

7. SURFACE AREA AND VOLUME ①

Class: IX. Mathematics

FOUNDATION

TEACHING TASKS

01. for cylinder

$$r = \frac{21}{2}, h = 5 \text{ m}$$

$$\begin{aligned} \text{T.S.A} &= 2\pi r (r + h) \\ &= 2 \times \frac{22}{7} \times \frac{21}{2} \left(\frac{21}{2} + 5 \right) \\ &= 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{121}{2} \end{aligned}$$

total cost per 10 per m²

$$= 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{121}{2} \times 10 = \text{Rs } 399.30$$

(Approx)

Ans: B

02 C.S.A. of cylinder = $2\pi rh$

C.S.A of cone = πrl

$$\frac{2\pi rh}{\pi rl} = \frac{6}{5}$$

$$\Rightarrow \frac{h}{\sqrt{r^2 + h^2}} = \frac{3}{5}$$

$$\Rightarrow \frac{h^2}{r^2 + h^2} = \frac{9}{25}$$

$$25h^2 = 9r^2 + 9h^2$$

$$16h^2 = 9r^2$$

$$\Rightarrow \frac{h^2}{r^2} = \frac{9}{16}$$

$$\Rightarrow h:r = 3:4$$

Q3 let Initial edge = a | final edge = a' (2)
 Initial Surface Area = A | final s. A = A'

Given $a' = a + 50\% \text{ of } a$
 $= a + \frac{a}{2} = \frac{3a}{2}$

$A = 6a^2 =$

$A' = 6\left(\frac{3a}{2}\right)^2 = \frac{9}{4}(6a^2) = \frac{9}{4} \cdot A$

percentage Increase = $\frac{A' - A}{A} \times 100$

$= \frac{\frac{9}{4}A - A}{A} \times 100$

$= \frac{5}{4} \times 100$

$= 125\%$

Ans: A

Q4 Area of hall = $\frac{\text{Total cost of decoration}}{\text{cost of decoration per sq. mt}}$

$= \frac{4050}{4.50} = 900 \text{ sq. mt}$

$l : b = 3 : 2 \Rightarrow l = 3x, b = 2x$

$2(lb + bh + hl) = 900$
 $2(3x \cdot 2x + 2x \cdot (4.5) + (4.5) \cdot 3x)$

L.S.A of hall = $2h(l+b)$

$900 = 2 \times 4.5 \times (3x + 2x)$

$\Rightarrow x = 20$

$\therefore l = 60 \text{ mt}, b = 40 \text{ mt}$

Ans: C

05 C.S.A of cone $= A = \pi r l$ (3)

$l = p\%$ increased

New slant height $L = l + p\%$ of l

$$= l + \frac{p}{100} l$$

$$L = l \left(\frac{100+p}{100} \right)$$

$$\left[\text{S.A. increase } (\%) = \frac{l \left(\frac{100+p}{100} \right) - \pi r l}{\pi r l} \right]$$

New S.A $= \pi r L$

$$= \pi r \cdot \left(\frac{l(100+p)}{100} \right)$$

$$\therefore \% \text{ increase in C.S.A} = \frac{\text{Final} - \text{original}}{\text{original}} \times 100$$

$$= \frac{\pi r \left(\frac{l(100+p)}{100} \right) - \pi r l}{\pi r l} \times 100$$

Ans: A

$$= p\%$$

06. Surface Area of sphere $= 4\pi r^2$

$r \rightarrow p\%$ of r

$$r \rightarrow \frac{p}{100} \times r$$

$$\text{New radius} = r + \frac{p}{100} r = r \left(\frac{100+p}{100} \right)$$

$$\text{New S.A} = 4\pi \left(r \left(\frac{100+p}{100} \right) \right)^2$$

$$\% \text{ change in S.A} = \frac{\text{Final} - \text{original}}{\text{original}} \times 100$$

$$= \frac{4\pi \left[r \left(\frac{100+p}{100} \right) \right]^2 - 4\pi r^2}{4\pi r^2} \times 100 = \left(2p + \frac{p^2}{100} \right) \%$$

Ans: C



07. $l = 5x$, $b = 5x$, $h = 4x$ (4)

$$2(lb + bh + hl) = 3300$$

$$\Rightarrow 2(5x \cdot 5x + 5x \cdot 4x + 4x \cdot 5x) = 3300$$

$$\Rightarrow x = 15$$

$$\therefore l = 90, b = 75, h = 60$$

Ans: D

08 $R = \frac{25}{2} \text{ cm}$, $r = \frac{24}{2} \text{ cm}$

T.S.A of hollow hemispherical vessel

$$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$$

$$= 2 \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2 + 2 \times \frac{22}{7} \times (12)^2 + \frac{22}{7} \left(\left(\frac{25}{2}\right)^2 - (12)^2\right)$$

$$= 1925.78 \text{ cm}^2$$

$$\text{Total cost of painting} = 1925.78 \times 0.05$$

$$= \text{Rs } 96.29$$

Ans: D

09 slant height of cone

$$l^2 = h^2 + r^2 = 144 + 25 = 169$$

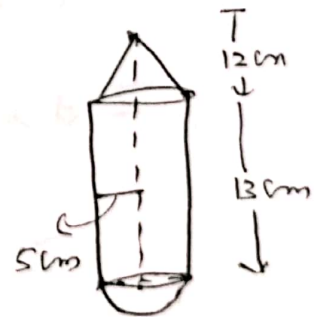
$$\therefore l = 13 \text{ cm}$$

S.A. of the toy

$$= 2\pi rh + \pi r l + 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times 5 \times 13 + \frac{22}{7} \times 5 \times 13 + 2 \times \frac{22}{7} \times 5^2$$

$$= 770 \text{ cm}^2$$



Ans: A

10 Given $l = 6\text{m}$, $b = 5\text{m}$, $h = 3.5\text{m}$ (5)

door $\rightarrow l = 1.2\text{mt}$, $b = 2\text{m}$

window $\rightarrow l = 1\text{m}$, $b = 1.9\text{mt}$

C.S.A of cuboid $= 2h(l+b)$

Required Area

$$= 2h(l+b) - 2 \times \text{doors} - 3 \times \text{windows}$$

$$= 2 \times 3.5(6+5) - 2 \times 1.2 \times 2 - 3 \times 1 \times 1.9$$

$$= 7 \times 11 - 4.8 - 5.7$$

$$= 77 - 10.5$$

$$= 66.5 \text{ sq.mt.}$$

Given wall paper $l = 70\text{cm} = 0.7\text{mt}$

$$l = \cancel{2.04} \text{mt.}$$

Cost per $1\text{mt}^2 \rightarrow \text{Rs. } 6.50$.

$$\text{Total paper required} = \frac{66.5}{1 \times 0.7} \text{ mt}$$

$$\text{Cost of the paper} = \frac{66.5}{0.7} \times 6.5$$

$$= \text{Rs } 617.5$$

Ans: A

1

$$l = 4x, b = 3x, h = 5.5 \text{ m} \quad (6)$$

$$\text{Cost of decorating} = \text{Rs } 5082$$

$$2h(l+b) =$$

$$\text{Cost per m}^2 \times \text{Area} = 5082$$

$$\Rightarrow 6.60 \times \text{Area} = 5082$$

$$\Rightarrow \text{Area} = 770$$

$$\therefore 2h(l+b) = 770$$

$$\Rightarrow 2 \times 5.5 (4x+3x) = 770$$

$$\Rightarrow x = 10$$

$$\therefore l = 40, b = 30$$

Ans: A, D

$$2 \text{ Given } a = 4 \text{ cm}$$

$$\text{Volume of } 4 \text{ cm cube} = a^3 = 4^3 = 64.$$

$$\text{Volume of cube having } 1 \text{ cm edge}$$

$$= \frac{64}{(2)^3} = 1^3 = 1$$

$$\text{No. of Small cubes} = \frac{V_1}{V_2} = \frac{64}{1} = 64.$$

$$\begin{aligned} \text{Surface area of Larger cube} &= 6a^2 \\ &= 6 \times 4^2 \\ &= 96 \end{aligned}$$

$$\text{Surface area of Small cube} = 6a^2 = 6 \times 1^2 = 6$$

$$\text{Ratio} = 96 : 6 = 16 : 1$$

Ans: A, B, D



3. Initial diameter = d ⑦
 decreased by 25% = $d - 25\% \text{ of } d$
 $= d - \frac{25}{100} \times d$

$\therefore$ New diameter = $\frac{3d}{4}$, $\therefore$ radius = $\frac{3d}{8}$

% of decreased C.S.A of ~~Initial~~ Sphere

$$= \frac{\text{Final} - \text{original}}{\text{original}} \times 100$$

$$= \frac{4\pi\left(\frac{3d}{8}\right)^2 - 4\pi\left(\frac{d}{2}\right)^2}{4\pi\left(\frac{d}{2}\right)^2} \times 100$$

$$= 43.75\% \text{ (True)}$$

Statement II: C.S.A increases when

diameter increases (False)

Ans: C

4 Statement I: $l = 25 \text{ cm}$
 $b = 3.5 \text{ cm}$

$$\begin{aligned} \text{C.S.A} &= 2\pi r h \\ &= 2 \times \frac{22}{7} \times 25 \times 3.5 \\ &= 550 \text{ cm}^2 \text{ (True)} \end{aligned}$$



Statement II: conceptual (True) Ans: A

7

for a cube.

(8)

original edge $\rightarrow a$ final edge $= a + 50\% \text{ of } a$

$$= a + \frac{50}{100} \times a$$

$$= \frac{3a}{2}$$

$\therefore$ Increase in surface area

$$= \frac{\text{Final} - \text{original}}{\text{original}} \times 100$$

$$= \frac{6\left(\frac{3a}{2}\right)^2 - 6a^2}{6a^2} \times 100$$

Ans: A

$$= 125\%$$

8

$$6a^2 = 150 \Rightarrow a^2 = 25 \Rightarrow a = 5$$

$$\text{length of diagonal} = \sqrt{3}a = \sqrt{3} \times 5 = 5\sqrt{3}$$

Ans: B

19

Surface area of spheres $= 616$

$$\Rightarrow 4\pi r^2 = 616$$

$$\Rightarrow r = 7$$

Ans: C

$$\therefore \text{diameter} = 14 \text{ cm}$$

10

$$4\pi r^2 = 616 \Rightarrow r = 7$$

$$\text{New area} = 616 - 75\% \text{ of } 616$$

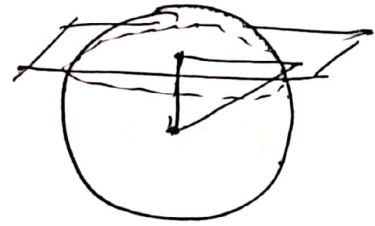
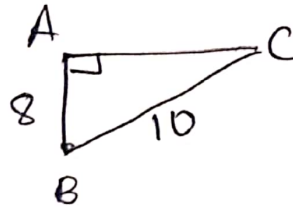
$$= 616 - 462$$

$$= 154 \text{ cm}^2$$

⑨

$$4\pi r^2 = 154 \Rightarrow r = 3.5 \text{ cm} \quad \text{Ans: C}$$

11



$$\therefore AC^2 + AB^2 = BC^2$$

$$\Rightarrow AC^2 + 64 = 100$$

$$\Rightarrow AC^2 = 36 \Rightarrow AC = 6 \quad \text{Ans: 6}$$

12



$$\text{T.S.A of each hemisphere} = 3\pi r^2$$

$$\text{Total T.S.A. of two hemispheres} = 6\pi r^2$$

$$\text{Rate of painting} = 6\pi r^2 \times 8$$

$$= 48\pi r^2 = a\pi r^2$$

$$\Rightarrow a = 48$$

Ans: 48

LEARNERS TASK

CUG'S

01.	Conceptual	Ans: D
02.	Conceptual	Ans: C
03.	Conceptual	Ans: D
04.	Conceptual	Ans: B
05.	Conceptual	Ans: A
06.	Conceptual	Ans: D
07.	Conceptual	Ans: D
08.	Conceptual	Ans: D
09.	Conceptual	Ans: D
10.	Conceptual	Ans: B

JEEMAINS LEVEL

01. $d = 7 \text{ m}$, $r = \frac{7}{2} \text{ m}$

$$S.A = 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 154 \text{ sq.m}$$

Ans: A



02

$$l:b:h = 2:3:4$$

$$\Rightarrow l = 2x, b = 3x, h = 4x$$

$$\begin{aligned} \text{C.S.A} &= 2(lb + bh + hl) \\ &= 2(2x \cdot 3x + 3x \cdot 4x + 4x \cdot 2x) \\ &= 52x^2 \text{ Sq.mt} \end{aligned}$$

$$\begin{aligned} \text{Total cost} &= \text{Area} \times \text{cost per 1 Sq.mt} \\ &= 52x^2 \times (4.50) = 52x^2 \cdot (4) \\ &= 26x^2 = 416 \end{aligned}$$

$$\Rightarrow x = 4.$$

$\therefore$ The dimensions are 8, 12, 16

Ans: C

03

$$2\pi r = 17.6$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 17.6$$

$$\Rightarrow r = 2.8 \text{ mt}$$

$$\text{C.S.A} = 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times (2.8)^2$$

$$= 49.28$$

Cost of painting $\rightarrow$ per $100 \text{ cm}^2 \rightarrow \text{Rs } 5.$

$$\begin{aligned} \text{Now C.S.A} &= 49.28 \text{ mt}^2 \\ &= 49.28 \times 100 \times 100 \text{ cm}^2 \\ &= 492800 \text{ cm}^2 \end{aligned}$$

$$\text{Cost of painting} = \frac{5}{100} \times 492800$$

$$= \text{Rs } 24,640.$$

Ans: D

04

$$\text{original S.A} = 6a^2$$

$$\text{New S.A} = 6(2a)^2 = 24a^2$$

$$\% \text{ increase} = \frac{\text{New} - \text{original}}{\text{original}} \times 100$$

$$= \frac{18a^2}{6a^2} \times 100 = 300\%$$

Ans: D



05 for cone

$$r = 7, h = 24$$

$$l^2 = h^2 + r^2$$

$$= 24^2 + 7^2$$

$$l^2 = 625 \Rightarrow l = 25$$

$$C.S.A = \pi r l$$

$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

$$\text{Now Area} = l \times b$$
$$= l \times 5$$

$$\therefore 5l = 550$$

$$\Rightarrow l = 110 \text{ m}$$

Ans: B

06 $l = 20, b = 16, h = 12$

$$\text{length of the rod} = \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{20^2 + 16^2 + 12^2}$$

$$= \sqrt{400 + 256 + 144}$$

$$= \sqrt{800}$$

$$= 10\sqrt{8} = 10 \times 2\sqrt{2}$$

$$= 20 \times 1.414$$

$$= 28.28$$

Ans: D

07 diagonal of floor = $\sqrt{l^2 + b^2}$

$$\therefore \sqrt{l^2 + b^2} = 10 \Rightarrow l^2 + b^2 = 100 \rightarrow \text{①}$$

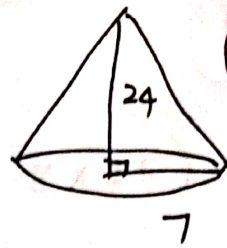
$$\text{diagonal} = \sqrt{l^2 + b^2 + h^2} = 10\sqrt{2}$$

$$\Rightarrow l^2 + b^2 + h^2 = 200$$

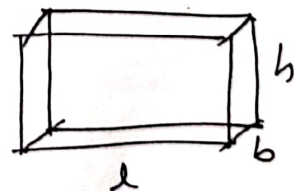
$$\Rightarrow 100 + h^2 = 200$$

$$\Rightarrow h = 10 \text{ m}$$

Ans: D



(14)



08 $l = 25\text{cm}, b = 15\text{cm}, h = 8\text{cm}$

(15)

$$T.S.A = 2(lb + bh + hl)$$

$$= 2(25 \times 15 + 15 \times 8 + 8 \times 25)$$

$$= 1390\text{ cm}^2$$

Ans: C

10 $r = 14\text{cm}$

$$C.S.A = 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times 14 \times 14$$

$$= 1232\text{ sq. m}$$

$$\text{Total cost} = 1232 \times 3$$

$$= \text{Rs } 3696$$

Ans: D



1 $r = 14\text{m}, d = 7\text{cm}$

$$\therefore R = r + d = 14 + 7 = 21\text{cm.}$$



$$R = r + d$$

T.S.A. of hollow hemisphere

$$= 2\pi r^2 + 2\pi R^2 + \pi R^2 - \pi r^2$$

$$= \pi r^2 + 3\pi R^2$$

$$= \frac{22}{7} \times 14^2 + 3 \times \frac{22}{7} \times (21)^2$$

$$= 4774$$

Ans: A

2. $l + b + h = 5\sqrt{3}$ (16)

$$\sqrt{l^2 + b^2 + h^2} = 3\sqrt{5}$$

$$\Rightarrow l^2 + b^2 + h^2 = 45$$

$$(l + b + h)^2 - 2(lb + bh + hl) = 45$$

$$\Rightarrow lb + bh + hl = 15$$

$$\therefore T.S.A = 2(lb + bh + hl)$$

$$= 2 \times 15$$

$$= 30 \text{ sq. cm}$$

Ans A

3 Statement I: $r = 15 \text{ m}$

$$d = 7 \text{ m}$$

$$\therefore R = r + d$$

$$= 15 + 7$$

$$= 22 \text{ m}$$

$$\text{Area of path} = \pi R^2 - \pi r^2$$

$$= \frac{22}{7} \times 22^2 - \frac{22}{7} \times 15^2$$

$$= \frac{22}{7} (22 + 15) (22 - 15)$$

$$= \frac{22}{7} \times 37 \times 7$$

$$= \frac{814}{1} \cdot 814$$

$$\text{Total cost} = \frac{814}{1} \times 10$$

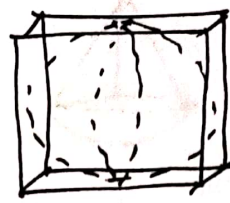
$$= \text{Rs. } 8140 \text{ (True)}$$

Statement II: Conceptual (True)

Ans A

4 Statement I:

$$r = 3.5 = \frac{7}{2} \text{ cm}$$



(17)

$$\begin{aligned} \text{C.S.A of sphere} &= 4\pi r^2 \\ &= 4 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \end{aligned}$$

$$\begin{aligned} \text{C.S.A of Cub} &= 6a^2 \\ &= 6 \times (7)^2 \end{aligned} \quad \begin{aligned} \text{Since } a &= 2r \\ &= 2 \times \frac{7}{2} \end{aligned}$$

$$\begin{aligned} \text{Difference in S.A} &= 6 \times 7^2 - 4 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = 7 \\ &= 140 \text{ cm}^2 \end{aligned}$$

Statement II: Conceptual (True)

Ans. A

7 $l = 3 \text{ mt}, b = 2 \text{ mt}, h = 1 \text{ mt}$

$$\begin{aligned} \text{T.S.A} &= 2(lb + bh + hl) \\ &= 2(3 \times 2 + 2 \times 1 + 1 \times 3) \\ &= 22 \text{ mt}^2 \end{aligned}$$

$$\begin{aligned} \text{Total cost} &= \text{Rs } 22 \times 10 \\ &= \text{Rs } 220. \end{aligned}$$

Ans. B

8

S.A. of Remaining box = T.S.A — Area of Side removed

$$= 2(lb + bh + hl) - 2 \times b$$

$$\begin{aligned} &= 2b + 2bh + 2hl \\ &= 3 \times 2 + 2 \times 2 \times 1 + 2 \times 1 \times 3 \\ &= 6 + 4 + 6 = \end{aligned}$$

$$= 22 - 2 \times 1$$

$$= 20$$

Ans. C



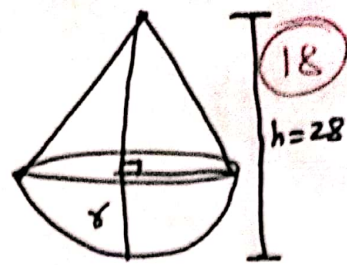
9

for cone hemisphere

$$r = 14 \text{ m}$$

for cone

$$r = 14 \text{ m}, h = 28 - 14 = 14 \text{ m}$$



$$\begin{aligned} \text{Ans. } l^2 &= h^2 + r^2 & \therefore l &= \sqrt{2 \times 14^2} \\ &= 14^2 + 14^2 & &= 14\sqrt{2} \\ &= 2 \times 14^2 & & \end{aligned}$$

Area of Canvas required = ~~Area~~ C.S.A.

$$\begin{aligned} \text{of cone} &= \pi r l \\ &= \frac{22}{7} \times 14 \times 14\sqrt{2} \\ &= 616\sqrt{2}. \end{aligned}$$

Ans: D

10.

Area of metal sheet required

= C.S.A. of hemisphere

$$\begin{aligned} &= 2\pi r^2 \\ &= 2 \times \frac{22}{7} \times 14^2 \\ &= 1232 \end{aligned}$$

Ans: D

11

for cube

$$a = 4$$

for cuboid

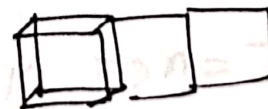
$$l = 12, b = 4, h = 4$$

$$\begin{aligned} \text{T.S.A} &= 2(lb + bh + hl) \\ &= 2(12 \times 4 + 4 \times 4 + 12 \times 4) = 224. \end{aligned}$$

$$\begin{aligned} \text{T.S.A of 3 cubes} &= 3 \times 6a^2 = 18 \times 4^2 \\ &= 288 \end{aligned}$$

$$\therefore \text{Ratio} = \frac{288}{224} = \frac{9}{7} = 7:9$$

Ans: 7



$$a = 4$$



12.

for cylinder

(19)

$$h = 8 \text{ cm}$$

$$C.S.A = 2\pi r h = 704$$

$$2 \times \frac{22}{7} \times r \times 8 = 704$$

$$r = 14$$

$$\text{Sum of the digits} = 1 + 4 = 5$$

Ans: 5

Teaching Task (Surface Areas & Volumes) (JEE Advanced)

05

Assertion: Conceptual (False)

Reason: Conceptual (True)

Ans: D

06

Assertion: $S_1 = 6a^2$

$$S_2 = 6(2a)^2 = 24a^2 = 4(6a^2) = 4S_1 \text{ (True)}$$

Reason: $S_1 = 6a^2 \Rightarrow S_1 \propto a^2$ (True) Ans: A

13

a) Conceptual (P)

c) Conceptual (r)

b) Conceptual (Q)

d) Conceptual (S)

Ans: P, Q, r, S

14

a) $V = 10 \times 5 \times 2 = 100 \text{ cm}^3$

b) $V = 12 \times 8 \times 6 = 576 \text{ cm}^3$ (r)

c) $V = 25 \times 2 \times 1 = 50 \text{ cm}^3$

d) $V = 20 \times 5 \times 5 = 500 \text{ cm}^3$ (P, S)

LEARNERS TASK (JEE Mains)

09

Cylinder $\rightarrow r = 2 \text{ m}, h = 7 \text{ m}$

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 7 = 88 \text{ m}^3$$

Ans: D

JEE ADVANCED

05

Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans: A

06

Assertion: Conceptual (False)

sphere has smaller surface area when compared to cube

Reason: Conceptual (True)

Ans: D

13 a) Conceptual (p) | c) Conceptual (r)
b) Conceptual (q) | d) Conceptual (s) Ans. p, q, r, s

14. a) $V = \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 = 462 \text{ cm}^3 (p)$
b) $V = \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 20 = 3080 \text{ cm}^3 (q)$
c) $V = \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 10 = 125.6 \text{ cm}^3 (r)$
d) $V = \pi r^2 h = \frac{22}{7} \times 35 \times 35 \times 100 = 38500 \text{ cm}^3 (s)$
Ans. p, q, r, s

$\Rightarrow$ THE END $\in$