  $\vec{C} = \hat{A}$  ( $|\hat{A}| = 1$ )

we can write it as  $\vec{C} = |\vec{B}| \times \hat{A}$

$$\Rightarrow \vec{C} = |\vec{B}| \times \frac{\vec{A}}{|\vec{A}|} \rightarrow \text{①}$$

$$\begin{aligned} |\vec{B}| &= \sqrt{7^2 + 24^2} \quad \therefore \vec{C} = \frac{25}{5} (3\hat{i} + 4\hat{j}) \quad |\vec{A}| = \sqrt{3^2 + 4^2} \\ &= \sqrt{49 + 576} \quad \vec{C} = 5(3\hat{i} + 4\hat{j}) \quad = \sqrt{9 + 16} \\ &= \sqrt{625} \quad \vec{C} = 15\hat{i} + 20\hat{j} \quad = \sqrt{25} \\ &= 25 \quad \vec{C} = 15\hat{i} + 20\hat{j} \quad = 5 \end{aligned}$$

④ Given force  $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$  ;  $a = 1 \text{ m/s}^2$

$$\begin{aligned} |\vec{F}| &= \sqrt{6^2 + (-8)^2 + 10^2} = \sqrt{36 + 64 + 100} \\ &= \sqrt{200} = 10\sqrt{2} \end{aligned}$$

$\therefore$  The mass of the body  $m = \frac{|\vec{F}|}{a} = 10\sqrt{2} \text{ kg}$

⑤ The velocity of the body  $\vec{u} = 2\hat{i} - 3\hat{j} + 11\hat{k} \text{ m/s}$   
 $\vec{a} = 10\hat{i} + 10\hat{j} + 10\hat{k} \text{ m/s}^2$

After  $t = 0.25 \text{ sec}$  the velocity of the body is

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\begin{aligned} \Rightarrow \vec{v} &= 2\hat{i} - 3\hat{j} + 11\hat{k} + (10\hat{i} + 10\hat{j} + 10\hat{k})0.25 \\ &= 2\hat{i} - 3\hat{j} + 11\hat{k} + 2.5\hat{i} + 2.5\hat{j} + 2.5\hat{k} \end{aligned}$$

$$|\vec{v}| = 4.5\hat{i} - 0.5\hat{j} + 13.5\hat{k}$$

$$\begin{aligned}
 |\vec{V}| &= \sqrt{(4 \cdot 5)^2 + (-0 \cdot 5)^2 + (13 \cdot 5)^2} \\
 &= \sqrt{\left(\frac{9}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + \left(\frac{27}{2}\right)^2} \\
 &= \sqrt{\left(\frac{1}{2}\right)^2 [9^2 + (-1)^2 + (27)^2]} = \frac{1}{2} \sqrt{81 + 1 + 729} \\
 |\vec{V}| &= \frac{1}{2} \sqrt{811} \text{ m/s}
 \end{aligned}$$

⑥ Given two vectors are

$$\text{let } \vec{A} = \overset{\hat{i}}{l_1} + \overset{\hat{j}}{m_1} + \overset{\hat{k}}{n_1} \quad \& \quad \vec{B} = \overset{\hat{i}}{l_2} + \overset{\hat{j}}{m_2} + \overset{\hat{k}}{n_2}$$

If any two vectors are parallel then the

$$\text{condition is } \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

$$\Rightarrow \frac{1}{3} = \frac{2}{m} = \frac{-3}{-9}$$

$$\therefore \text{Either from } \frac{1}{3} = \frac{2}{m} \quad (\text{or}) \quad \frac{2}{m} = \frac{+3}{+9}$$

$$\Rightarrow m = 6$$

$$\Rightarrow \frac{2}{m} = \frac{1}{3} \Rightarrow m = \underline{6}$$

⑦ let the two vectors are given by

$$\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k} \quad \& \quad \vec{b} = 5\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\text{Given condition is } 3\vec{a} + 2\vec{b} - \vec{c} = 0$$

$$\therefore 3(2\hat{i} + \hat{j} - 3\hat{k}) + 2(5\hat{i} + 3\hat{j} - 2\hat{k}) - \vec{c} = 0$$

$$\Rightarrow 6\hat{i} + 3\hat{j} - 9\hat{k} + 10\hat{i} + 6\hat{j} - 4\hat{k} - \vec{c} = 0$$

$$\Rightarrow 16\hat{i} + 9\hat{j} - 13\hat{k} - \vec{c} = 0$$

$$\Rightarrow \vec{c} = 16\hat{i} + 9\hat{j} - 13\hat{k}$$

⑧ Given three forces are

$$\vec{F}_1 = 3\hat{i} - 4\hat{j} + 2\hat{k} ; \vec{F}_2 = 2\hat{i} + 3\hat{j} - \hat{k} ; \vec{F}_3 = 2\hat{i} + 4\hat{j} - 5\hat{k}$$

∴ The resultant force is  $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$$\therefore \vec{F} = 3\hat{i} - 4\hat{j} + 2\hat{k} + 2\hat{i} + 3\hat{j} - \hat{k} + 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\Rightarrow \vec{F} = 7\hat{i} + 3\hat{j} - 4\hat{k}$$

⑨ Given  $\vec{A} = 6\hat{i} - 2\hat{k}$  Given relation is  
 $\vec{A} - 2\vec{B} = -3(\vec{A} + \vec{B})$

$$\Rightarrow \vec{A} - 2\vec{B} = -3\vec{A} - 3\vec{B}$$

$$\Rightarrow 3\vec{B} - 2\vec{B} = -3\vec{A} - \vec{A}$$

$$\Rightarrow \vec{B} = -4\vec{A} = -4[6\hat{i} - 2\hat{k}]$$

$$\vec{B} = -24\hat{i} + 8\hat{k}$$

⑩ Given  $\vec{A} = 3\hat{i} + 5\hat{j} - 2\hat{k}$  and  $\vec{B} = -3\hat{j} + 6\hat{k}$

Given condition is  $2\vec{A} + 7\vec{B} + 4\vec{C} = 0$

$$\therefore 2(3\hat{i} + 5\hat{j} - 2\hat{k}) + 7(-3\hat{j} + 6\hat{k}) + 4\vec{C} = 0$$

$$\Rightarrow 6\hat{i} + 10\hat{j} - 4\hat{k} - 21\hat{j} + 42\hat{k} + 4\vec{C} = 0$$

$$\Rightarrow 6\hat{i} - 11\hat{j} + 38\hat{k} + 4\vec{C} = 0$$

$$\Rightarrow 4\vec{C} = -6\hat{i} + 11\hat{j} - 38\hat{k}$$

$$\Rightarrow \vec{C} = -\frac{6}{4}\hat{i} + \frac{11}{4}\hat{j} - \frac{38}{4}\hat{k}$$

$$\vec{C} = -1.5\hat{i} + 2.75\hat{j} - 9.5\hat{k}$$

⑬ Given  $\vec{A} = 3\hat{i} + 4\hat{j}$  &  $\vec{B} = 12\hat{i} + 5\hat{j}$

$$|\vec{A}| = \text{magnitude} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\therefore |\vec{B}| = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

$$\text{unit vector of } \vec{A} = \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{3\hat{i} + 4\hat{j}}{5}$$

$$\text{unit vector of } \vec{B} = \hat{B} = \frac{\vec{B}}{|\vec{B}|} = \frac{12\hat{i} + 5\hat{j}}{13} \text{ is not a unit vector.}$$

⑭ Given  $\vec{A} = 2\hat{i} + 2\hat{j}$  &  $\vec{B} = 2\hat{i} - 4\hat{j}$

$$\begin{aligned} \text{magnitude of } \vec{B} = |\vec{B}| &= \sqrt{2^2 + (-4)^2} \\ &= \sqrt{4 + 16} = \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

⑮ magnitude of  $\vec{A} = |\vec{A}| = \sqrt{2^2 + 2^2}$   
 $= \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

⑯ unit vector of  $\vec{A} = \hat{A} = \frac{\vec{A}}{|\vec{A}|}$

$$\begin{aligned} &= \frac{2\hat{i} + 2\hat{j}}{2\sqrt{2}} = \frac{2\hat{i} + 2\hat{j}}{\sqrt{8}} \\ &= \frac{2(\hat{i} + \hat{j})}{2\sqrt{2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} \end{aligned}$$

⑰ Given  $\vec{A} = 3\hat{i} + 4\hat{j} + 10\hat{k}$

It's magnitude along x-axis is 3

## Task Ques

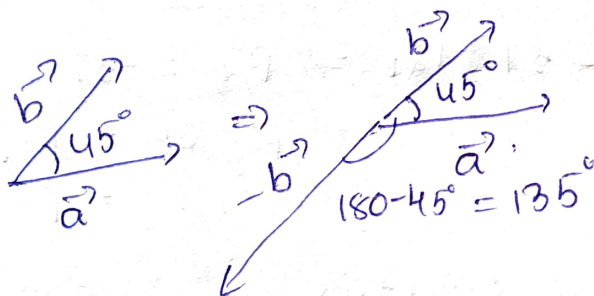
Q Given  $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$  then the magnitude of  $\vec{A} = |\vec{A}| = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{9 + 16 + 25} = \sqrt{50}$

$$|\vec{A}| = 5\sqrt{2}$$

12 Given  $\vec{Q} = 10\hat{i} + 8\hat{j} + 6\hat{k}$

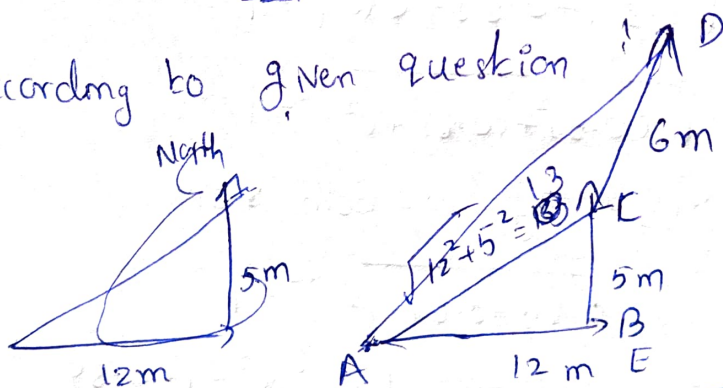
$$\begin{aligned}\text{magnitude in } yz\text{-plane} &= \sqrt{y^2 + z^2} \\ &= \sqrt{8^2 + 6^2} = \sqrt{64 + 36} \\ &= \sqrt{100} = 10\end{aligned}$$

13 The angle between  $\vec{a}$  and  $\vec{b}$  is  $45^\circ$



SAG's

1 According to given question



$$\begin{aligned}\text{The displacement of a particle} &= \sqrt{12^2 + 5^2 + 6^2} \\ &= \sqrt{144 + 25 + 36} \\ &= \sqrt{205} \\ &= 14.32 \text{ m.}\end{aligned}$$

(2) Given vector is  $\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

The magnitude is  $= \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$   
 $= \sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{9}{9}} = 1$

(3) Given Vector let it be  $\vec{A} = 5\hat{i} + p\hat{j} + 4\sqrt{2}\hat{k}$

Given that  $|\vec{A}| = 11$ .

$\Rightarrow \sqrt{5^2 + p^2 + (4\sqrt{2})^2} = 11$  By squaring on both sides

$\Rightarrow \left\{ \sqrt{25 + p^2 + 32} \right\}^2 = 11^2$

$\Rightarrow p^2 + 57 = 121 \Rightarrow p^2 = 121 - 57$

$\Rightarrow p^2 = 64 \Rightarrow p = \sqrt{64}$

$\Rightarrow p = \pm 8$

(4) Given vector  $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$  is a unit vector

$\Rightarrow |0.5\hat{i} + 0.8\hat{j} + c\hat{k}| = 1$

$\Rightarrow \sqrt{(0.5)^2 + (0.8)^2 + c^2} = 1$

By squaring on both sides.

$\Rightarrow \left( \sqrt{0.25 + 0.64 + c^2} \right)^2 = 1$

$\Rightarrow 0.89 + c^2 = 1 \Rightarrow c^2 = 1 - 0.89$

$\Rightarrow c^2 = 0.11$

$\Rightarrow c = \sqrt{0.11}$

(5) Given vector is  $3\hat{i} + \hat{j} + 2\hat{k}$

It's length in xy plane  $= \sqrt{x^2 + y^2}$

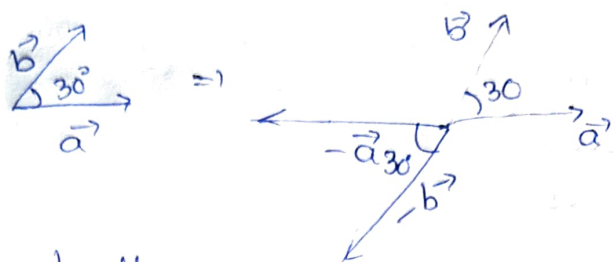
$= \sqrt{3^2 + 1}$

$= \sqrt{9 + 1}$

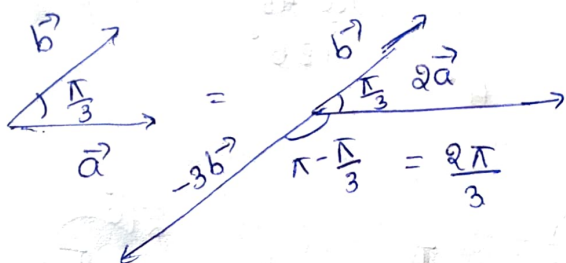
$= \sqrt{10}$

(6) Given angle between  $\vec{a}$  and  $\vec{b}$  is  $30^\circ$

(i.e)



(7) Graphically



(8) Given  $|x\hat{i} - 3\hat{j} + 5\hat{k}| = \sqrt{98}$

i.e  $\sqrt{x^2 + (-3)^2 + 5^2} = \sqrt{98}$

By doing squaring on both sides we get

$$\Rightarrow (\sqrt{x^2 + 9 + 25})^2 = (\sqrt{98})^2$$

$$\Rightarrow x^2 + 34 = 98$$

$$\Rightarrow x^2 = 64 \Rightarrow x = \pm 8$$

(9)

(a)  $|3\hat{i} + 4\hat{j}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

(b)  $|2\hat{i} + 4\hat{j} + 6\hat{k}| = \sqrt{2^2 + 4^2 + 6^2} = \sqrt{4 + 16 + 36} = \sqrt{56}$

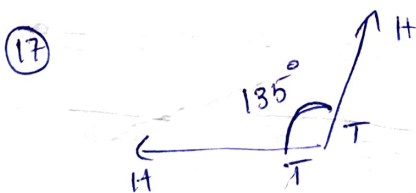
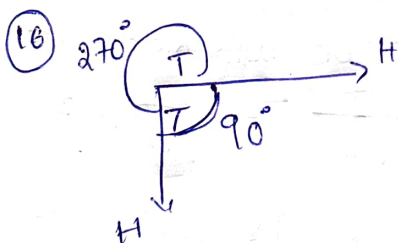
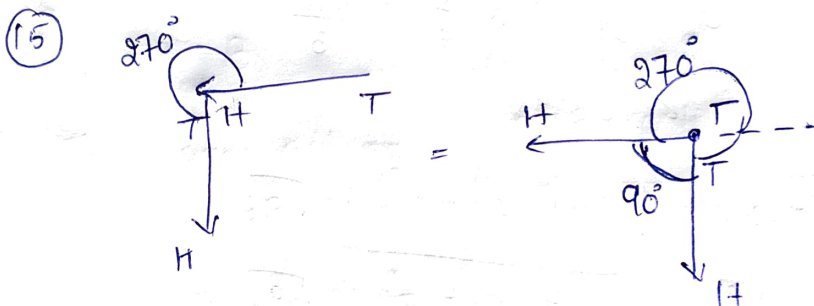
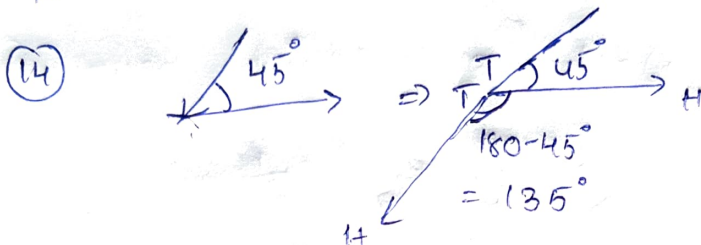
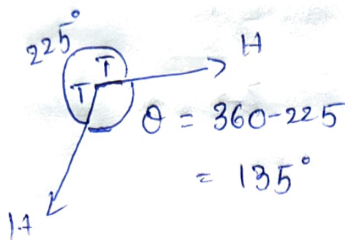
(c)  $|2\hat{i} + 2\hat{j} + 2\hat{k}| = \sqrt{2^2 + 2^2 + 2^2} = \sqrt{4 + 4 + 4} = \sqrt{12}$

(10) The vector is  $3\hat{i} + 4\hat{j} + 5\hat{k}$

The unit vector =  $\frac{\text{Vector}}{\text{magnitude}} = \frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{\sqrt{3^2 + 4^2 + 5^2}}$

$$= \frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{\sqrt{9 + 16 + 25}} = \frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$$

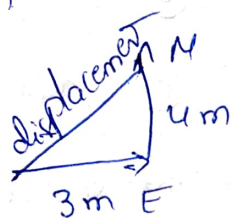
(13) use Head-Head (or) Tail-Tail method



(18) According to given question

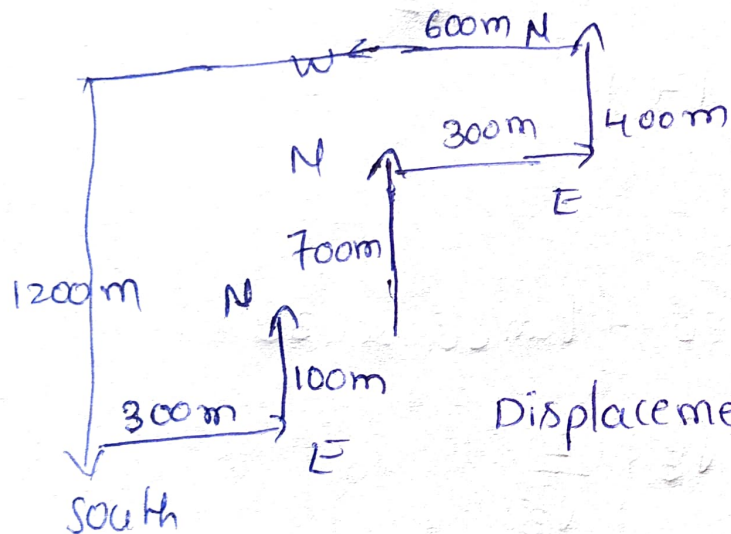
$$\therefore \text{displacement} = 3\hat{i} + 4\hat{j}$$

$$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ m}$$



East is represented by a unit vector  $\hat{i}$  and North by  $\hat{j}$

(19) Acc to given question



$\hat{i}$  → unit vector along East

$\hat{j}$  → " " " North

$-\hat{i}$  → unit vector along west

$-\hat{j}$  → unit vector along south

$$\text{Displacement} = 700\hat{j} + 300\hat{i} + 400\hat{j} - 600\hat{i} - 1200\hat{j} + 300\hat{i} + 100\hat{j}$$

$$\vec{S} = -400\hat{j} + 400\hat{j} \Rightarrow \underline{\vec{S} = 0}$$

$\vec{S}$  is a vector