FOUNDATION^{+} (V_2)

Class: 9th. MATHEMATICS

1. FOOT OF THE PERPENDICULAR

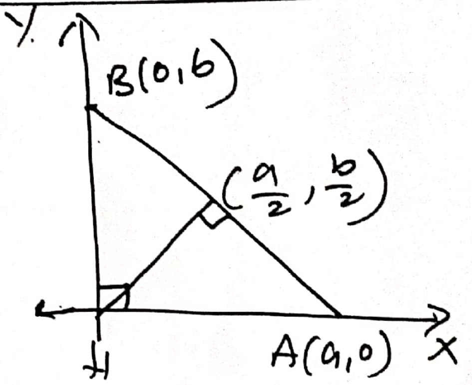
TEACHING TASK (JEE MAINS)

01. Given the points A $(a\cos\theta, a\sin\theta)$, B $(a\cos\phi, a\sin\phi)$

Let $\theta = 0^\circ$, and $\phi = 90^\circ$

$\therefore$ $A(a,0)$, $B(0,a)$

Foot of the perpendicular $= \left(\frac{a}{2}, \frac{a}{2}\right)$



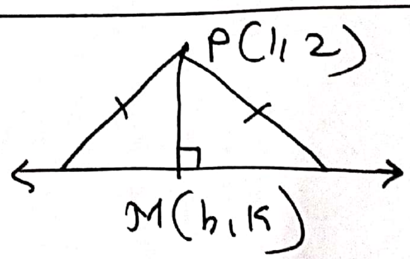
option: C: $\left(\frac{a(\cos\theta+\cos\phi)}{2}, \frac{a(\sin\theta+\sin\phi)}{2}\right)$

$= \left(\frac{a}{2}, \frac{a}{2}\right)$

Ans: C

02 $2x - y + 3 = 0$, $P(1,2)$

M is the foot of the $\perp$r from $P(1,2)$ on $2x-y+3=0$



Let $M(h,k)$

$\therefore \frac{h-1}{2} = \frac{k-2}{-1} = \frac{-2(2.1-2+3)}{2^2+(-1)^2}$

$\therefore (h,k) = \left(\frac{-1}{5}, \frac{13}{5}\right)$

Ans: A

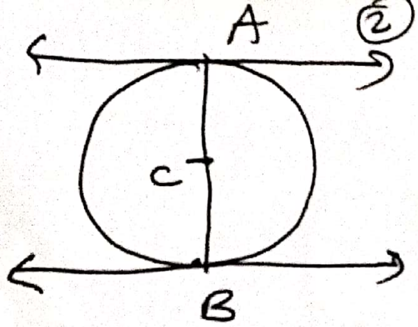
03 The Images of $(7,2)$, $(-3,2)$ w.r.t X-axis are $(7,-2)$ and $(-3,-2)$ respectively.

clearly, Eqn of the line passing through $(7,-2)$ and $(-3,-2)$ is $y = -2$

Ans: D

04 $3x+4y-14=0 \rightarrow (1)$

$6x+8y+7=0 \rightarrow (2)$



$(1) \times 2 \Rightarrow 6x+8y-28=0 \rightarrow (3)$

Distance b/w the lines (2) & (3)

$$AB = \frac{|7+28|}{\sqrt{36+64}} = \frac{35}{10} = \frac{7}{2}$$

$\therefore$ Diameter $= \frac{7}{2} \Rightarrow$ radius $= \frac{7}{4}$ or

Ans:

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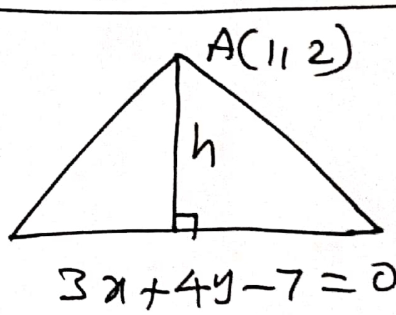
05 $$h = \frac{3(1)+4(2)-7}{\sqrt{9+16}} = \frac{4}{5}$$



$$\therefore \frac{\sqrt{3}}{2}a = \frac{4}{5} \Rightarrow a = \frac{8\sqrt{3}}{15}$$

Ans: C

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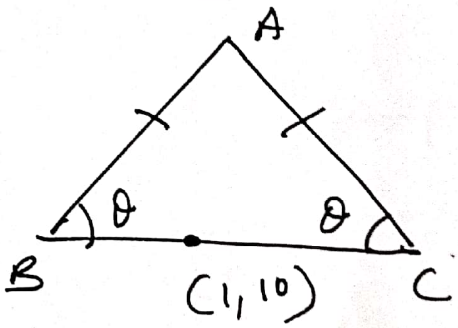
06 $AB \Rightarrow 7x-y+3=0$

Slope of AB $= 7$

$AC \Rightarrow x+y-3=0$



slope of AC $= -1$

Let slope of BC $= m$

We have, $$\left|\frac{m-7}{1+7m}\right| = \left|\frac{m+1}{1-m}\right|$$

$$\Rightarrow \frac{m-7}{1+7m} = \frac{m+1}{1-m} \quad \text{(or)} \quad \frac{m-7}{1+7m} = -\left(\frac{m+1}{1-m}\right)$$

$\Rightarrow 8m^2+8=0$ $\qquad \Rightarrow 3m^2+8m-3=0$

$\Rightarrow m^2=-1$

not possible

Ans: C

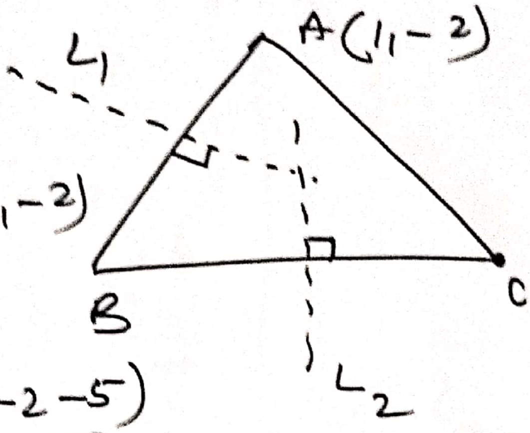
07. We know, the image of (a,b) w.r.t the line $y=x$ is (b,a)

$\therefore$ The image of $(-1,1)$ wrt $y=x$ is $(1,-1)$

Ans: B

08. $L_1 \Rightarrow \quad x-y-5=0$

$L_2 \Rightarrow \quad x+2y=0$



B is the image of $A(1,-2)$ w.r.t $x-y-5=0$

$$\frac{h-1}{1}=\frac{k+2}{-1}=-2\frac{(1+2-5)}{1+(-1)^2}$$

$\therefore B(3,-4)$.

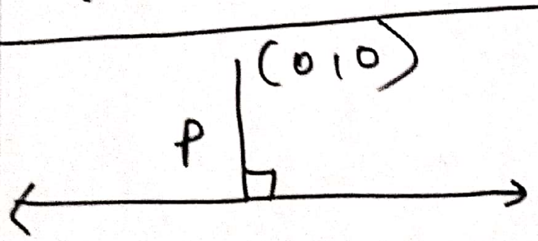
C is the image of $B(3,-4)$ w.r.t $x+2y=0$

$$\therefore \frac{h-3}{1}=\frac{k+4}{2}=-\frac{2(3+2(-4))}{1+4}=2$$

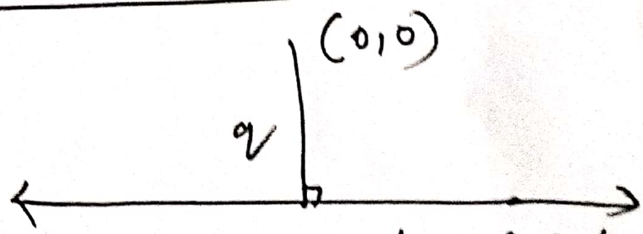
$\therefore C(5,0)$

Ans: C

09.



$x\sec\alpha - y\,\mathrm{cosec}\,\alpha - a = 0$



$x\cos\alpha + y\sin\alpha - a\cos 2\alpha = 0$

$$\therefore p=\frac{|-a|}{\sqrt{\sec^2\alpha+\mathrm{cosec}^2\alpha}}$$

$\therefore p = a\cos\alpha\sin\alpha$

$p=\frac{a}{2}\sin 2\alpha$

$\Rightarrow 2p = a\sin 2\alpha$

$$q=\frac{|-a\cos 2\alpha|}{\sqrt{\cos^2\alpha+\sin^2\alpha}}$$

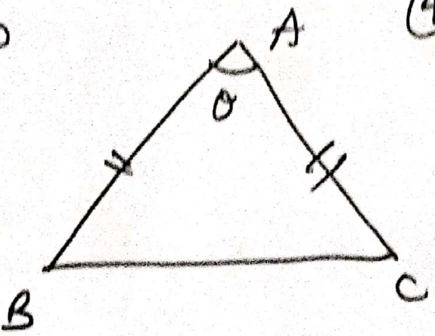
$\therefore q = a\cos 2\alpha$

$\Rightarrow 4p^2+q^2=a^2$

Ans. A

10 $AB \Rightarrow (a+b)x+(a-b)y-2ab=0$

$m_{AB} = -\left(\frac{a+b}{a-b}\right)$



$AC \Rightarrow (a-b)x+(a+b)y-2ab=0$

$m_{AC} = -\left(\frac{a-b}{a+b}\right)$

$$\tan\theta = \left| \frac{-\left(\frac{a+b}{a-b}\right) + \left(\frac{a-b}{a+b}\right)}{1+\left(\frac{a+b}{a-b}\right)\left(\frac{a-b}{a+b}\right)} \right|$$

$$\therefore \tan\theta = \left(\frac{2ab}{a^2-b^2}\right)$$

Ans: B

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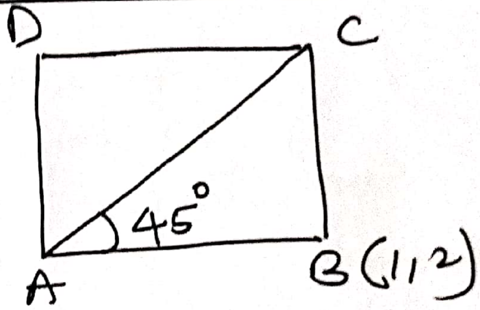
11) $AC \rightarrow 8x-15y=0$



$M_{AC} = \frac{8}{15}$

let slope of $AB = m$

$$\tan 45^\circ = \left| \frac{\frac{8}{15}-m}{1+\frac{8m}{15}} \right|$$

$$\Rightarrow 1 = \left| \frac{8-15m}{15+8m} \right|$$

$$\Rightarrow m = -\frac{7}{23} \text{ or } \frac{23}{7}$$

opt A $\rightarrow$ $23x-7y=9$

$m = \frac{23}{7}$

opt B: $7x+23y=53$

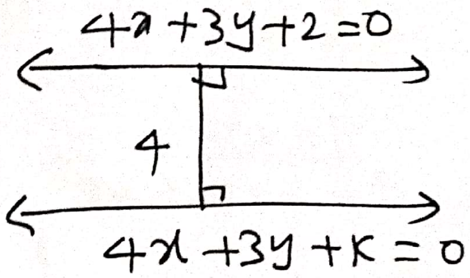
$m = -\frac{7}{23}$

Ans: A, B

II method: By direct options verification

(1,2) satisfies $23x-7y=9$ and $7x+23y=53$

12. $$4 = \frac{|K-2|}{\sqrt{9+16}}$$



$\Rightarrow |K-2| = 20$

$\Rightarrow K = 22, -18$

Equations are $4x+3y+22=0$, $4x+3y-18=0$

Ans.. A, C

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13. Statement I: $x\cos\alpha + y\sin\alpha - p = 0$

Let $x\cos\alpha + y\sin\alpha + K = 0$

$$\text{Distance} = \frac{|K+p|}{\sqrt{\cos^2\alpha + \sin^2\alpha}} = p' - p$$

$\Rightarrow K + p = \pm(p' - p)$

$\Rightarrow K = p' - 2p$ or $K = -p'$

Hence, the eqn of the second line is

$x\cos\alpha + y\sin\alpha + p' - 2p = 0$ (True)

Ans. A

Statement II: Conceptual (True)

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14. Statement I: $2x + 3y + 4 = 0$

$\lambda x + Ky + 2 = 0$

Identical lines $\Rightarrow \frac{2}{\lambda} = \frac{3}{K} = \frac{4}{2}$

$\Rightarrow \lambda = 1, K = \frac{3}{2}$

Now $3\lambda - 2K = 3(1) - 2\left(\frac{3}{2}\right) = 0$ (True)

Ans. A

Statement II: Conceptual (True)

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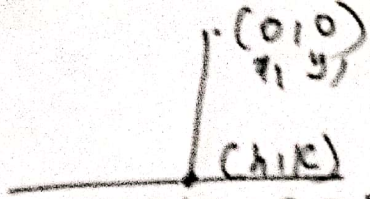
15. Assertion: Conceptual (True)

Ans.. A

Reason: Conceptual (True)

~~16. ASS~~

16 **Assertion:**



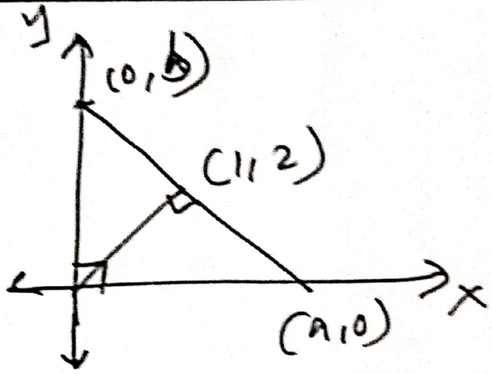
$$\frac{h-0}{a} = \frac{k-0}{b} = \frac{-(a\cdot 0+b\cdot 0+c)}{a^2+b^2}$$

$$\Rightarrow \frac{h}{a} = \frac{k}{b} = \frac{-c}{a^2+b^2}$$

$$\Rightarrow (h,k) = \left(\frac{-ac}{a^2+b^2}, \frac{-bc}{a^2+b^2}\right) \quad \text{(True).}$$

Reason: conceptual (~~[illegible]~~) (True) Ans: B

17\. $\left(\frac{a}{2}, \frac{b}{2}\right) = (1,2)$



$\Rightarrow a=2, \quad b=4$

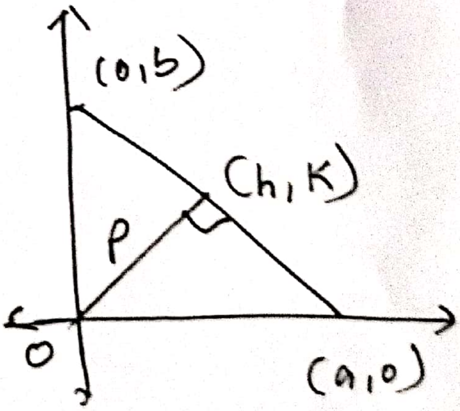
$\therefore$ Eqn of the line

$$\frac{x}{2}+\frac{y}{4}=1$$

$$\Rightarrow 2x+y-4=0$$

$\perp$r distance from origin $= \frac{|-4|}{\sqrt{4+1}} = \frac{4}{\sqrt{5}}$ Ans: C

18\. $\left(\frac{a}{2}, \frac{b}{2}\right) = (h,k)$



$\Rightarrow a=2h, \quad b=2k$

Eqn of line

$$\Rightarrow \frac{x}{2h}+\frac{y}{2k}=1$$

$$\Rightarrow \frac{x}{2h}+\frac{y}{2k}-1=0$$

$\perp$r. distance from the origin $= p$

$$\Rightarrow \frac{|-1|}{\sqrt{\left(\frac{1}{2h}\right)^2+\left(\frac{1}{2k}\right)^2}} = p$$

$$\Rightarrow \frac{1}{h^2}+\frac{1}{k^2}=\frac{4}{p^2}$$

$\therefore$ Eqn of the locus

$$\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$$

Ans. C

19. $\frac{q^3-p^3}{q-r} = \frac{r^3-p^3}{r-p} = \frac{p^3-q^3}{p-q}$

$\Rightarrow q^2+qr+r^2 = r^2+rp+p^2 = p^2+pq+q^3$

$\Rightarrow q^2+qr-rp-p^2=0$

$\Rightarrow (q^2-p^2)+r(q-p)=0$

$\Rightarrow (q+p)(q-p)+r(q-p)=0$

$\Rightarrow (q-p)(p+q+r)=0$

$\Rightarrow q-p=0$ or $\boxed{p+q+r=0}$

$\Rightarrow \boxed{q=p}$

Similarly we can prove $\boxed{p=r}$

$\therefore$ i.e $p=q=r$

This can not be possible

Ans. A

20 $P(2,-1)$, $Q(5,3)$, $R(-2,6)$

slope of $PQ = m_1 = \frac{3+1}{5-2} = \frac{4}{3}$

slope of $QR = m_2 = \frac{6-3}{-2-5} = \frac{-3}{7}$

$\therefore \tan\theta = \left| \frac{\frac{4}{3}+\frac{3}{7}}{1+\frac{4}{3}\cdot\frac{3}{7}} \right| = \frac{37}{9}$

Ans: D

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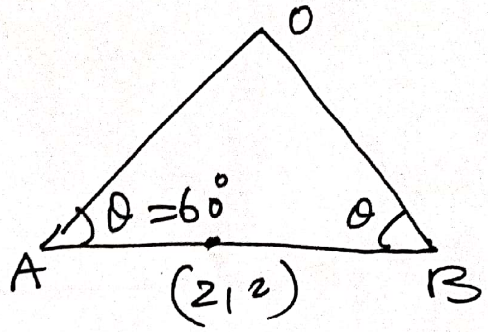
21. $OA \rightarrow \sqrt{3}x + y = 0$

$m_{OA} = -\sqrt{3}$



$OB \rightarrow \sqrt{3}x - y = 0$

$m_{OB} = \sqrt{3}$

$$\tan 60° = \left| \frac{\sqrt{3} - m}{1 + \sqrt{3}m} \right|$$

$$\Rightarrow \sqrt{3} = \left| \frac{\sqrt{3} - m}{1 + \sqrt{3}m} \right|$$

$$\Rightarrow \pm\sqrt{3} = \frac{\sqrt{3} - m}{1 + \sqrt{3}m}$$

Case(i) $\sqrt{3} = \dfrac{\sqrt{3} - m}{1 + \sqrt{3}m}$

$\Rightarrow m = 0$

i.e. The line is parallel to X-axis

$\therefore$ Eqn of the line $y = K$

$(2,2) \Rightarrow y = 2$

$\therefore K = 2$ Ans: 2

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22. $4x - 3y - 8 = 0$; $2x + 5y - 4 = 0$

$m_1 = \dfrac{4}{3}$ $\qquad m_2 = -\dfrac{2}{5}$

$$\tan\theta = \left| \frac{\frac{4}{3} + \frac{2}{5}}{1 - \frac{8}{15}} \right| \Rightarrow \tan\theta = \left| \frac{26}{7} \right|$$

$\therefore \alpha = \dfrac{26}{7} = 3.7$

Ans: 3

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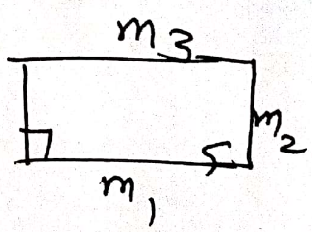
23 a) $x = -3 \Rightarrow x + 3 = 0$ (s)

b) $\left. \begin{array}{l} 6x + 8y + 6 = 0 \\ 6x + 8y + 11 = 0 \end{array} \right\}$ $d = \dfrac{|6 - 11|}{\sqrt{36 + 64}} = \dfrac{5}{10} = \dfrac{1}{2}$ (p)

c) ~~y = 4x~~ $y = 4 - 3x \rightarrow m_1 = -3$

$ay = x + 10 \rightarrow m_2 = \dfrac{1}{a}$

$2y + bx + 9 = 0 \rightarrow m_3 = \dfrac{-b}{2}$

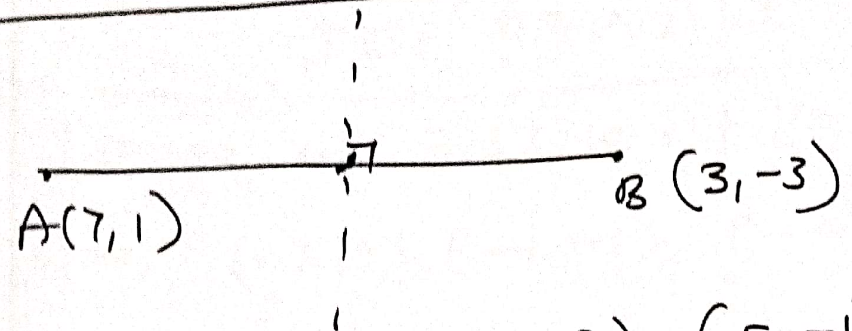


$m_1 \times m_2 = -1$	$m_1 = m_3$	$ab = 18$ (q)
$\Rightarrow -3 \times \frac{1}{a} = -1$	$-3 = -\frac{b}{2}$	
$\Rightarrow a = 3$	$\Rightarrow b = 6$	

Ans: 18

Ans: s, p, q

d)



$\cdot\, m = \left(\frac{7+3}{2}, \frac{1-3}{2}\right) = (5,-1)$ (r)

Ans: s, p, q, r

24 A) $(2,-3)$ (p)
B) $(-4,-5)$ (q)
C) $(2,1)$ (r)
D) $(-4,-3)$ (s)

Ans: p, q, r, s

LEARNERS TASK (CUO'S)

01.	Conceptual	Ans: B
02	Conceptual	Ans: C
03	Conceptual	Ans: A
04	Conceptual	Ans: C
05	Conceptual	Ans: D
06	Conceptual	Ans: D
07	$2P = A + B$	Ans: C

$\longleftarrow P \overset{A}{\underset{B}{|}} \longrightarrow$

08	Conceptual	Ans: B
09.	$f(x, -y) = 0$	Ans: A
10	$f(-b, -a) = 0$	Ans: C

JEE MAINS LEVEL

01. Image of (2,3) w.r.t y-axis is (-2,3)

Now, (-2,3), (-1,-1), (-4, n) are collinear

$$\Rightarrow \frac{-1-3}{-1+2} = \frac{n+1}{-4+1} \Rightarrow n = 11$$

Ans: C

02
$$\frac{x}{p\sec\alpha} + \frac{y}{p\,\text{cosec}\,\alpha} = 1$$

$$\Rightarrow \left(\frac{\cos\alpha}{p}\right)x + \left(\frac{\sin\alpha}{p}\right)y - 1 = 0$$

$\perp$r distance from origin $= \dfrac{|-1|}{\sqrt{\left(\frac{\cos\alpha}{p}\right)^2 + \left(\frac{\sin\alpha}{p}\right)^2}} = |p|$

Ans: D

03 Let $L \equiv 3x + 4y - 7 = 0$, $(1,2), (k, 1)$
$\quad\quad (x_1, y_1), (x_2, y_2)$

$L_{11} = 3(1) + 4(2) - 7 = 4$

$L_{22} = 3k + 4(1) - 7 = 3k - 3$

Required Ratio $= -L_{11} : L_{22} = -4 : (3k-3)$

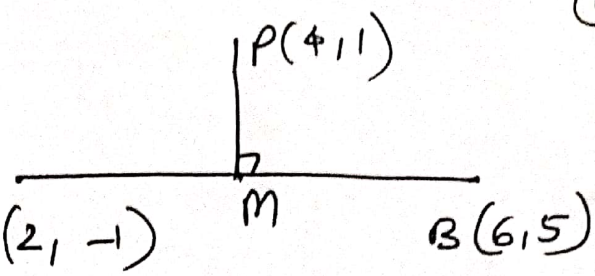
Given Ratio $= 4 : 9$

$$\therefore \frac{-4}{3k-3} = \frac{4}{9}$$

$$\Rightarrow k = -2$$

Ans: B

04. Slope of AB $= \frac{5+1}{6-2} = \frac{3}{2}$



$\therefore$ slope of PM $= -\frac{2}{3}$

Eqn of PM

$y - 1 = -\frac{2}{3}(x-4)$

$\Rightarrow \quad 2x + 3y - 11 = 0$

Required ratio $= -L_{11} : L_{22}$

$= -\left[2(2) + 3(-1) - 11\right] : \left(2(6) + 3(5) - 11\right)$

$= \quad 5 : 8$

Ans.. B

05

$x \cos\alpha + y \sin\alpha - p_1 = 0$

$x \cos\beta + y \sin\beta - p_2 = 0$

$\cos\theta = \dfrac{\cos\alpha \cdot \cos\beta + \sin\alpha \sin\beta}{\sqrt{\cos^2\alpha + \sin^2\alpha} \cdot \sqrt{\cos^2\beta + \sin^2\beta}} = \cos(\alpha - \beta)$

Ans.. B

$\therefore \Rightarrow \quad \theta = \alpha - \beta$

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06 Foot of the $\perp$r of (4,5) on X-axis = (4, 0)

Image of (4,5) w.r.t y-axis = (-4, 5)

Eqn of the line $\quad y - 0 = \frac{5-0}{-4-4}(x-4)$

$\Rightarrow \quad 5x + 8y - 20 = 0$ Ans.. B

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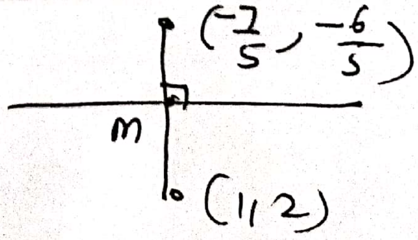
07 $M = \left(\frac{-1}{5}, \frac{2}{5}\right)$



opt: B : $3x + 4y - 1 = 0$ satisfies.

$\left(-\frac{1}{5}, \frac{2}{5}\right)$

Ans: B

08 Given line $4x-y-2=0 \rightarrow (1)$

$\Rightarrow y = 4x-2$

Let the point on (1) be $P(x, 4x-2)$.

Let $A(-5,6)$, $B(3,2)$.

Given $PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow (x+5)^2 + (4x-8)^2 = (x-3)^2 + (4x-4)^2$

$\Rightarrow x = 4$

$\therefore$ Required point $= (4, 14)$

Ans: B

09 Foot of the ⊥r from $(0,0)$ on the line

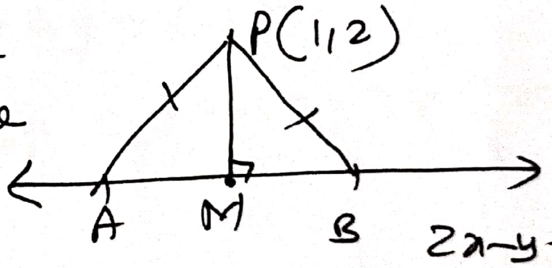
$2x-y+5=0$

$$\frac{h-0}{2} = \frac{k-0}{-1} = -\frac{(2\cdot 0 - 0 + 5)}{4+1}$$

Ans: C

$\Rightarrow (h,k) = (-2,1)$

10 M is the foot of the ⊥r from $P(1,2)$ on the line $2x-y+3=0$



$$\frac{h-1}{2} = \frac{k-2}{-1} = -\frac{(2\cdot 1 - 2 + 3)}{4+1}$$

$\therefore M(h,k) = \left(-\frac{1}{5}, \frac{13}{5}\right)$

Ans: A

JEE ADVANCED LEVEL

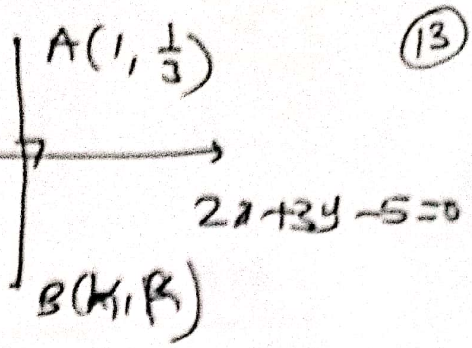
11. $4x-y-7=0$; $kx-5y-9=0$

$$\cos 45^\circ = \frac{|4\cdot k+5|}{\sqrt{16+1}\cdot\sqrt{k^2+25}}$$

$k = -\frac{25}{3}$ or 3

Ans: A, B

12. $B(h,k)$ is the Image of $A(1,\frac{1}{3})$ w.r.t $2x+3y-5=0$



$$\frac{h-1}{2} = \frac{k-\frac{1}{3}}{3} = \frac{-2\left(2.1+3.\frac{1}{3}-5\right)}{4+9}$$

$\therefore B(h,k) = \left(\frac{21}{13}, \frac{49}{39}\right)$

Ans: A, B

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13 Statement I: $y-3kx+4=0 \Rightarrow 3kx-y-4=0 \rightarrow (1)$

Also, $(2k-1)x-(8k-1)y-6=0 \rightarrow (2)$

Lines are perpendicular $\Rightarrow a_1a_2+b_1b_2=0$

$(3k)(2k-1)+(-1)(-(8k-1))=0$

$\Rightarrow k=\frac{1}{6}$ (True)

Statement II: Conceptual (True)

Ans. A

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14 Statement I: $(1,1)$, $3x+4y+c=0$

$$7 = \frac{|3(1)+4(1)+c|}{\sqrt{9+16}} \Rightarrow c=-42, 28 \text{ (True)}$$

Statement II: Conceptual (True)

Ans. A

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15 Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans. A

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16 Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans. A

17. $(4, -13)$; $5x + y + 6 = 0$

$$\frac{h-4}{5} = \frac{k+13}{1} = \frac{-2(5\cdot 4 - 13 + 6)}{25+1}$$

$\therefore (h, k) = (-1, -14)$

Ans : A

18 $(3, 5)$; $x - y + 1 = 0$

$$\frac{h-3}{1} = \frac{k-5}{-1} = \frac{-2(3-5+1)}{1^2+(-1)^2} \; ; \; (h, k) = (4, 4)$$

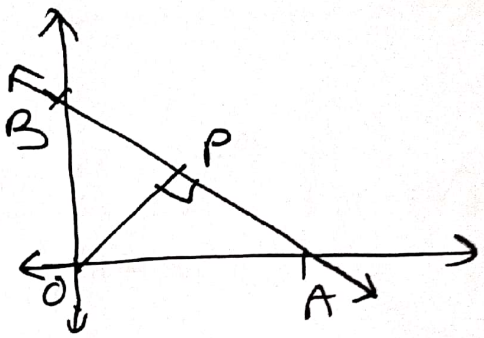
option B: $(x-2)^2 + (y-4)^2$

$= (4-2)^2 + (4-4)^2 = 4$

Ans. B

19 $3x + y - \lambda = 0 \longrightarrow (1)$

$A\left(\frac{\lambda}{3}, 0\right)$, $B(0, \lambda)$



slope of $AB = -3$

$\therefore$ slope of $OP = \frac{1}{3}$

Eqn of $OP \Rightarrow y = mx$

$\Rightarrow y = \frac{1}{3}x$

$\Rightarrow x - 3y = 0 \longrightarrow (2)$

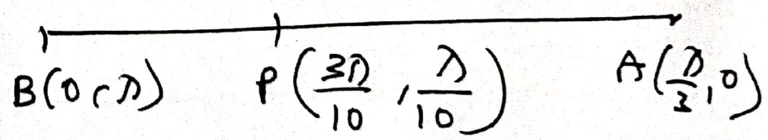
$\Rightarrow x = 3y$

sloving (1) & (2) for P

$(1) \Rightarrow 3(3y) + y - \lambda = 0 \Rightarrow y = \frac{\lambda}{10}$

$\therefore x = 3y = 3 \cdot \frac{\lambda}{10}$

$\therefore P\left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$

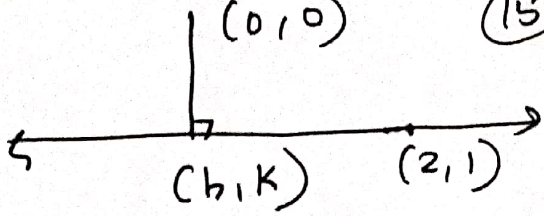


$BP : PA = 0 - \frac{3\lambda}{10} : \frac{3\lambda}{10} - \frac{\lambda}{3}$

$= 9 : 1$

Ans: A

20 $m_1 \times m_2 = -1$



$$\frac{k}{h} \times \frac{k-1}{h-2} = -1$$

$$\Rightarrow k^2 - k = -h^2 + 2h$$

$$\Rightarrow h^2 + k^2 - 2h - k = 0$$

Eqn of the locus $\Rightarrow x^2 + y^2 - 2x - y = 0$ Ans.- B

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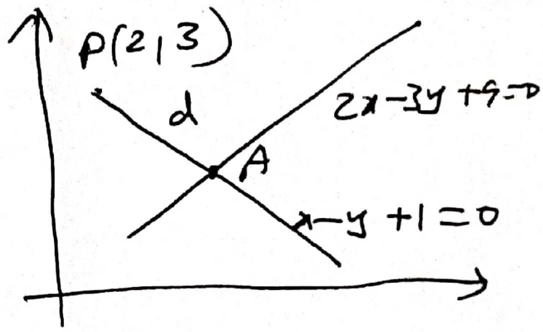
21. $x - y + 1 = 0$



$$\Rightarrow m = -\frac{a}{b} = \frac{-1}{-1} = 1$$

Slope $= m = 1$

$\tan\theta = 1$

$\Rightarrow \theta = 45^\circ$

$$\text{Distance} = \left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right| = \left|\frac{2(2) - 3(3) + 9}{2\left(\frac{1}{\sqrt{2}}\right) + 3\left(\frac{1}{\sqrt{2}}\right)}\right| = 4\sqrt{2}$$

$\therefore d = 4\sqrt{2} \Rightarrow d^2 = 32$ Ans: 32

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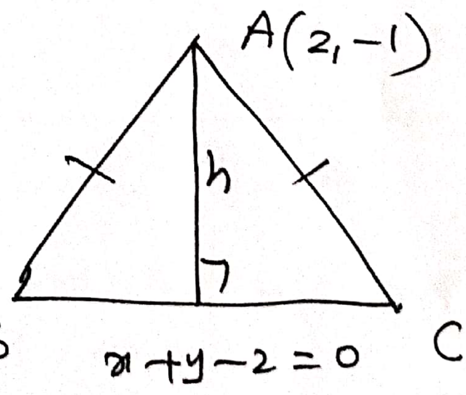
22 $A(2,-1)$, $x + y - 2 = 0$

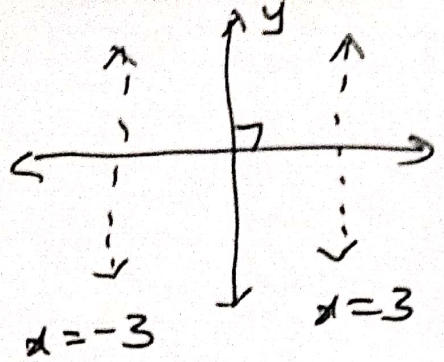


$$h = \frac{|2-1-2|}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

$$\therefore \frac{\sqrt{3}}{2} d = \frac{1}{\sqrt{2}}$$

$$\Rightarrow d = \frac{2}{\sqrt{6}} \Rightarrow d^2 = \frac{4}{6} = 0.6$$ Ans. 0

23 i)



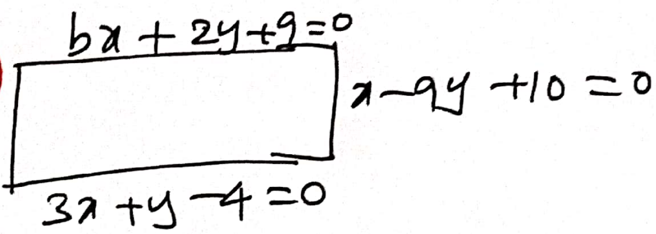
(s)

ii) $3x+4y+3=0 \Rightarrow 6x+8y+6=0$

$6x+8y+11=0$

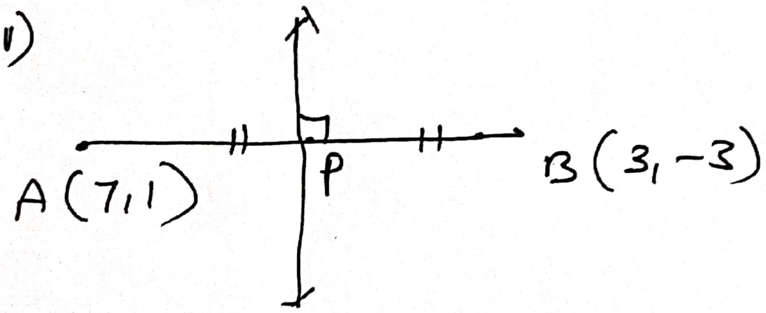
$$\therefore d = \frac{|6-11|}{\sqrt{36+64}} = \frac{1}{2} \text{ (p)}$$

iii) (Repeated)



$ab = 18$ (q)

iv)

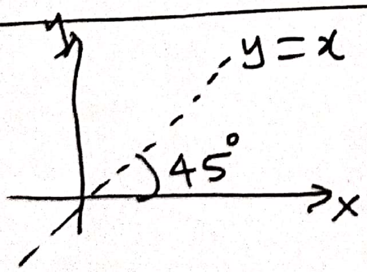


$$P = \left(\frac{7+3}{2}, \frac{1-3}{2}\right)$$

$$= (5, -1) \text{ (r)}$$

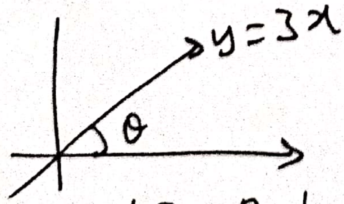
Ans:- s, p, q, r

24 i)



(p)

ii)



$$\tan\theta = \left|\frac{3-0}{1+3\cdot 0}\right|$$

$$\therefore \theta = \tan^{-1}(3) \text{ (q)}$$

iii) $m_1 = -1$, $m_2 = 2$

$$\tan\theta = \left|\frac{-1-2}{1-2}\right| = 3 \text{ (q)}$$

iv) $m_1 = -2$, $m_2 = \frac{1}{2}$

$$m_1 \times m_2 = -2 \times \frac{1}{2} = -1$$

$$\therefore \theta = 90^\circ \text{ (r)}$$

Ans: p, q, q, r

⟹ THE END ⟸