4. QUADRATIC EXPRESSIONS-II

(1)

Class: IX, MATHEMATICS

FOUNDATION: SOLUTION

2025-26

JEE MAINS
TEACHING TASK

01. $-5x^2+2x+3$

ax^2+bx+c

$\therefore a=-5<0$

The expression will have "maximum" value

$$\text{Max. value} = \frac{4ac-b^2}{4a}$$

$$= \frac{4(-5)\cdot 3-(2)^2}{4(-5)} = \frac{16}{5}$$

Ans: A

02. $$\text{max. of } \frac{1}{4x^2+2x+1} = \frac{1}{\text{min. } 4x^2+2x+1}$$

Now, min of $4x^2+2x+1$

$$\text{min. value} = \frac{4ac-b^2}{4a} = \frac{4.4.1-2^2}{4.4} = \frac{3}{4}$$

$$\therefore \text{Ans} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

Ans.

03. $x^2+3|x|+2=0$

$\Rightarrow x^2+2=-3|x|$

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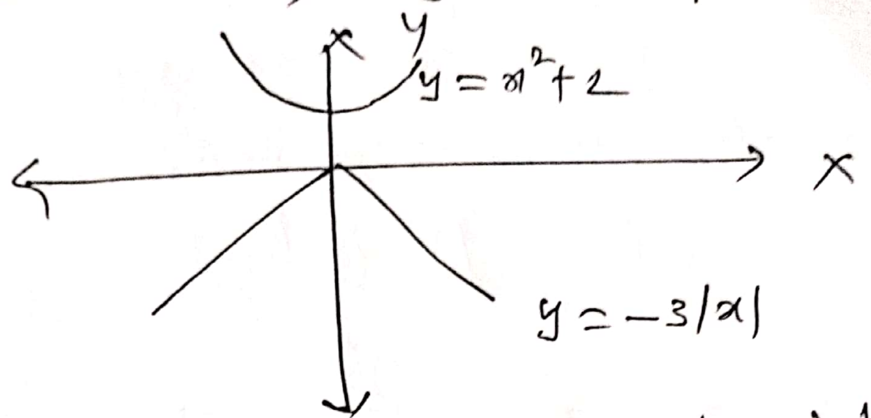
$y = x^2 + 2$; $y = -3|x|$



These two curves do not intersect each other, hence there are no real roots

Ans. B

04 $x^2 - 2|x| = 0 \Rightarrow \cancel{x^2 = 2|x|}$.

$\Rightarrow |x|^2 - 2|x| = 0$

$\Rightarrow |x|(|x| - 2) = 0$

$\Rightarrow |x| = 0$ or $|x| = 2$

$\Rightarrow x = 0$ or $x = \pm 2$

$\therefore$ Three solutions

Ans : D

05 $y = -x^2 + 2x + 3$

$ax^2 + bx + c$

Since $a = -1 < 0$, curve is open downwards

and $b^2 - 4ac = (2)^2 - 4(-1).3$

$= 16 > 0$.

It has two real roots.

The parabola intersects the x-axis a in two distinct points

Ans : D

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3

06 $y = x^2 + 2x + 3$

$ax^2 + bx + C$

max. value occurs at $x = -\frac{b}{2a}$

$= -\frac{2}{2 \cdot 1} = -1$

Ans. B

07. a, b are the roots of $x^2 + ax + b = 0$

$\therefore a + b = -a$

$\Rightarrow 1 + b = -1$

$\Rightarrow \boxed{b = -2}$

$a \cdot b = b$

$\boxed{a = 1}$

Least value of $x^2 + ax + b$

$= \frac{4 \cdot 1 \cdot b - a^2}{4 \cdot 1} = \frac{4(-2) - 1^2}{4}$

$= \frac{-9}{4}$

Ans: B

08 $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a-2)^2 + 2(a+1)$

$= a^2 - 2a + 6$

$= (a-1)^2 + 5$

We know $(a-1)^2 \geq 0$

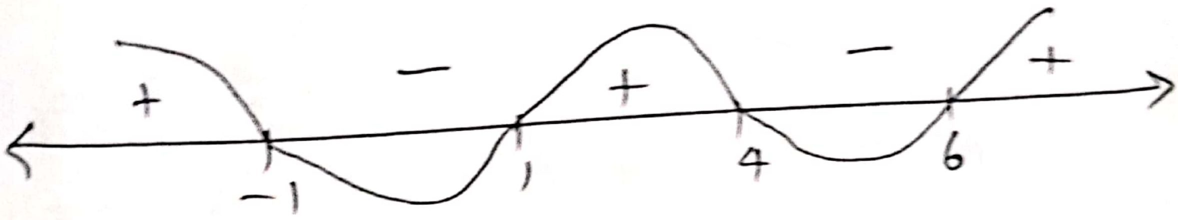
$\Rightarrow (a-1)^2 + 5 \geq 5$

$\therefore$ Least value $= 5$

Ans: C

09. $\dfrac{14x}{x+1} - \dfrac{9x-30}{x-4} < 0$ (4)

$\Rightarrow \dfrac{(x-1)(x-6)}{(x+1)(x-4)} < 0$



$x \in (-1,1) \cup (4,6)$

Ans: D

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10 $\min\{f(x)\} > \max.\{g(x)\}$

$\dfrac{4\cdot 1\cdot 2c^2 - 4b^2}{4\cdot 1} > \dfrac{4\cdot(-1)\cdot b^2 - 4c^2}{4(-1)}$

$\Rightarrow 2c^2 - b^2 > b^2 + c^2$

$\Rightarrow c^2 > 2b^2$

$\Rightarrow |c| > |b|\sqrt{2}$

Ans: B

JEE ADVANCED LEVEL

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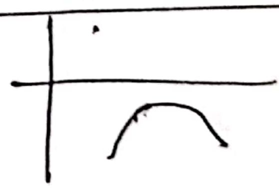
01. $y = -x^2 + 3x + 4$



(i) It is a parabola

(ii) since $a = -1 < 0$, it is open downwards

It attain maximum value

Ans: A, B

02 $(3+2\sqrt{2})^{x^2-4}+(3-2\sqrt{2})^{x^2-4}=6$ (5)

clearly, $(3+2\sqrt{2})+(3-2\sqrt{2})=6$

and $(3+2\sqrt{2}).(3-2\sqrt{2})=9-8=1$

$\therefore x^2-4=\pm 1$

$\Rightarrow x^2-4=1$ or $x^2-4=-1$

$\Rightarrow x^2=5$ or $x^2=3$

$\Rightarrow x=\pm\sqrt{5}$ or $x=\pm\sqrt{3}$

Ans: A, B, C, D

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03 Statement I:

max. value $=\frac{4(-1).2-3^2}{4(-1)}=\frac{17}{4}$ (True)

Statement II: Conceptual (True)

Ans: A

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04 Statement I:

$(\log_5 x)^2+\log_5 x<2$

let $\log_5 x=a$

$\Rightarrow a^2+a-2<0$

$\Rightarrow (a+2)(a-1)<0$

$\Rightarrow -2<a<1$

$\Rightarrow -2<\log_5 x<1$

$\Rightarrow \frac{1}{25}<x<5 \Rightarrow x\in(\frac{1}{25},5)$ (True)

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Statement II: Conceptual (True) Ans. A

05 Assertion: $y = -x^2 + 4x$ 6

$$\text{Max. value} = \frac{4.(-1).0 - 4^2}{4.(-1)} = 4$$

$\therefore$ Range $= (-\infty, 4]$ (True)

Reason: Conceptual (True) Ans A

06 Assertion: $x^2 + 4x + 4 < 0$

$\Rightarrow (x+2)^2 < 0$ (which is False)

Reason: $(x+2)^2 = 0$

$\Rightarrow x = -2$ (True) Ans: D

$\boxed{07}$ $y = \dfrac{x^2 - 6x + 5}{x^2 + 2x - 1}$

$\Rightarrow yx^2 + 2yx - y - x^2 + 6x - 5 = 0$

$\Rightarrow (y-1)x^2 + (2y+6)x - (y+5) = 0$

$\Delta \geq 0$

$\Rightarrow (2y+6)^2 + 4(y-1)(y+5) \geq 0$

$\Rightarrow y^2 + 5y + 2 \geq 0$

$\Rightarrow y^2 + 2.\frac{5}{2}y + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 2 \geq 0$

$\Rightarrow \left(y + \frac{5}{2}\right)^2 - \frac{17}{4} \geq 0$

$\Rightarrow y \in \left(-\infty, \frac{-5-\sqrt{17}}{2}\right) \cup \left(\frac{-5+\sqrt{17}}{2}, \infty\right)$

There ~~won't~~ won't be any least value $\boxed{\text{Ans:}}$

08 $\frac{A}{B} = \frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}} = \frac{\left(x-\frac{1}{x}\right)^2+2}{x-\frac{1}{x}}$ 7

$\Rightarrow \left(x-\frac{1}{x}\right) + \frac{2}{\left(x-\frac{1}{x}\right)}$

We know, $A \cdot M \geq G.M.$ $\frac{a+b}{2} \geq \sqrt{ab}$

$\Rightarrow \frac{\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}}{2} \geq \sqrt{\left(x-\frac{1}{x}\right)\cdot\frac{2}{\left(x-\frac{1}{x}\right)}}$

$\Rightarrow \left(x-\frac{1}{x}\right) + \frac{2}{x-\frac{1}{x}} \geq 2\sqrt{2}$

$\therefore$ least value $= 2\sqrt{2}$ Ans. D

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09 $2^{x^2-5x+8} + 3^{x^2-5x+8} = 13$

clearly, $2^2+3^2 = 13$

$\therefore x^2-5x+8 = 2$

$\Rightarrow x^2-5x+6=0$

$\Rightarrow (x-2)(x-3)=0$

$\Rightarrow x = 2, 3$ Ans. A

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10 $(2\sqrt{2})^{x^2-4x+6} + (3\sqrt{2})^{x^2-4x+6} = 26$

clearly, $(2\sqrt{2})^2+(3\sqrt{2})^2 = 26$

$\therefore x^2-4x+6 = 2$

$\Rightarrow x^2-4x+4=0$

$\Rightarrow x = 2, 2$

Ans: D

11. $(x-19)(x-97)-p=0$

$x^2-(19+97)x+19.97-p=0 \longrightarrow (1)$

$\alpha+\beta=19+97, \quad \alpha.\beta=19.97-p$

Now, given $(x-\alpha)(x-\beta)+p=0$

$\Rightarrow x^2-(\alpha+\beta)x+\alpha\beta+p=0$

$\Rightarrow x^2-(\alpha+\beta)x+19.97-p+p=0$

$\Rightarrow x^2-(\alpha+\beta)x+19.97=0$

$\Rightarrow x^2-(19+97)x+19.97=0$

$\Rightarrow (x-19)(x-97)=0$

$\Rightarrow x=19$ or 97

Ans. 19

minimum root $=19$

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12 $x=\dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}\times\dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}=\dfrac{(\sqrt{7}+\sqrt{3})^2}{7-3}=\dfrac{5+\sqrt{21}}{2}$

$y=\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}\times\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}=\dfrac{5-\sqrt{21}}{2}$

$x^4+y^4+(x+y)^4$

$=\left(\dfrac{5+\sqrt{21}}{2}\right)^4+\left(\dfrac{5+\sqrt{21}}{2}\right)^4+\left(\dfrac{5+\sqrt{21}}{2}+\dfrac{5-\sqrt{21}}{2}\right)^4$

$=1152$

Ans. 1152

13(a)

13 a) $y = x^2+3x+4 \Rightarrow a=1>0$, open upward parabola

b) $y = -x^2+x+2 \Rightarrow a=-1<0$, open downward parabola

c) $y = x^2+5x+6$

$$\text{min. value} = \frac{4\cdot1\cdot6-5^2}{4\cdot1} = -\frac{1}{4}$$

d) $y = -x^2-5x+6$

$$\text{max. value} = \frac{4(-1)6-(-5)^2}{4(-1)} = \frac{49}{4}$$

Ans: t, s, r, q

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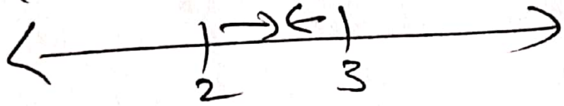
14 a) $x^2-5x+6<0$

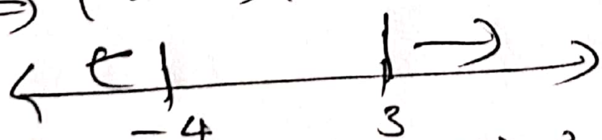
$\Rightarrow (x-2)(x-3)<0$

$\Rightarrow$ 

$2<x<3$

b) $x^2+x-12>0$

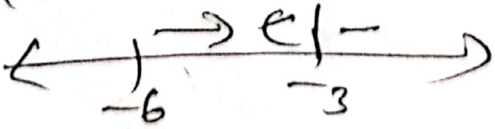
$\Rightarrow (x+4)(x-3)>0$



$x<-4$ or $x>3$

c) $x^2+9x+18<0$

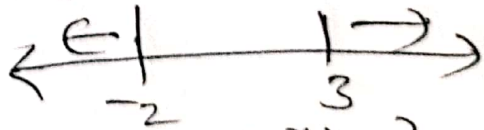
$(x+3)(x+6)<0$



$-6<x<-3$

d) $x^2-x-6>0$

$\Rightarrow (x-3)(x+2)>0$



$x<-2$ or $x>3$

Ans: p, r, t, q

LEARNER'S TASK

CUQ'S

01. Conceptual — Ans: B

02 Conceptual — Ans: B

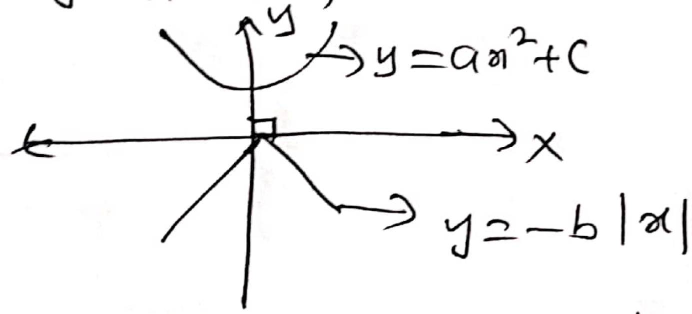
03 Conceptual — Ans: A

04 Conceptual — Ans: A

05 $ax^2 + b|x| + c = 0$

$\Rightarrow ax^2 + c = -b|x|$

$\Rightarrow y = ax^2 + c, \quad y = -b|x|$



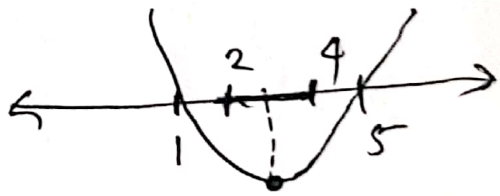
Both the curves do not intersect eachother,

Hence, there are no real roots — Ans: C

06 Conceptual — Ans. C

07 $x^2 - 6x + 5$

$= (x-1)(x-5)$



min. value $= \dfrac{4ac - b^2}{4a} = -4$

At $x = -\dfrac{b}{2a} = \dfrac{-(-6)}{2\cdot 1} = \dfrac{6}{2} = 3$

$\therefore$ At $x = 3$, the minimum value $= -4$

Ans: A

08	Conceptual	Ans: C
09.	Conceptual	Ans: D
10.	Conceptual	Ans: A

JEE MAINS LEVEL

01. $y = (x-2)(x-3) = x^2 - 5x + 6 \rightarrow$ parabola

Ans. B

02. $y = x^2 - 2x + 3$

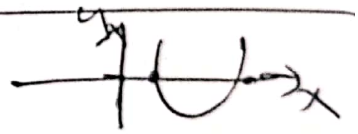
min. value $= \dfrac{4.1.3 - (-2)^2}{4.1} = \dfrac{12-4}{4} = 2$

accurs at $x = \dfrac{-b}{2a} = \dfrac{-(-2)}{2.1} = 1$ Ans. C

03. $y = -x^2 + 2x + 3$

max. value $= \dfrac{4(-1).3 - 2^2}{4(-1)} = 4$ Ans. C

04. $y = x^2 - 4x + 3$; $a = 1 > 0$



open upward parabola

$\Delta = (-4)^2 - 4.1.3 = 16 - 12 = 4 > 0$

The curve intersects X-axis at two distinct points

Ans. B

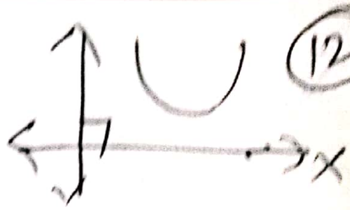
05. $y = -x^2 + 3x + 4$

$a = -1 < 0$, open downward parabola

$\Delta = (3)^2 - 4.(-1).4 = 9 + 16 = 25 > 0$

Ans. A

06 strictly above the x-axis



$\Leftrightarrow$ No-real roots

$\Leftrightarrow$ $\Delta < 0$

$\Leftrightarrow$ $[8(a+5)]^2 - 4 \cdot 16 \cdot (-7a-5) < 0$

$\Leftrightarrow$ $a^2 + 17a + 30 < 0$

$\Leftrightarrow$ $(a+2)(a+15) < 0$

$\Leftrightarrow$ $-15 < a < -2$

Ans. B

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07 $|x|^2 - 3|x| + 2 = 0$

$\Rightarrow (|x| - 2)(|x| - 1) = 0$

$\Rightarrow |x| = 2$ or $|x| = 1$

$\Rightarrow x = \pm 2$ or $x = \pm 1$

Ans. C

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08 $-x^2 + x + 6 > 0$

$\Rightarrow x^2 - x - 6 < 0$

$\Rightarrow (x-3)(x+2) < 0$

$\Rightarrow -2 < x < 3$

Ans. D

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09. $2^{x^2-3} = 2^1 \Rightarrow x^2 - 3 = 1 \Rightarrow x = \pm 2$ Ans A

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10. $(x-2)^2 - 2|x-2| - 15 = 0$

$\Rightarrow |x-2|^2 - 2|x-2| - 15 = 0$

Let $|x-2| = a$

$\therefore a^2 - 2a - 15 = 0$

$\Rightarrow (a+3)(a-5) = 0$

$a + 3 = 0$ or $a - 5 = 0$

$a = -3$ | $a = 5$

$|x-2| = -3$ Not possible | $|x-2| = 5$

$x - 2 = \pm 5$

$\Rightarrow x = -3$ or 7

sum $= -3 + 7 = 4$

Ans. A

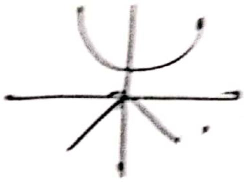
JEE ADVANCED LEVEL

01 Conceptual — Ans: A, B, C

02 Conceptual

$x^2+2|x|+5=0$

$x^2+5=-2|x|$



Ans: A, B

03 Statement I: Conceptual (True)

Statement II: Conceptual (True)

Ans. A

04 Statement I:

$(\cos p-1)x^2+\cos p\,x+\sin p=0$

$\therefore \Delta\geq 0$

$\Rightarrow \cos^2 p-4(\cos p-1)(\sin p)\geq 0$

clearly $\cos^2 p\geq 0$, $-1\leq \cos p\leq 1$

$\Rightarrow \cos p-1\leq 0$

i.e. $\cos^2 p$ is positive, $\cos p-1$ is negative

$\Rightarrow \underbrace{\cos^2 p}_{\text{positive}}-4(\underbrace{\cos p-1}_{\text{negative}})\sin p\geq 0$

$\Rightarrow \sin p\geq 0$

$\Rightarrow$ p lies in either I or II quadrants

$\Rightarrow p\in[0,\pi]$

$\Rightarrow p\in(0,\frac{\pi}{2})$ (True)

Statement II: Conceptual (False)

Ans. C

05 **Assertion:** $y = -x^2 + 6x - 5$

$$\text{max. value} = \frac{4(-1)(-5) - 6^2}{4(-1)}$$

$$= \frac{20 - 36}{-4} = \frac{-16}{-4} = 4 \text{ (True)}$$

Reason: Conceptual (True) Ans: A

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06 **Assertion:** $y = -x^2 + 4x - 3$

$a = -1 < 0$, maximum value (False)

Reason: Conceptual (True) Ans: D

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07 **NOTE:** please, Change the question Numbers here onwards.

$y = -x^2 + 3x + 4$

$$\text{max. value} = \frac{4(-1).4 - 3^2}{4.(-1)} = \frac{25}{4}$$ Ans: B

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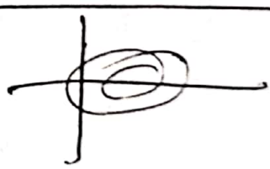
08 $y = -x^2 + x + 3$



$$\text{Turning point} = \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$$

$$= \left(\frac{1}{2}, \frac{-13}{4}\right)$$ Ans: B

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09 $-x^2 + 5x + 24 > 0$

$\Rightarrow x^2 - 5x - 24 < 0$

$\Rightarrow (x-8)(x+3) < 0$

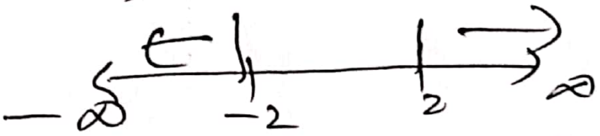
$\Rightarrow -3 < x < 8$ Ans: C

10 option Verification

option: B

$$x^2-4 \geq 0$$

$$\Rightarrow (x+2)(x-2) \geq 0$$

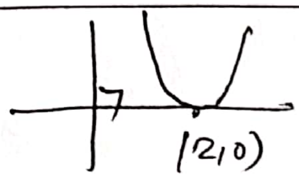


$x \leq -2$ or $x \geq 2$

Ans: B

11 $y = x^2-4x+4$

$= (x-2)^2$



$y=0 \Rightarrow x=2$

Ans: 2

12 $(x+2)(x-3)=0 \Rightarrow x^2-x-6=0$

$$\text{min. value} = \frac{4(1)(-6)-(-1)^2}{4\cdot 1} = \frac{-25}{4} = \alpha$$

$\Rightarrow 4\alpha+25=0$

Ans.. D

13 a) Never touches X-axis

b) Intersects X-axis at two distinct points

c) Touches the X-axis

d) open upward

Ans: t, s, r, q

14. a) $-3<x<4$

$(x+3)(x-4)<0$

$\Rightarrow x^2-x-12<0$

b) $x<-3,\ x>4$

$\Rightarrow (x+3)(x-4)>0$

$\Rightarrow x^2-x-12>0$

c) $-4<x<-3$

$\Rightarrow (x+4)(x+3)<0$

$\Rightarrow x^2+7x+12<0$

d) $x<-4$ or $x>-3$

$\Rightarrow (x+4)(x+3)>0$

$\Rightarrow x^2+7x+12>0$

Ans. P, Q, R, S

$\Longrightarrow$ THE END $\Longleftarrow$