

ws-7 vertically projected body

Task

①

Given, velocity of body = u

It is at the same point at $t=4s$ as well as at $t=6s$
so let $t_1=4s$ and $t_2=6s$, body is at a height h .

$$\text{so } h = ut_1 - \frac{1}{2} g t_1^2 \rightarrow (1) \quad \text{and}$$

$$h = ut_2 - \frac{1}{2} g t_2^2 \rightarrow (2)$$

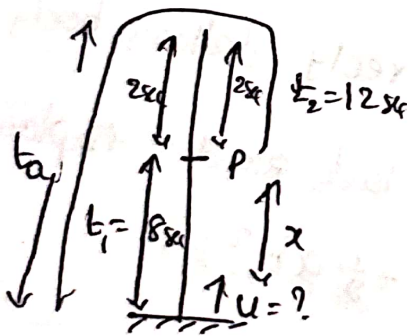
$$\text{By solving } ut_1 - \frac{1}{2} g t_1^2 = ut_2 - \frac{1}{2} g t_2^2$$

$$\Rightarrow u(t_1 - t_2) = \frac{1}{2} g (t_1^2 - t_2^2)$$

$$\Rightarrow u(t_1 - t_2) = \frac{1}{2} g (t_1 + t_2)(t_1 - t_2)$$

$$\Rightarrow u = \frac{1}{2} \times 10 (4+6) = 5 \times 10 = 50 \text{ m/s}$$

②



$$a = \frac{u}{g}$$

$$\Rightarrow u = t_a g = 10 \times 10 = 100 \text{ m/s}$$

$$\text{Hence From } s = ut + \frac{1}{2} at^2$$

$$\Rightarrow x = 100 \times 10 + \frac{1}{2} (-g)(10)^2$$

$$= 1000 - \frac{1}{2} \times 10 \times 100$$

$$\Rightarrow 500 \text{ m} \approx 480 \text{ m}$$

$$\text{For } (g = 9.8 \text{ m/s}^2)$$

Given $u = 20 \text{ m/s}$; $t = 3 \text{ sec}$; $a = -g = -10 \text{ m/s}^2$

distance travelled by the body in 3 sec

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow s = 20 \times 3 + \frac{1}{2}(-10)(3)^2$$

$$= 60 - 45 = 15 \text{ m}$$

(4)

Given distance covered in 5th sec = 2 Distance covered in 6th sec

$$\Rightarrow s_5 = 2s_6 \quad [\text{Here } a = -g]$$

$$\Rightarrow u - \frac{g}{2}(2(5)-1) = 2 \left[u - \frac{g}{2}(2(6)-1) \right]$$

$$\Rightarrow u - \frac{9g}{2} = 2u - \frac{22g}{2}$$

$$\Rightarrow 2u - u = \frac{22g}{2} - \frac{9g}{2} \Rightarrow u = \frac{13 \times g}{2} = \frac{13}{2} \times 10 = 65 \text{ m/s}$$

(5) (6)

Let h be max height reached by the body $h = \frac{u^2}{2g}$

$u=0$
 $t=1 \text{ sec}$
 $\downarrow$ FFB
 $u=0$ From $s = ut + \frac{1}{2}at^2$ $a = -g$

$$\Rightarrow (-h) = 0(1) + \frac{1}{2}(-g)(1)^2$$

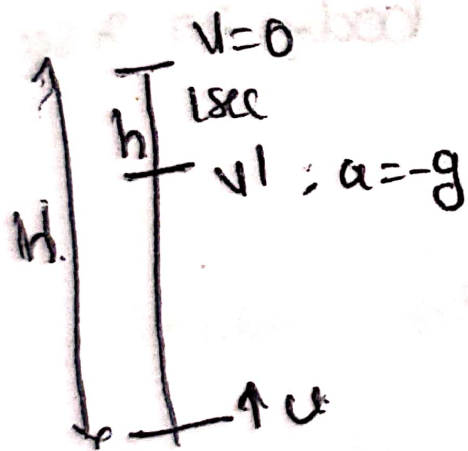
$$\Rightarrow -h = -\frac{g}{2} \Rightarrow h = \frac{g}{2} = \frac{9.8}{2} = 4.9 \text{ m}$$

The last second of vertically projected body acts as 1st sec of freely falling body

⑥. ⑤ continuation

From $H_{\max} = \frac{u^2}{2g}$

From $u^2 - v^2 = 2as$



$h =$

$\rightarrow u^2 - v^2 = 2(-g)h$

$\rightarrow -v^2 = -2g \frac{g}{2}$

$\rightarrow v = g$

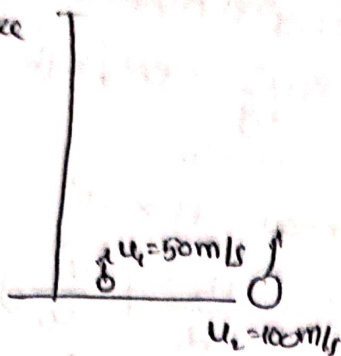
⑦

Given $u = 50 \text{ m/s}$

$\therefore u = 100 \text{ m/s}$

(7)

(2)

 $t = 2 \text{ sec}$ 

$$u_1 = u_1 - gt = 50 - 10 \times 2$$

$$= 50 - 20 = 30 \text{ m/s}$$

$$u_2 = 100 \text{ m/s}$$

$$u_{\text{rel}} = u_2 - u_1 = 100 - 30 = 70 \text{ m/s}$$

in 2nd sec distance travelled by 1st body

$$H = ut + \frac{1}{2} at^2$$

$$= 50(2) - \frac{1}{2} \times 10(2)^2$$

$$= 100 - 20$$

$$= 80 \text{ m}$$

t' be the time of meeting

$$t' = \frac{H}{u_{\text{rel}}} = \frac{80}{70}$$

$$= \frac{8}{7}$$

$$\text{Total time taken to meet} = 2 + t' = 2 + \frac{8}{7} = \frac{22}{7} \text{ sec}$$

(8)

Given $u = 4.9 \text{ m/s}$

They are asking time taken to reach the ground

$$T = \frac{2u}{g} = 2 \times \frac{4.9}{9.8} = 1 \text{ sec}$$

(9)

After 6th sec the stone is at maximum height. in 7th sec the stone act as FFB.

$\therefore$ distance travelled by the vertically projected body in last sec = distance travelled by it in 7th or distance travelled by it in 1st sec which act as FFB for return journey $\Rightarrow \frac{g}{2}$. $\therefore \frac{S_1}{S_7} = \frac{\frac{g}{2}}{\frac{g}{2}} = \frac{1}{1} = 1:1$

(10)

let u be the velocity of Projection

$$H_{\max} = \frac{u^2}{2g} \Rightarrow u = \sqrt{2gh} = \sqrt{2 \times 10 \times 40}$$

$$u' = 3u$$

we know $H \propto u^2$

$$\Rightarrow \frac{H_2}{H_1} = \left(\frac{u_2}{u_1} \right)^2$$

$$\Rightarrow \frac{H_2}{40} = \left(\frac{3u}{u} \right)^2 \Rightarrow \frac{H_2}{40} = 9$$

$$\Rightarrow H_2 = 360 \text{ m.}$$

(12)

For vertically projected body

use $v^2 - u^2 = -2gs$

$\Rightarrow v^2 = u^2 - 2gs$

we know $h = \frac{u^2}{2g}$
 $\Rightarrow 2gh = u^2$

(a) For $h = \frac{h}{2}$ $\Rightarrow v_1^2 = u^2 - 2g \frac{h}{2} = u^2 - \frac{u^2}{2} = \frac{u^2}{2}$
 $\Rightarrow v_1 = \frac{u}{\sqrt{2}}$

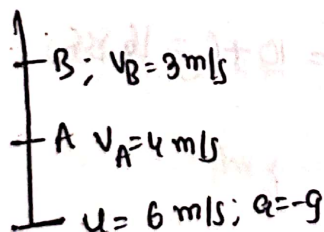
(b) For $h = \frac{3h}{4}$ $\Rightarrow v_2^2 = u^2 - 2g(\frac{3h}{4}) = u^2 - \frac{3u^2}{2} = \frac{u^2}{2}$
 $\Rightarrow v_2 = \frac{u}{\sqrt{2}}$

(c) For $h = \frac{h}{3}$ $\Rightarrow v_3^2 = u^2 - 2g \frac{h}{3} = u^2 - \frac{u^2}{3} = \frac{2u^2}{3}$
 $\Rightarrow v_3 = \sqrt{\frac{2}{3}} u$

(d) For $h = \frac{h}{4}$ $\Rightarrow v_4^2 = u^2 - 2g \frac{h}{4} = u^2 - \frac{u^2}{2} = \frac{u^2}{2}$
 $\Rightarrow v_4 = \frac{u}{\sqrt{2}}$

(14)

Given $u = 6 \text{ m/s}$



height of A $\Rightarrow v_A^2 - u^2 = -2gh_A$
 $\Rightarrow 4^2 - 6^2 = -2 \times 10 \times h_A$
 $\Rightarrow (16) - (36) = -20 h_A$
 $\Rightarrow h_A = 1 \text{ m}$

height of B $\Rightarrow v_B^2 - u^2 = -2gh_B$
 $\Rightarrow 9^2 - 6^2 = -2 \times 10 \times h_B$
 $\Rightarrow 81 - 36 = -20 h_B$
 $\Rightarrow h_B = \frac{27}{20} = 1.35 \text{ m}$

$h_B - h_A = 1.35 - 1$
 $= 0.35 \text{ m}$

(15)

For a vertically projected body

Time of flight

$$= \frac{2u}{g} = 2 \Rightarrow u = g$$

$$H_{\max} = \frac{u^2}{2g} = \frac{g^2}{2g} = \frac{g}{2} = \frac{9.8}{2} = 4.9 \text{ m}$$

(16)

clearly Time of flight $T = \frac{2u}{g} = 3 \text{ sec.}$

$$u = \frac{3g}{2}$$

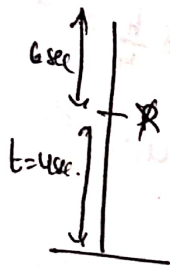
$$H_{\max} = \frac{u^2}{2g} = \frac{\left(\frac{3g}{2}\right)^2}{2g} = \frac{\frac{3g}{2} \times \frac{3g}{2}}{2g} = \frac{9 \times g}{2} = 45 \text{ m}$$

(17)

Given

$$u = 98 \text{ m/s}$$

$$T_a = \frac{u}{g} = \frac{98}{9} = \frac{98}{9.8} = 10 \text{ sec.}$$



To reach maximum height from ground

Time taken = 10 sec

From x to top = 6 sec

From top to x → 6 sec

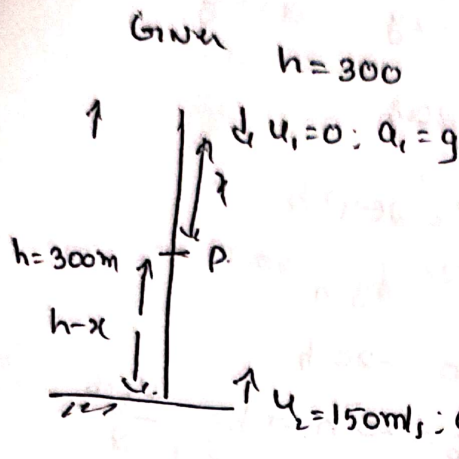
∴ Total time from ground to x in downward

$$\text{Journey} = 10 + 6 = 16 \text{ sec}$$

LTank
see main

⑤

①



Let P is a meeting point

From FFB

$$S = ut + \frac{1}{2}at^2$$

For object 1: $S = x; u = 0; a = g$

$$x = 0 \times t + \frac{1}{2}gt^2 \Rightarrow x = \frac{1}{2}gt^2 \quad \text{--- (1)}$$

For object 2: $S = 300 - x; u = 150; a = -g$

$$300 - x = 150t - \frac{1}{2}gt^2 \quad \text{--- (2)}$$

From (1) and (2)

$$300 - \frac{1}{2}gt^2 = 150t - \frac{1}{2}gt^2$$

$$\Rightarrow 300 = 150t \Rightarrow t = 2 \text{ sec.}$$

From top height $x = \frac{1}{2} \times g \times t^2$

$$= \frac{1}{2} \times 9.8 \times 2^2$$

$$= 2 \times 9.8 = 19.6 \text{ m.}$$

②

Given $H_{\text{max}} = g$

From $H = \frac{u^2}{2g} \Rightarrow u^2 = 2gH \Rightarrow u^2 = 2g \times g$

$$\Rightarrow u = \sqrt{2g} = 14.14 \text{ m/s}$$

③

Given $u = 5 \text{ m}$

They ask long maximum height reached

$$H_{\text{max}} = \frac{u^2}{2g} = \frac{5^2}{2 \times 10} = \frac{25}{20} = 1.25 \text{ m.}$$

④

Given $u = 100 \text{ m/s}$

Time of flight $T = \frac{2u}{g} = \frac{2 \times 100}{10} = 20 \text{ sec.}$

(5)

Time of ascent = 2 sec ; $v = 2 \text{ sec}$

$$\frac{u}{g} = 2 \Rightarrow u = 2g ; a = -g$$

$$\text{From } v^2 - u^2 = 2ax$$

$$\Rightarrow 2^2 - (2g)^2 = 2(-g)h$$

$$\Rightarrow 4 - 4g^2 = -2gh$$

$$\Rightarrow 4 - 4 \times 100 = -20h$$

$$\Rightarrow -396 = -20h \Rightarrow h = \frac{39.6}{2}$$

Given $H = a$
we

(6)

Given

$$T = 4 \text{ sec}$$

$$\Rightarrow \frac{2u}{g} = 4$$

$$\Rightarrow u = 2g = 19.6 \text{ m}$$

(7)

$$H_e = 12 \text{ m} ; g_e = g$$

$$g_m = \frac{1}{6}g$$

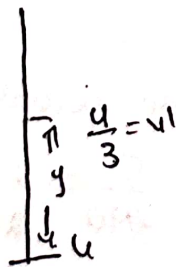
$$H_{\text{men}} = \frac{u^2}{2g} \times u = \text{same}$$

$$\Rightarrow \frac{H_m}{H_e} = \frac{g_e}{g_m}$$

$$\Rightarrow \frac{H_m}{12} = \frac{g}{\frac{1}{6}g}$$

$$\Rightarrow H_m = 12 \times 6 = 72 \text{ m}$$

(8)



$$\text{From } v^2 - u^2 = 2ax$$

$$\Rightarrow \left(\frac{u}{3}\right)^2 - u^2 = 2(-g)y$$

$$\Rightarrow -\frac{8u^2}{9} = -2gy$$

$$\Rightarrow y = \frac{8u^2}{9 \cdot 2g}$$

$$\text{We know } H = \frac{u^2}{2g}$$

$$\Rightarrow y = \frac{8}{9} H$$

$$\Rightarrow H = \frac{9y}{8}$$

6

Given $H = 9.8 \text{ m}$.

we know

$$H = \frac{u^2}{2g}$$

$$\Rightarrow u = \sqrt{2gH} = \sqrt{2 \times 9.8 \times 9.8} = 9.8\sqrt{2} \text{ m/s}$$

$$\text{Time of flight} = \frac{2u}{g} = \frac{2 \times 9.8\sqrt{2}}{9.8} = 2\sqrt{2} \text{ sec.}$$

(10)

let u be velocity of projection.

at $s = \frac{H}{2}$, $u = 10 \text{ m/s}$, $a = -g$

From $v^2 - u^2 = 2as$

$$\Rightarrow 10^2 - u^2 = -2g\left(\frac{H}{2}\right)$$

$$\Rightarrow 100 - u^2 = -g \frac{u^2}{2g}$$

$$\Rightarrow 100 = -\frac{u^2}{2} + u^2 \Rightarrow \frac{u^2}{2} = 100$$

$$u^2 = 200 \text{ m}$$

$$H = \frac{u^2}{2g} = \frac{200}{2 \times 10} = 10 \text{ m.}$$

(4)

(i) Given $u = 100 \text{ m/s}$

$$\text{Time of flight} = T = \frac{2u}{g} = \frac{2 \times 100}{10} = 20 \text{ sec.}$$

(ii)

Given $\frac{u_1}{u_2} = \frac{2}{3}$: we know $H \propto u^2$.

$$\frac{H_1}{H_2} = \left[\frac{u_1}{u_2} \right]^2 = \left[\frac{2}{3} \right]^2 = \frac{4}{9}$$

Given $H_{\max} = 45\text{m}$

$$u' = \frac{u}{2}$$

$$t = \text{Time} = \frac{u}{g} = \frac{30}{10} = 3\text{ sec.}$$

$$H_{\max} = \frac{u^2}{2g} \Rightarrow u = \sqrt{2gh}$$

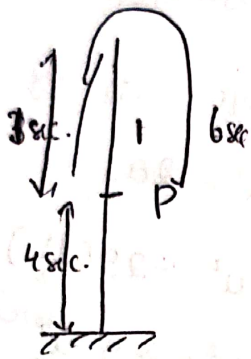
$$= \sqrt{2 \times 10 \times 45}$$

$$= 30\text{ m/s}$$

$$\frac{t'}{t} = \frac{u'}{u}$$

$$\Rightarrow \frac{t'}{3} = \frac{\frac{u}{2}}{u} \Rightarrow t' = \frac{3}{2} = 1.5\text{ sec}$$

(15)



Total Time of ascent $T = 3 + 4 = 7\text{ sec.}$

$$\Rightarrow \frac{u}{g} = 7 \Rightarrow u = 7g$$

$\therefore$ Maximum height reached by it

$$H = \frac{u^2}{2g} = \frac{(7g)^2}{2g} = \frac{49g^2}{2g}$$

$$= \frac{25g^2}{2g} = \frac{25 \times 10}{2} = 125\text{m.}$$

(16)

Given $u = 20\text{m/s}$

From $v^2 - u^2 = 2as$

$$\Rightarrow v^2 - (20)^2 = 2(-g)\left(\frac{3}{4}H\right)$$

$$\Rightarrow v^2 - 400 = -2 \times 10 \times 15$$

$$\Rightarrow v^2 - 400 = -300$$

$$\Rightarrow v^2 = 100$$

$$\Rightarrow v = 10\text{ m/s}$$

$$a = -g : s = H = \frac{u^2}{2g}$$

$$s = \frac{3}{4} H = \frac{3}{4} \times \frac{u^2}{2g}$$

$$= \frac{3}{4} \times \frac{20^2}{2 \times 10}$$

$$= 15\text{m}$$