

8. HEIGHTS AND

①

DISTANCES

Class: VIII. MATHEMATICS
FOUNDATION! SOLUTIONS

TEACHING TASK
JEE MAINS LEVEL

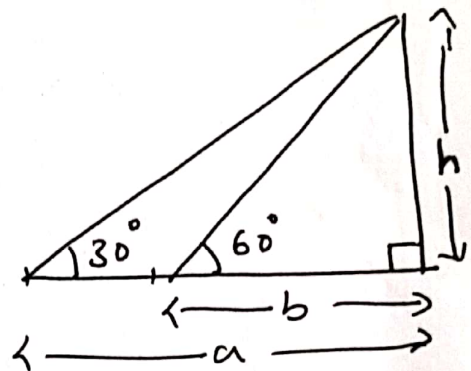
2025/26

01. From diagram

$$\tan 30^\circ = \frac{h}{a}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a}$$

$$\Rightarrow h = \frac{a}{\sqrt{3}} \rightarrow (i)$$



Again, $\tan 60^\circ = \frac{h}{b}$

$$\Rightarrow \sqrt{3} = \frac{h}{b}$$

$$\Rightarrow h = \sqrt{3}b \rightarrow (ii)$$

$$\text{Now } (i) \& (ii) \Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$$

Ans: B

02. From the diagram

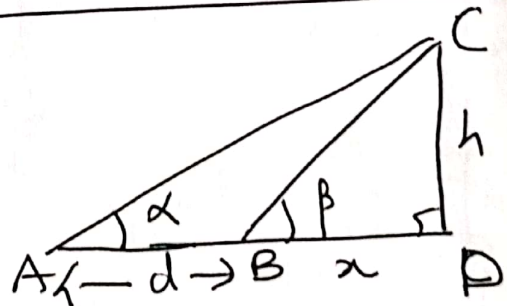
$$\tan \beta = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\tan \beta}$$

$$\Rightarrow x = h \cot \beta \rightarrow (i)$$

Now, $\tan \alpha = \frac{h}{d+x}$

$$\Rightarrow d+x = h \cot \alpha$$



$$\Rightarrow x = h \cot \alpha - d \rightarrow (ii)$$

From (i) and (ii)

$$\Rightarrow h \cot \beta = h \cot \alpha - d$$

$$\Rightarrow h \cot \alpha - h \cot \beta = d$$

$$\Rightarrow h = \frac{d}{\cot \alpha - \cot \beta}$$

Ans: A

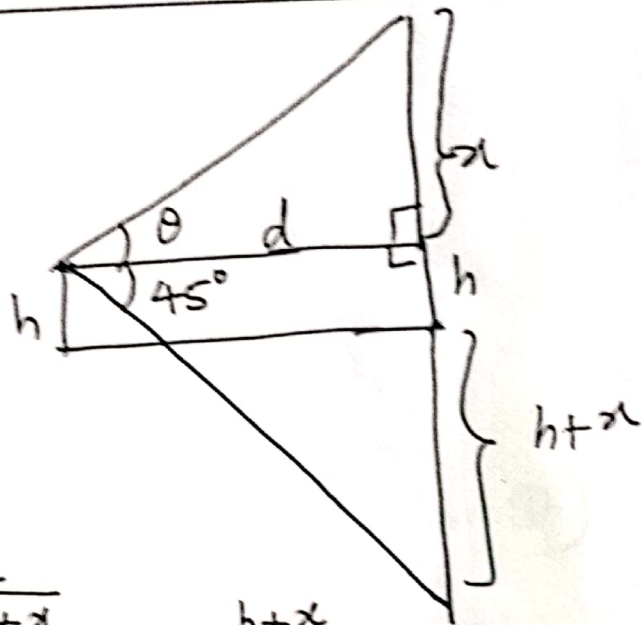
03 $\tan \theta = \frac{x}{d} \rightarrow (1)$

$$\tan 45^\circ = \frac{2h+x}{d}$$

$$\Rightarrow d = 2h+x \rightarrow (2)$$

Now, $\therefore \tan \theta = \frac{x}{2h+x}$

$$\begin{aligned} \tan(45^\circ + \theta) &= \frac{1 + \tan \theta}{1 - \tan \theta} \\ &= \frac{1 + \frac{x}{2h+x}}{1 - \frac{x}{2h+x}} = \frac{h+x}{h} \end{aligned}$$



$$\therefore h+x = h(\tan 45^\circ + \tan \theta)$$

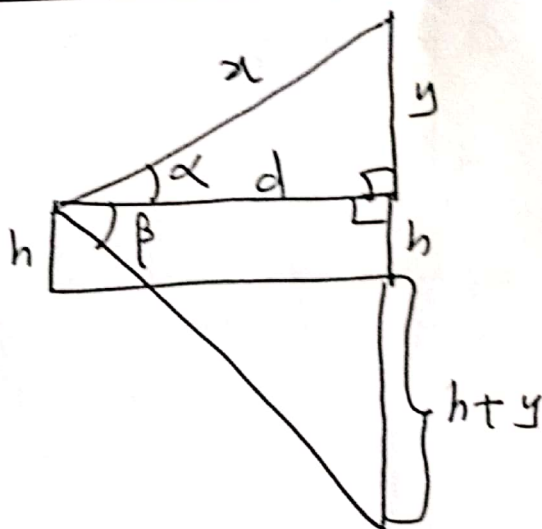
$\therefore$ The height of the cloud = $h(\tan 45^\circ + \tan \theta)$
Ans: A

04 $\tan \alpha = \frac{y}{d} \Rightarrow d = \frac{y}{\tan \alpha}$

$$\tan \beta = \frac{2h+y}{d} \Rightarrow d = \frac{2h+y}{\tan \beta}$$

$$\therefore \frac{y}{\tan \alpha} = \frac{2h+y}{\tan \beta}$$

$$\Rightarrow \frac{y}{\tan \alpha} = \frac{2h}{\tan \beta} + \frac{y}{\tan \beta}$$



$$\Rightarrow \frac{y}{\tan \alpha} - \frac{y}{\tan \beta} = \frac{2h}{\tan \beta}$$

(3)

$$\Rightarrow y \left(\frac{\tan \beta - \tan \alpha}{\tan \alpha \cdot \tan \beta} \right) = \frac{2h}{\tan \beta}$$

$$\Rightarrow \frac{y}{\tan \alpha} = \frac{2h}{\tan \beta - \tan \alpha}$$

$$\Rightarrow d = \frac{2h}{\tan \beta - \tan \alpha} \rightarrow \textcircled{1}$$

Now, $\sec \alpha = \frac{x}{d}$

$$\Rightarrow x = d \sec \alpha$$

$$\Rightarrow x = \frac{2h \sec \alpha}{\tan \beta - \tan \alpha} \rightarrow (\because \text{from } \textcircled{1})$$

Hence, The distance b/w the cloud and the observer is $\frac{2h \sec \alpha}{\tan \beta - \tan \alpha}$

Ans: D

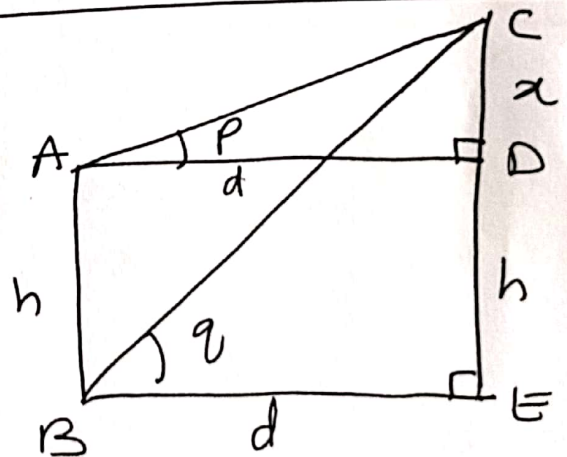
05

From the diagram

AB = Building

CE = Hill

A, B = Position of the observer



Now, $\tan p = \frac{x}{d}$ and

$$\tan q = \frac{x+h}{d}$$

$$\Rightarrow d = x \cot p \text{ also } d = (x+h) \cot q$$

$$\therefore x \cot p = (x+h) \cot q$$

$$\Rightarrow x(\cot p - \cot q) = h \cot q$$

$$\Rightarrow x = \frac{h \cot q}{\cot p - \cot q}$$

Now, height of the hill

$$x+h = \frac{h \cot q}{\cot p - \cot q} + h$$

$$= \frac{h \cot q + h \cot p - h \cot q}{\cot p - \cot q}$$

$$= \frac{h \cot p}{\cot p - \cot q}$$

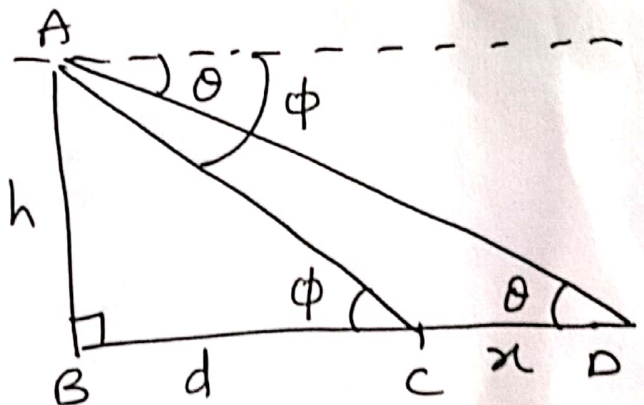
Ans. C

06.

In the diagram
C, D = Position of the
boats

A = position of the
observer

AB = Light house



Now, $\tan \phi = \frac{h}{d} \Rightarrow d = \frac{h}{\tan \phi}$

$$\tan \theta = \frac{h}{d+x} \Rightarrow d+x = \frac{h}{\tan \theta}$$

$$\therefore \frac{h}{\tan \phi} + x = \frac{h}{\tan \theta}$$

$$\Rightarrow x = \frac{h}{\tan \theta} - \frac{h}{\tan \phi}$$

$$\Rightarrow x = h \left(\frac{\tan \phi - \tan \theta}{\tan \theta \cdot \tan \phi} \right)$$

∴ The distance between the two boats is $\frac{h(\tan \phi - \tan \alpha)}{\tan \alpha \cdot \tan \phi}$ (5)

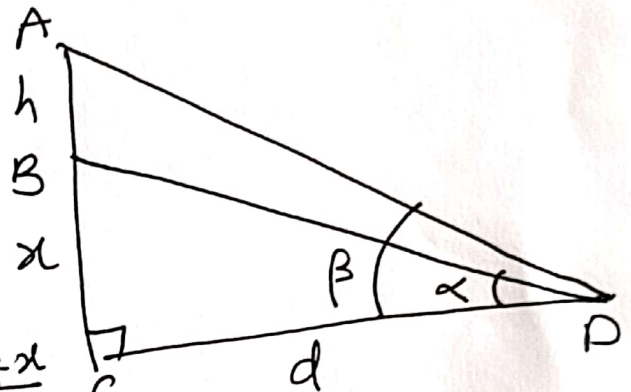
Ans. B

07

D = position of the observer

AB = Flag staff

BC = Vertical tower



$$\text{Now, } \tan \alpha = \frac{x}{d} \quad \left| \quad \tan \beta = \frac{h+x}{d} \right.$$

$$\Rightarrow d = \frac{x}{\tan \alpha} \quad \left| \quad \Rightarrow d = \frac{h+x}{\tan \beta} \right.$$

$$\therefore \frac{x}{\tan \alpha} = \frac{h+x}{\tan \beta}$$

$$\Rightarrow \frac{x}{\tan \alpha} = \frac{h}{\tan \beta} + \frac{x}{\tan \beta}$$

$$\Rightarrow x \left(\frac{1}{\tan \alpha} - \frac{1}{\tan \beta} \right) = \frac{h}{\tan \beta}$$

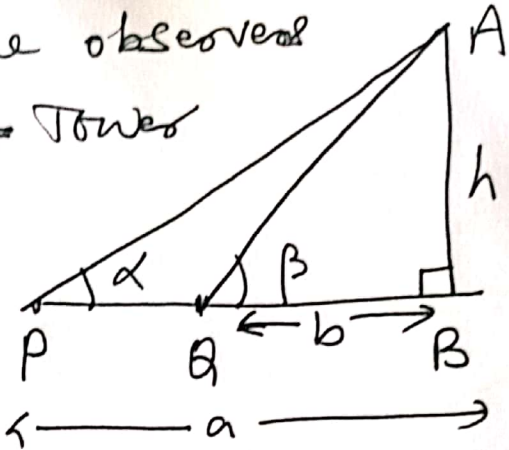
$$\Rightarrow x \left(\frac{\tan \beta - \tan \alpha}{\tan \alpha \cdot \tan \beta} \right) = \frac{h}{\tan \beta}$$

$$\Rightarrow x = \frac{h \tan \alpha}{\tan \beta - \tan \alpha}$$

∴ Height of the tower is $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$

Ans: B

08 $P, Q \rightarrow$ position of the observer
 $AB =$ ~~position of the~~ Tower
 Given $\alpha + \beta = 90^\circ$



$\Rightarrow \tan(\alpha + \beta) = \tan 90^\circ$

$\Rightarrow \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \text{not defined}$

$\Rightarrow 1 - \tan \alpha \cdot \tan \beta = 0$

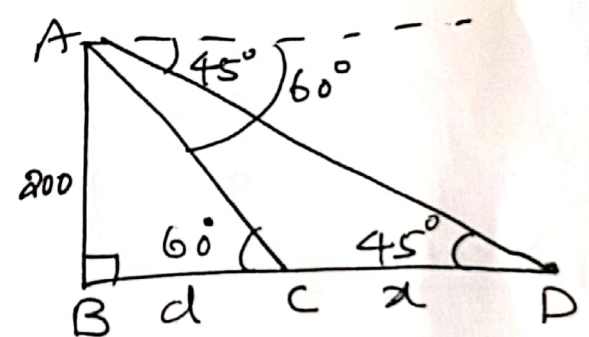
$\Rightarrow \tan \alpha \cdot \tan \beta = 1$

$\Rightarrow \frac{h}{a} \cdot \frac{h}{b} = 1$

$\Rightarrow h = \sqrt{ab}$

$\therefore$ Height of the tower is $\sqrt{ab}$ Ans. A

09 $AB =$ Altitude of the aeroplane
 $CD =$ ~~opposite~~ sides of the banks of a river
 $CD = x =$ width of the river



Now, $\tan 60^\circ = \frac{200}{d} \Rightarrow d = \frac{200}{\sqrt{3}} \rightarrow (1)$

Also, $\tan 45^\circ = \frac{200}{d+x}$

$\Rightarrow d+x = 200$

$\Rightarrow \frac{200}{\sqrt{3}} + x = 200$ ($\because$ from (1))

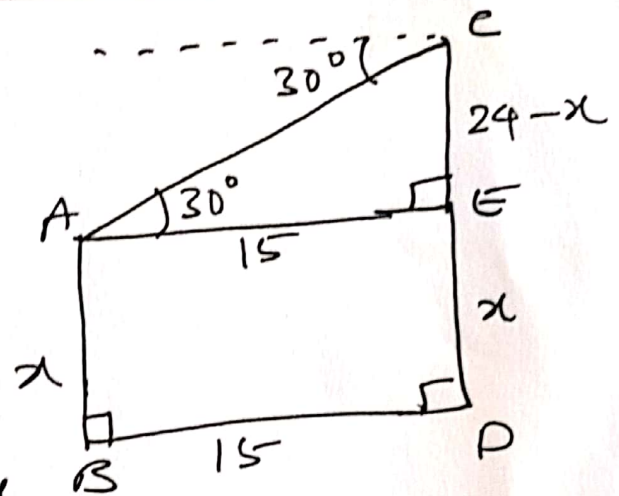
$$\Rightarrow x = 200 - \frac{200}{\sqrt{3}}$$

(9)

$$\Rightarrow x = 200 \left(\frac{\sqrt{3} - 1}{\sqrt{3}} \right)$$

Ans: B

10. AB $\rightarrow$ First pole
 CD $\rightarrow$ Second pole
 BD = 15 mt, CD = 24 mt
 Let AB = x mt
 $\therefore CE = (24 - x)$ mt



In $\triangle AEC$, $\tan 30^\circ = \frac{24 - x}{15}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{24 - x}{15}$$

$$\Rightarrow 24 - x = \frac{15}{\sqrt{3}} = 5\sqrt{3} = 8.67$$

$$\Rightarrow x = 24 - 8.67$$

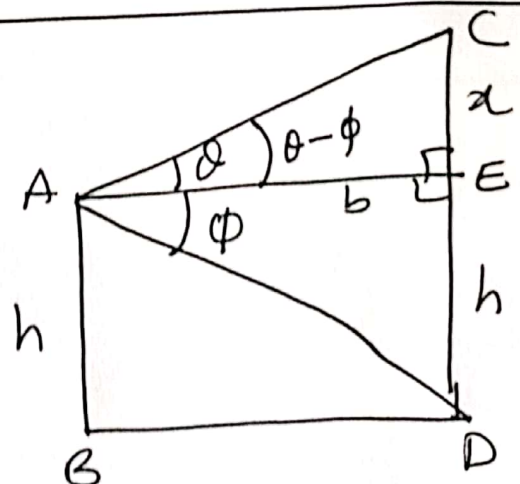
$$\Rightarrow x = 15.34 \text{ (Approx)}$$

Ans: C

JEE ADVANCED LEVEL

01. AB = window
 CD = Building
 BD = Road
 AB = h

$$\tan \phi = \frac{h}{b} \rightarrow (1)$$



$$\therefore \tan(\theta - \phi) = \frac{x}{b}$$

(8)

$$\Rightarrow \frac{\tan \theta - \tan \phi}{1 + \tan \theta \cdot \tan \phi} = \frac{x}{b}$$

$$\Rightarrow \frac{\frac{\sin \theta}{\cos \theta} - \frac{h}{b}}{1 + \frac{\sin \theta}{\cos \theta} \cdot \frac{h}{b}} = \frac{x}{b} \quad (\because \text{from (1)})$$

$$\Rightarrow \frac{b \sin \theta - h \cos \theta}{b \cos \theta + h \sin \theta} = \frac{x}{b}$$

$$\Rightarrow x = \frac{b(b \sin \theta - h \cos \theta)}{b \cos \theta + h \sin \theta}$$

Height of the building = $h + x$

$$\therefore h + x = h + \frac{b(b \sin \theta - h \cos \theta)}{b \cos \theta + h \sin \theta}$$

$$= \frac{bh \cos \theta + h^2 \sin \theta + b^2 \sin \theta - bh \cos \theta}{b \cos \theta + h \sin \theta}$$

$$= \frac{(h^2 + b^2) \sin \theta}{b \cos \theta + h \sin \theta}$$

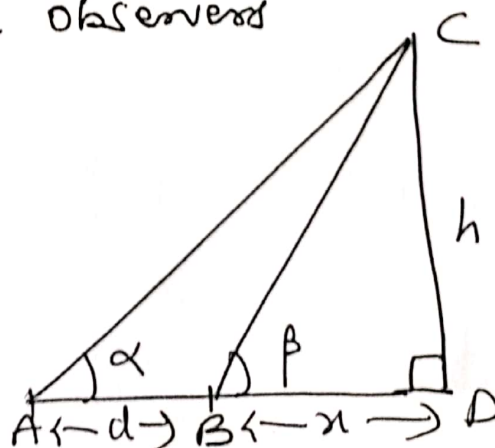
Ans: C

Q2 A B $\rightarrow$ Position of the observer

CD = tower

AB = d

Let BD = x, CD = h



Now, $\tan \beta = \frac{h}{x}$ and $\tan \alpha = \frac{h}{d+x}$ (9)

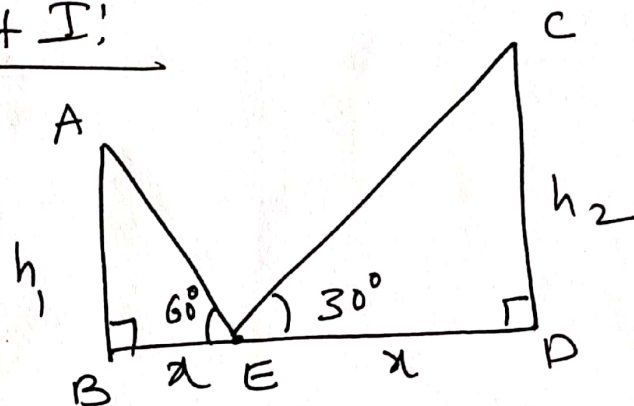
$\Rightarrow x = h \cot \beta$ and $d+x = h \cot \alpha$

$\Rightarrow d + h \cot \beta = h \cot \alpha$

$\Rightarrow h = \frac{d}{\cot \alpha - \cot \beta}$

$\therefore$ Height of the tower = $\frac{d}{\cot \alpha - \cot \beta}$ Ans B

03 statement I:



$\tan 60^\circ = \frac{h_1}{x}$

$\Rightarrow x = \frac{h_1}{\sqrt{3}} \rightarrow (i)$

$\tan 30^\circ = \frac{h_2}{x}$

$\Rightarrow x = \sqrt{3} h_2 \rightarrow (ii)$

From (i) and (ii)

$\Rightarrow \frac{h_1}{\sqrt{3}} = \sqrt{3} h_2 \Rightarrow h_1 = 3 h_2$

$\Rightarrow h_1 : h_2 = 3 : 1$ (True)

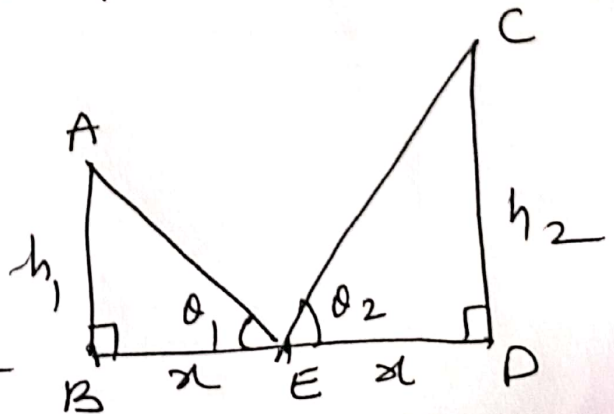
Statement II:

$\tan \theta_1 = \frac{h_1}{x}$

$\Rightarrow x = \frac{h_1}{\tan \theta_1}$

$\tan \theta_2 = \frac{h_2}{x}$

$\Rightarrow x = \frac{h_2}{\tan \theta_2}$



$$\Rightarrow \frac{h_1}{\tan \theta_1} = \frac{h_2}{\tan \theta_2}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

$$\Rightarrow h_1 : h_2 = \tan \theta_1 : \tan \theta_2 \text{ (True) } \text{Ans: A}$$

(10)

04. statement I:

$$\tan 30^\circ = \frac{h}{x+y}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y}$$

$$\Rightarrow x+y = \sqrt{3}h \rightarrow (1)$$

$$\text{Again, } \tan 60^\circ = \frac{h}{y} \Rightarrow \sqrt{3} = \frac{h}{y}$$

$$\Rightarrow y = \frac{h}{\sqrt{3}} \rightarrow (2)$$

$$\text{from (1)} \Rightarrow x + \frac{h}{\sqrt{3}} = \sqrt{3}h$$

It is not possible to find h value unless the value of x is given. (False)

statement II: Question Repeated. Please

Refer Q no: 2 (Advanced question) (True)

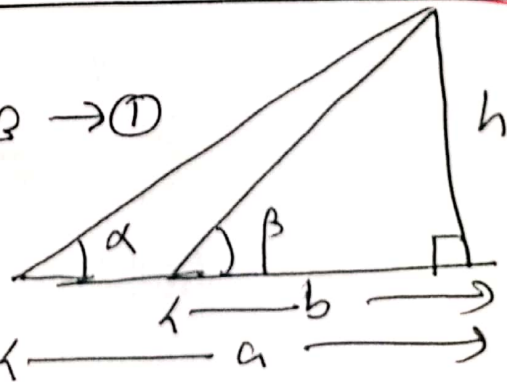
Ans: D

05. statement I:

$$\tan \beta = \frac{h}{b} \Rightarrow h = b \tan \beta \rightarrow (1)$$

$$\text{Also, } \tan \alpha = \frac{h}{a}$$

$$\Rightarrow h = a \tan \alpha \rightarrow (2)$$



Now, ① & ② $\Rightarrow h^2 = ab \tan \alpha \cdot \tan \beta$

$\Rightarrow h = \sqrt{ab \tan \alpha \cdot \tan \beta}$ (False)

Statement II: $a = 9 \text{ m}$, $b = 4 \text{ m}$
 $\alpha = 30^\circ$, $\beta = 60^\circ$

Now, $h = \sqrt{ab \tan \alpha \cdot \tan \beta}$

$= \sqrt{9 \cdot 4 \cdot \tan 30^\circ \cdot \tan 60^\circ}$

$= \sqrt{36 \times \frac{1}{\sqrt{3}} \times \sqrt{3}}$

$= 6 \text{ m}$ (True)

Ans: D

06 Statement I:

$\tan \alpha = \frac{h}{x}$ and $\tan \beta = \frac{h}{y}$

$\alpha + \beta = 90^\circ$

$\Rightarrow \tan(\alpha + \beta) = \tan 90^\circ$

$\Rightarrow \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \text{Not defined}$

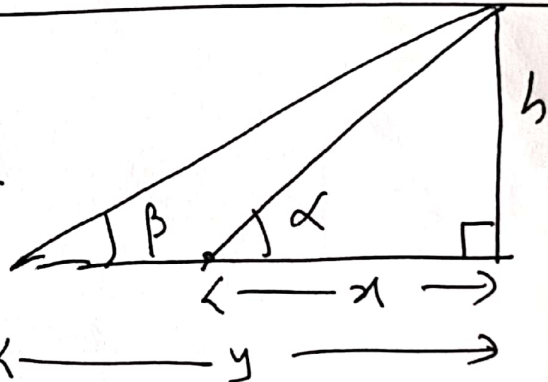
$\Rightarrow \tan \alpha \cdot \tan \beta = 1$

$\Rightarrow \frac{h}{x} \cdot \frac{h}{y} = 1$

$\Rightarrow h^2 = xy$

$\Rightarrow h = \sqrt{xy}$ (True)

Statement II: Given $x = a^2 + b^2$, $y = 2ab$



We have $h = \sqrt{xy}$

$$h = \sqrt{(a^2 + b^2) \cdot 2ab} \neq (a+b) \text{ (false)}$$

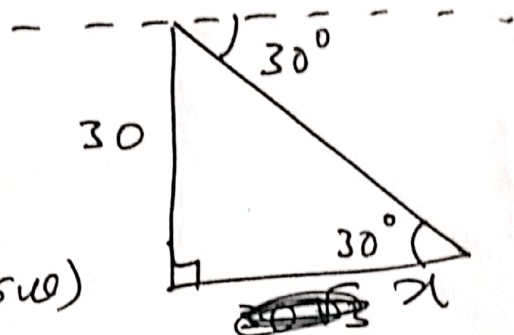
Ans. C

07 Assertion:

$$\tan 30^\circ = \frac{30}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{30}{x}$$

$$\Rightarrow x = 30\sqrt{3} \text{ mt (True)}$$



Ans. A

Reason: Conceptual (True)

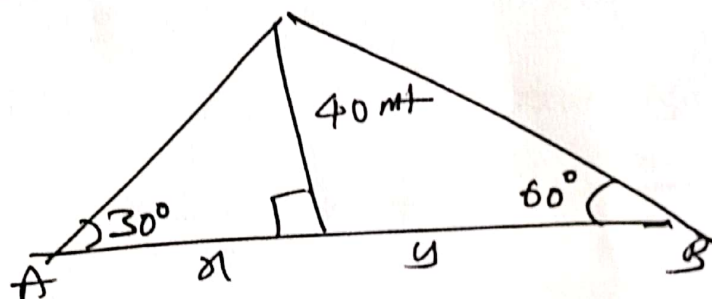
08 Assertion:

$$\tan 30^\circ = \frac{40}{x}$$

$$\Rightarrow x = 40\sqrt{3}$$

$$\tan 60^\circ = \frac{40}{y}$$

$$\Rightarrow y = \frac{40}{\sqrt{3}}$$



$$\begin{aligned} \text{Distance} &= x + y \\ &= 40\sqrt{3} + \frac{40}{\sqrt{3}} = \frac{160}{\sqrt{3}} \\ &= 92.38 \text{ (Approx)} \end{aligned}$$

(True)

Ans. A

Reason: Conceptual (True)

09

$$d = \frac{h(\tan \alpha + \tan \beta)}{\tan \alpha \cdot \tan \beta}$$

here $h = 10$, $\alpha = 45^\circ$, $\beta = 30^\circ$

$$d = \frac{10(\tan 45^\circ + \tan 30^\circ)}{\tan 45^\circ \cdot \tan 30^\circ}$$

(13)

$$= \frac{10\left(1 + \frac{1}{\sqrt{3}}\right)}{1 \cdot \frac{1}{\sqrt{3}}} = 10(\sqrt{3} + 1) \text{ m} \quad \text{Ans. A}$$

10. $d = 20\sqrt{3}$, $\alpha = 15^\circ$, $\beta = 30^\circ$

$$: 20\sqrt{3} = \frac{h(\tan 15^\circ + \tan 30^\circ)}{\tan 15^\circ \cdot \tan 30^\circ}$$

$$\Rightarrow 20\sqrt{3} = \frac{h\left((2-\sqrt{3}) + \frac{1}{\sqrt{3}}\right)}{(2-\sqrt{3}) \cdot \frac{1}{\sqrt{3}}}$$

$$\Rightarrow h = \frac{10\sqrt{3}(2-\sqrt{3})}{\sqrt{3}-1}$$

Ans. B

11 $\alpha = 45^\circ$, $d = 30\sqrt{2}$ $h = 2 \text{ m}$

We have $30\sqrt{2} = \frac{2(\tan 45^\circ + \tan \beta)}{\tan 45^\circ \cdot \tan \beta}$

$$\Rightarrow 30\sqrt{2} = \frac{2(1 + \tan \beta)}{\tan \beta}$$

$$\Rightarrow \tan \beta = \frac{2}{30\sqrt{2} - 2}$$

$$\Rightarrow \tan \beta = \frac{1}{15\sqrt{2} - 1}$$

Ans B

12

$$H = \frac{h(\tan \beta + \tan \alpha)}{\tan \beta - \tan \alpha}$$

(14)

$$\alpha = 30^\circ, \beta = 60^\circ$$

$$H = \frac{h(\sqrt{3} + \frac{1}{\sqrt{3}})}{\sqrt{3} - \frac{1}{\sqrt{3}}} \Rightarrow H = 2h$$

$$h : H = 1 : 2$$

Ans. A

13

$$H = \frac{h(\tan \beta + \tan \alpha)}{\tan \beta - \tan \alpha}$$

$$\Rightarrow h = \frac{H(\tan \beta - \tan \alpha)}{(\tan \beta + \tan \alpha)}$$

$$\Rightarrow h = H \left(\frac{1}{\cot \beta} - \frac{1}{\cot \alpha} \right)$$

$$\Rightarrow h = \frac{H(\cot \alpha - \cot \beta)}{(\cot \alpha + \cot \beta)}$$

Ans. A

14

$$\alpha = 0^\circ$$

$$\begin{aligned} \text{Now, } H &= \frac{h(\tan \beta + \tan \alpha)}{(\tan \beta - \tan \alpha)} \\ &= \frac{h(\tan \beta + \tan 0^\circ)}{(\tan \beta - \tan 0^\circ)} = h \end{aligned}$$

$$\Rightarrow H = h$$

$$\therefore h^2 + H^2 = 2h^2 \text{ or } 2H^2$$

Ans. —

15

AB, CD $\rightarrow$ poles

$$AB = 16 \text{ mt}$$

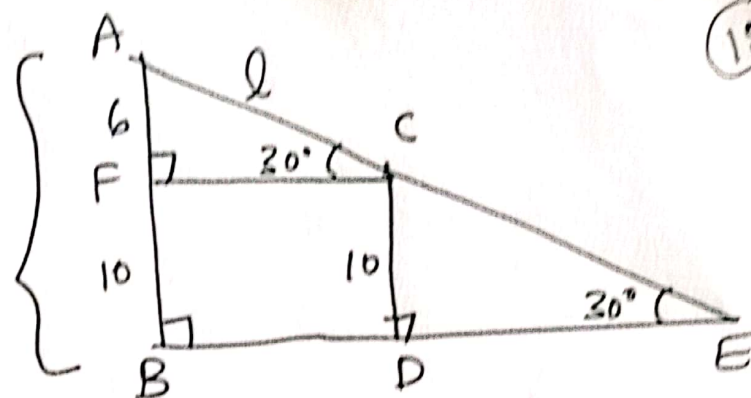
$$CD = 10 \text{ mt}$$

$$AF = 6 \text{ mt}$$

AC = length of wire

$$\text{In } \triangle AFC \rightarrow \sin 30^\circ = \frac{6}{l}$$

$$\Rightarrow \frac{1}{2} = \frac{6}{l} \Rightarrow l = 12 \text{ m} \quad \text{Ans. 12}$$

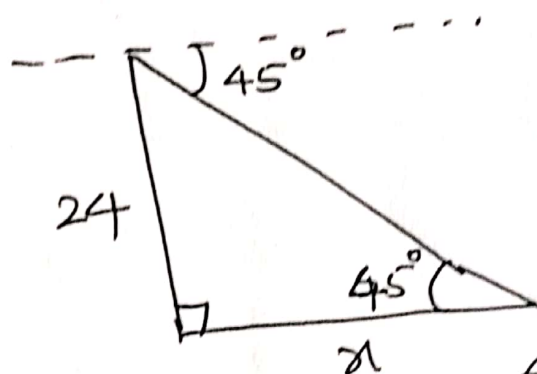


16

$$\tan 45^\circ = \frac{24}{x}$$

$$\Rightarrow 1 = \frac{24}{x}$$

$$\Rightarrow x = 24$$



Ans: 24

17

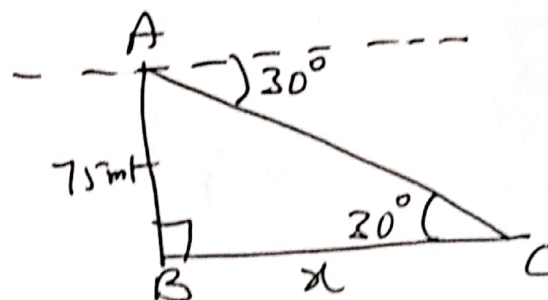
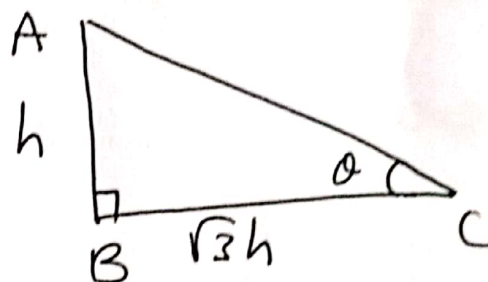
a) AB $\rightarrow$ TowerBC $\rightarrow$ shadow

$$\tan \theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^\circ \text{ (q)}$$

b) A $\rightarrow$ position of observerC $\rightarrow$ position of car

$$\tan 30^\circ = \frac{75}{x}$$



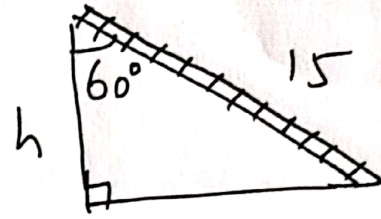
$$\Rightarrow x = 20\sqrt{3} (t)$$

(16)

c) $\cos 60^\circ = \frac{h}{15}$

$$\Rightarrow \frac{1}{2} = \frac{h}{15}$$

$$\Rightarrow h = \frac{15}{2} (x)$$

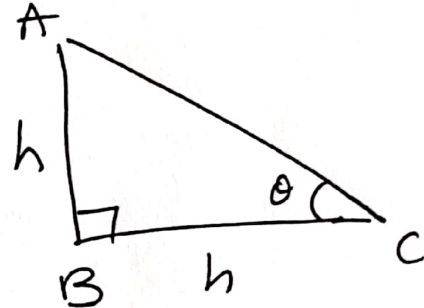


d) AB $\rightarrow$ Pole

BC $\rightarrow$ shadow

$$\tan \theta = \frac{AB}{BC} = 1$$

$$\theta = 45^\circ (s)$$



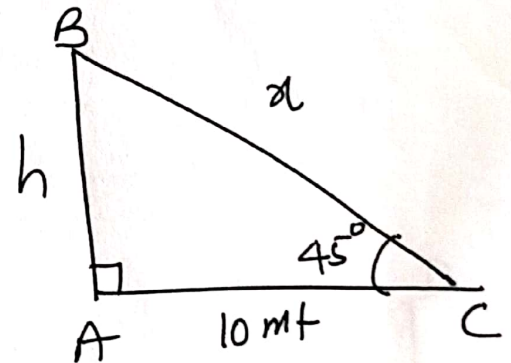
Ans: θ, t, x, s

18 a) $\tan 45^\circ = \frac{h}{10} \quad \left| \quad \sin 45^\circ = \frac{h}{x} \right.$

$$\Rightarrow h = 10$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{10}{x}$$

$$\Rightarrow x = 10\sqrt{2}$$

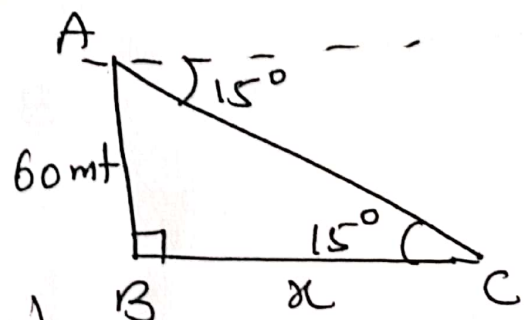


$\therefore$ ~~Len~~ Height of the tree before it was broken = $h + x = 10 + 10\sqrt{2} = 10(\sqrt{2} + 1) (t)$

b) $\tan 15^\circ = \frac{60}{x}$

$$\Rightarrow \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{60}{x}$$

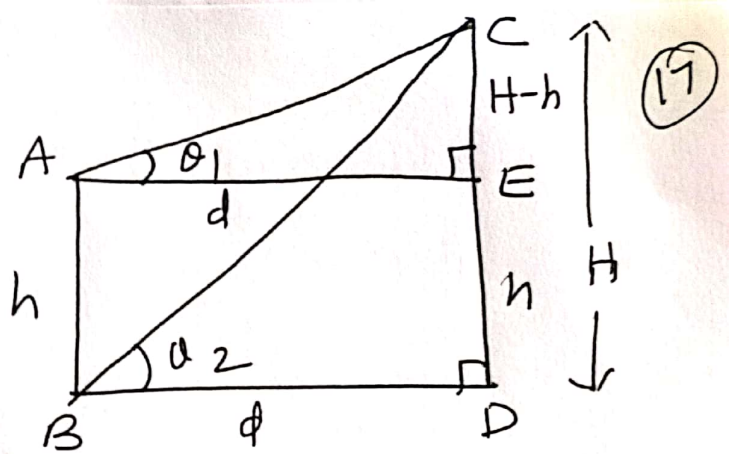
$$\Rightarrow x = 60 \left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1} \right) (q)$$



c) $AB \rightarrow$ Building

$CD \rightarrow$ hill

$A, B \rightarrow$ Position of observers



$$\tan \theta_1 = \frac{H-h}{d}$$

$$\tan \theta_2 = \frac{H}{d}$$

$$\Rightarrow d = \frac{H-h}{\tan \theta_1} \rightarrow (1)$$

$$\Rightarrow d = \frac{H}{\tan \theta_2} \rightarrow (2)$$

from (1) & (2)

$$\frac{H-h}{\tan \theta_1} = \frac{H}{\tan \theta_2}$$

$$\Rightarrow \frac{H}{\tan \theta_1} - \frac{H}{\tan \theta_2} = \frac{h}{\tan \theta_1}$$

$$\Rightarrow H \left(\frac{\tan \theta_2 - \tan \theta_1}{\tan \theta_1 \cdot \tan \theta_2} \right) = \frac{h}{\tan \theta_1}$$

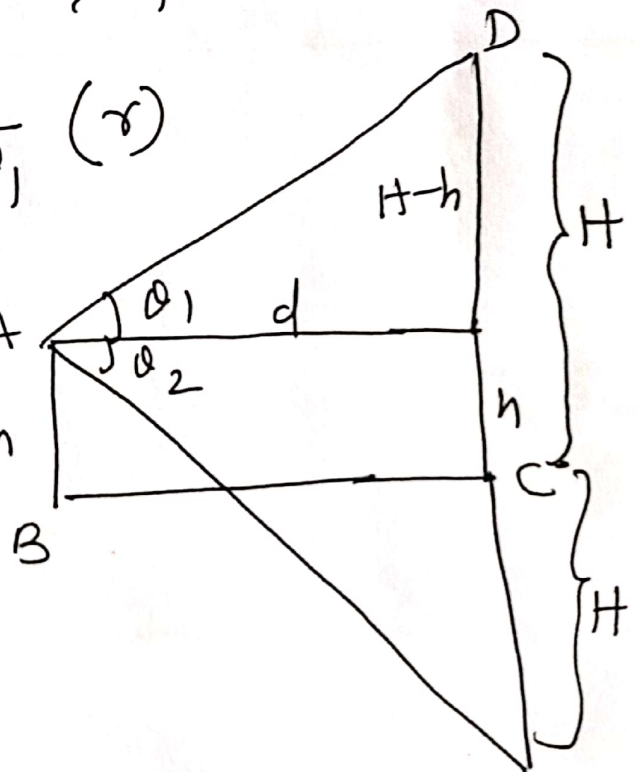
$$\Rightarrow H = \frac{h \cdot \tan \theta_2}{\tan \theta_2 - \tan \theta_1} \quad (3)$$

$$d) \tan \theta_1 = \frac{H-h}{d} \quad \tan \theta_2 = \frac{H+h}{d}$$

$$\Rightarrow d = \frac{H-h}{\tan \theta_1}$$

$$\Rightarrow d = \frac{H+h}{\tan \theta_2}$$

$$\therefore \frac{H-h}{\tan \theta_1} = \frac{H+h}{\tan \theta_2}$$



$$\Rightarrow \frac{H}{\tan \theta_1} - \frac{H}{\tan \theta_2} = \frac{h}{\tan \theta_2} + \frac{h}{\tan \theta_1}$$

18

$$\Rightarrow H(\tan \theta_2 - \tan \theta_1) = h(\tan \theta_1 + \tan \theta_2)$$

$$\Rightarrow H = \frac{h(\tan \theta_1 + \tan \theta_2)}{\tan \theta_2 - \tan \theta_1}$$

$$\Rightarrow H = \frac{h \left(\frac{\sin \theta_1}{\cos \theta_1} + \frac{\sin \theta_2}{\cos \theta_2} \right)}{\left(\frac{\sin \theta_2}{\cos \theta_2} - \frac{\sin \theta_1}{\cos \theta_1} \right)}$$

$$\Rightarrow H = \frac{h(\sin \theta_1 \cos \theta_2 + \cos \theta_1 \sin \theta_2)}{(\sin \theta_2 \cos \theta_1 - \cos \theta_2 \sin \theta_1)}$$

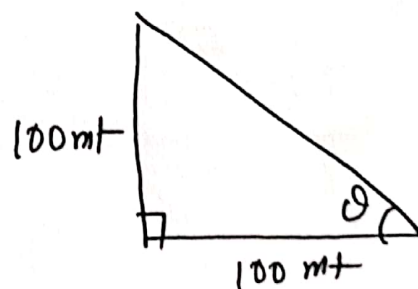
$$\Rightarrow H = \frac{h \sin(\theta_1 + \theta_2)}{\sin(\theta_2 - \theta_1)} \quad (s)$$

Ans: t, q, r, s

LEARNERS TASK (CUES)

01. $\tan \theta = \frac{100}{100} = 1$

$$\Rightarrow \theta = 45^\circ$$



Ans: C

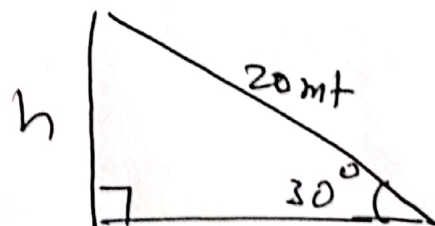
02

$\therefore \tan 30^\circ =$

$$\sin 30^\circ = \frac{h}{20}$$

$$\Rightarrow \frac{1}{2} = \frac{h}{20}$$

$$\Rightarrow h = 10$$



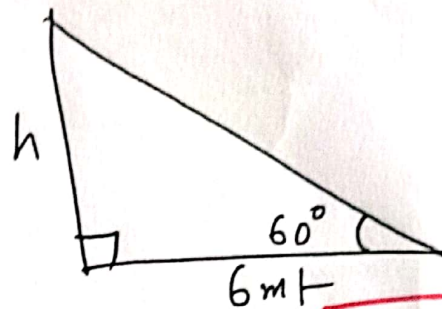
Ans: C

03

$$\tan 60^\circ = \frac{h}{6}$$

$$\Rightarrow \sqrt{3} = \frac{h}{6}$$

$$\Rightarrow h = 6\sqrt{3} \text{ m}$$

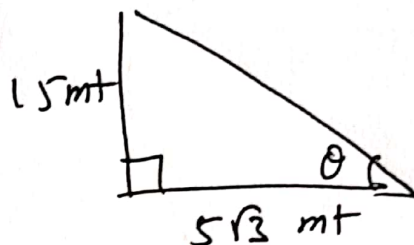


Am: B

04

$$\tan \theta = \frac{15}{5\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\theta = 60^\circ$$



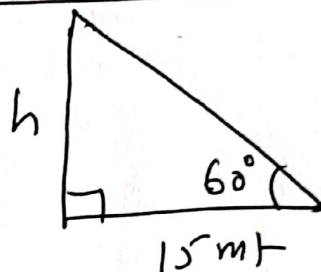
Am: A

05

$$\tan 60^\circ = \frac{h}{15}$$

$$\Rightarrow \sqrt{3} = \frac{h}{15}$$

$$\Rightarrow h = 15\sqrt{3}$$



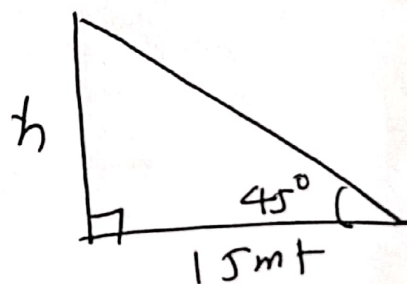
Am: A

06

$$\tan 45^\circ = \frac{h}{15}$$

$$\Rightarrow h = \frac{h}{15}$$

$$\Rightarrow h = 15 \text{ m}$$



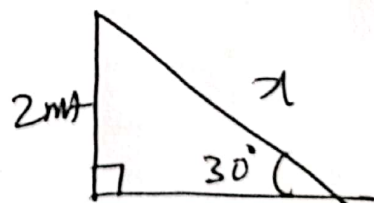
Am: D

07

$$\sin 30^\circ = \frac{2}{x}$$

$$\Rightarrow \frac{1}{2} = \frac{2}{x}$$

$$\Rightarrow x = 4$$



Am: C

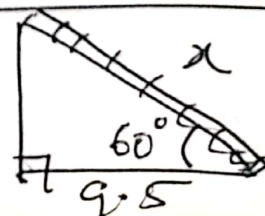
08

$$\cos 60^\circ = \frac{9.5}{x}$$

$$\Rightarrow \frac{1}{2} = \frac{9.5}{x}$$

$$x = 19 \text{ m}$$

Am: A

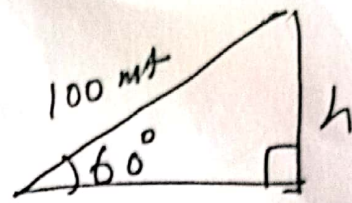


09.

$$\sin 60^\circ = \frac{h}{100}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{100}$$

$$\Rightarrow h = 50\sqrt{3}$$



(20)

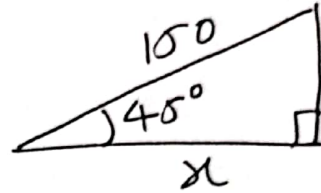
Ans: C

10

$$\cos 45^\circ = \frac{x}{150}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{x}{150}$$

$$\Rightarrow x = \frac{150}{\sqrt{2}} = 75\sqrt{2}$$



Ans: B

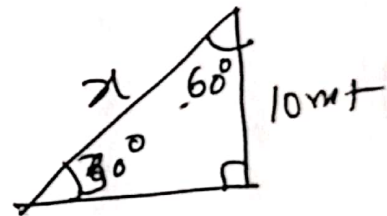
IEEE MAINS LEVEL

01.

$$\sin 30^\circ = \frac{10}{x}$$

$$\Rightarrow \frac{1}{2} = \frac{10}{x}$$

$$\Rightarrow x = 20$$



total length of the rope = $3 \times \frac{20}{\sin 30^\circ} = 60 \text{ m}$

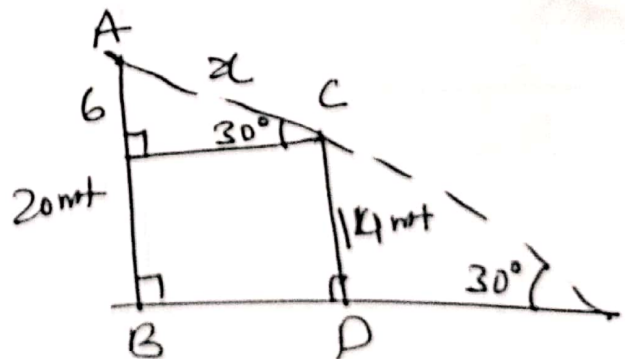
Ans: D

02

$$\sin 30^\circ = \frac{6}{x}$$

$$\Rightarrow \frac{1}{2} = \frac{6}{x}$$

$$\Rightarrow x = 12 \text{ m}$$



Ans: A

03

03 In the diagram

AB = tower

BC = Building

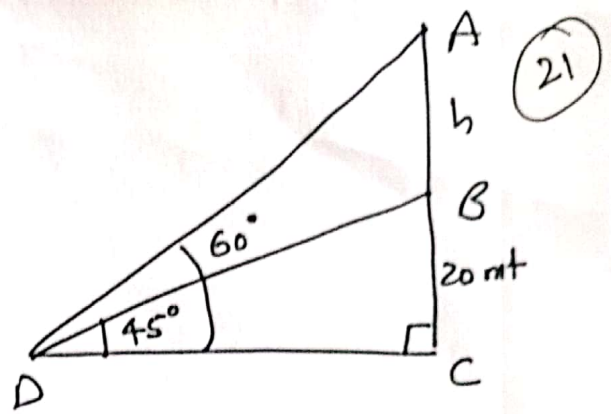
$$\tan 45^\circ = \frac{BC}{CD}$$

$$\Rightarrow 1 = \frac{20}{CD}$$

$$\Rightarrow CD = 20 \text{ mt}$$

$$\text{Now, } \tan 60^\circ = \frac{AC}{CD}$$

$$\Rightarrow \sqrt{3} = \frac{20+h}{20}$$



$$\Rightarrow 20 + h = 20\sqrt{3}$$

$$\Rightarrow h = 20\sqrt{3} - 20$$

$$\Rightarrow h = 20(\sqrt{3} - 1)$$

$$\Rightarrow h = 14 \text{ mt} \quad \text{Approx.}$$

Ans. D

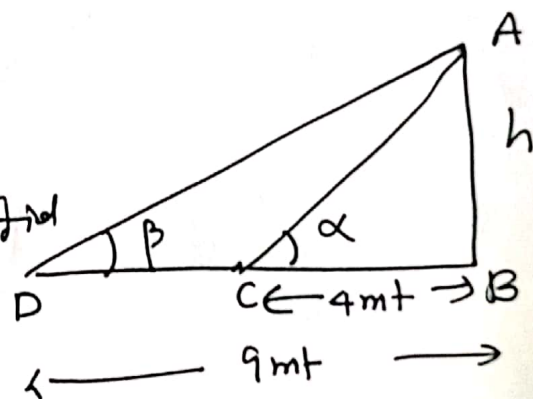
04 $\alpha + \beta = 90^\circ$

$$\Rightarrow \tan(\alpha + \beta) = \tan 90^\circ$$

$$\Rightarrow \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \text{not defined}$$

$$\Rightarrow \tan \alpha \cdot \tan \beta = 1$$

$$\Rightarrow \frac{h}{4} \cdot \frac{h}{9} = 1 \Rightarrow h^2 = 36 \Rightarrow h = 6 \text{ mt}$$



Ans: A

05 In the figure

AB = Tower

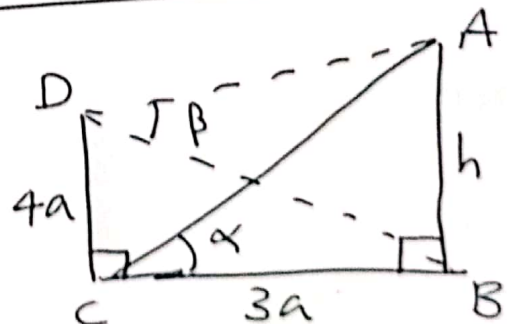
BC = First direction

CD = Second direction

Given $BC = 3a$ and $CD = 4a$

Now, In $\triangle BCD$

$$BD^2 = BC^2 + CD^2$$



$$\begin{aligned}
 &= (3a)^2 + (4a)^2 \\
 &= 9a^2 + 16a^2 \\
 &= 25a^2
 \end{aligned}$$

22

$$\therefore BD = 5a$$

Now, $\tan \alpha = \frac{h}{3a}$, $\tan \beta = \frac{h}{5a}$

$$\Rightarrow h = 5a \tan \beta \text{ also } h = 3a \tan \alpha$$

Ans: A, B

06 AB = mountain
Let BC = x km

$$\text{Now, } \tan 30^\circ = \frac{h}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow x = \sqrt{3}h$$

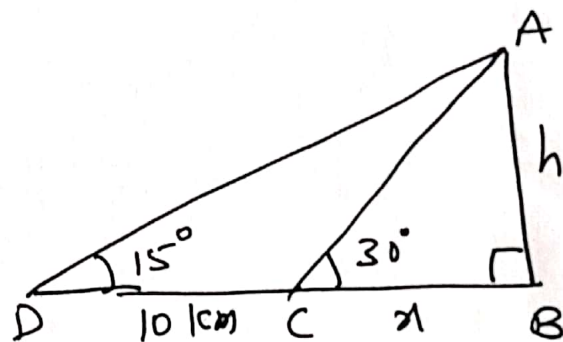
$$\tan 15^\circ = \frac{h}{10+x} \Rightarrow 2-\sqrt{3} = \frac{h}{10+x}$$

$$\Rightarrow 10+x = \frac{h}{2-\sqrt{3}}$$

$$\Rightarrow 10 + \sqrt{3}h = \frac{h}{2-\sqrt{3}}$$

$$\Rightarrow h = 5 \text{ mt}$$

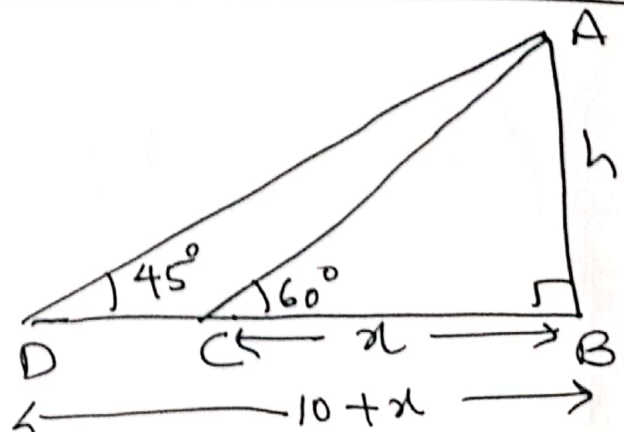
Ans: D



07 $\tan 60^\circ = \frac{h}{x}$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}}$$



$$\text{Also, } \tan 45^\circ = \frac{h}{10+x}$$

$$\Rightarrow 1 = \frac{h}{10+x}$$

$$\Rightarrow 10+x=h$$

$$\Rightarrow 10 + \frac{h}{\sqrt{3}} = h$$

$$h = \frac{10\sqrt{3}}{\sqrt{3}-1}$$

$$= 23.66 \text{ m}$$

(23)

Ans. C

08 From figure

$$\tan 45^\circ = \frac{h}{x}$$

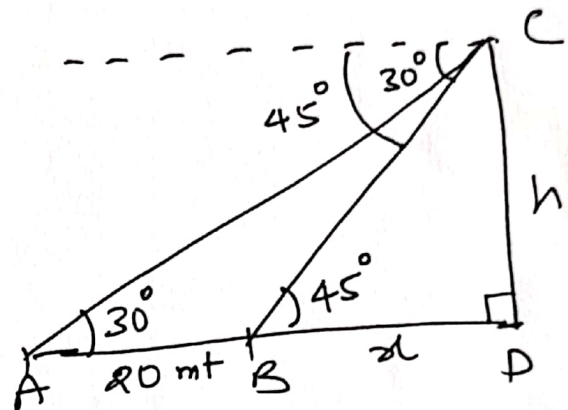
$$\Rightarrow 1 = \frac{h}{x}$$

$$\Rightarrow h = x \rightarrow (i)$$

$$\tan 30^\circ = \frac{h}{200+x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{200+h} \Rightarrow h = 100(\sqrt{3}+1) = 273.2 \text{ m}$$

Ans. B



09 AB, CD $\rightarrow$ two pillars of equal height

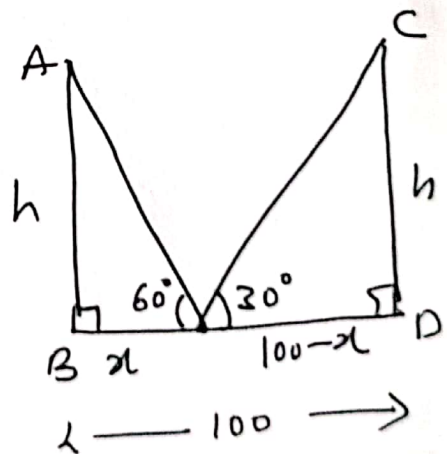
$$BD = 100 \text{ m}$$

$$\text{Now, } \tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \rightarrow (i)$$

$$\tan 30^\circ = \frac{h}{100-x}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100-x}$$

$$\Rightarrow 100-x = \sqrt{3}h$$

$$\Rightarrow 100 - \frac{h}{\sqrt{3}} = \sqrt{3}h \text{ (from (i))}$$

$$\Rightarrow 100\sqrt{3} - h = 3h$$

$$\Rightarrow h = 25\sqrt{3}$$

$$\Rightarrow h = 43.3 \text{ m}$$

(24)

Ans. A

10 AB $\rightarrow$ Tower

$$\tan \alpha = \frac{h}{x}$$

$$\Rightarrow x = h \cot \alpha \rightarrow (i)$$

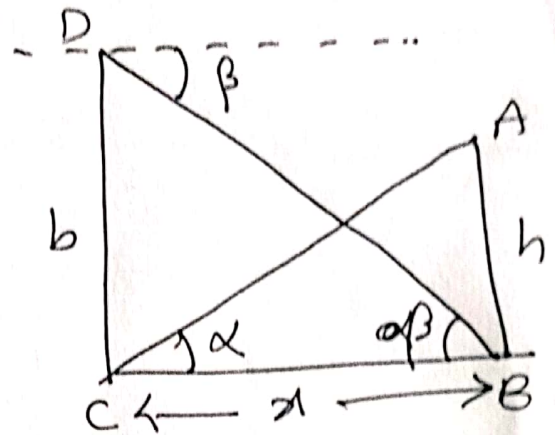
$$\Rightarrow \tan \beta = \frac{b}{x}$$

$$\Rightarrow x = b \cot \beta \rightarrow (ii)$$

$\therefore$ From (i) & (ii)

$$\Rightarrow h \cot \alpha = b \cot \beta$$

$$\Rightarrow h = b \cot \beta \tan \alpha$$



Ans. A

11 AB $\rightarrow$ Tower

CD $\rightarrow$ hill

$$\tan 30^\circ = \frac{50}{x}$$

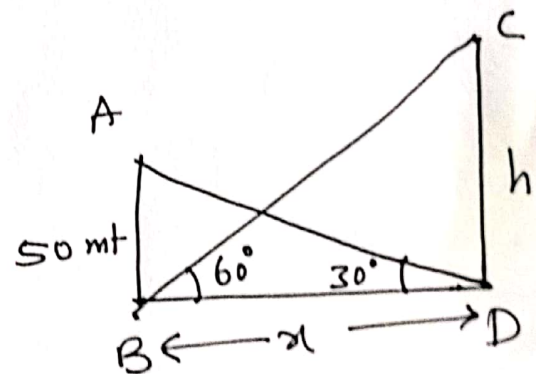
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{x}$$

$$\Rightarrow x = 50\sqrt{3} \rightarrow (i)$$

$$\tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{50\sqrt{3}} \quad (\because \text{from } i)$$

$$\Rightarrow h = 150 \text{ m}$$



Ans: C

JEE ADVANCED LEVEL QUESTIONS (25)

01 From the figure

$$\tan \alpha = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\tan \alpha} \rightarrow (i)$$

Again, $\tan \beta = \frac{h}{1-x}$

$$\Rightarrow 1-x = \frac{h}{\tan \beta}$$

$$\Rightarrow 1 - \frac{h}{\tan \alpha} = \frac{h}{\tan \beta}$$

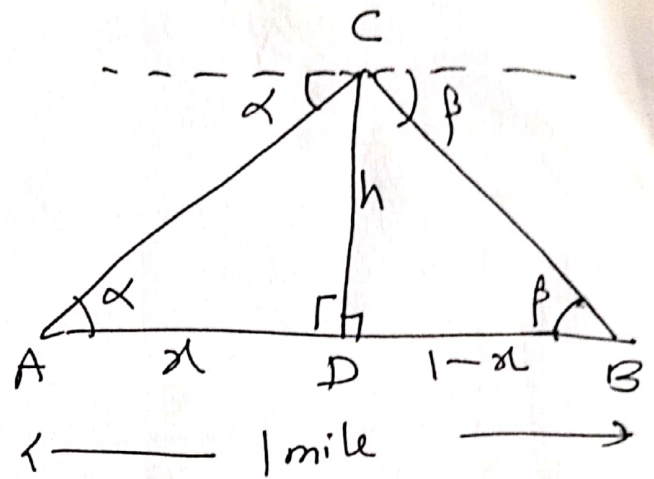
$$\Rightarrow \frac{h}{\tan \alpha} + \frac{h}{\tan \beta} = 1$$

$$\Rightarrow h \left(\frac{\tan \alpha + \tan \beta}{\tan \alpha \cdot \tan \beta} \right) = 1$$

$$\therefore h = \frac{\sin \alpha \cdot \sin \beta}{\sin \alpha \cos \beta + \cos \alpha \cdot \sin \beta} =$$

$$\frac{\sin \alpha \cdot \sin \beta}{\sin(\alpha + \beta)}$$

Ans: B, D



$$h = \frac{\tan \alpha \cdot \tan \beta}{\tan \alpha + \tan \beta}$$

(or)

$$h = \frac{\left(\frac{\sin \alpha}{\cos \alpha} \right) \left(\frac{\sin \beta}{\cos \beta} \right)}{\left(\frac{\sin \alpha}{\cos \alpha} \right) + \left(\frac{\sin \beta}{\cos \beta} \right)}$$

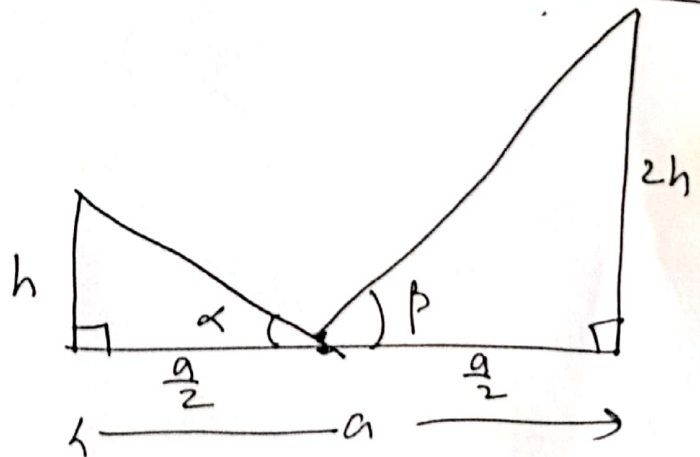
02 Given $\alpha + \beta = 90^\circ$

$$\Rightarrow \tan \alpha \cdot \tan \beta = 1$$

$$\Rightarrow \frac{h}{\left(\frac{a}{2} \right)} \cdot \frac{2h}{\left(\frac{a}{2} \right)} = 1$$

$$\Rightarrow 8h^2 = a^2$$

$$\Rightarrow h = \frac{a}{2\sqrt{2}}$$



∴ The height of the smaller pole - $\frac{a}{2\sqrt{2}}$ m (26)

The height of the larger pole - $2h = \frac{a}{\sqrt{2}}$ m

Ans. B, C

03

$$\tan 45^\circ = \frac{H+h}{d}$$

$$\Rightarrow 1 = \frac{H+h}{d}$$

$$\Rightarrow d = H+h \rightarrow (i)$$

Again, $\tan \theta = \frac{H-h}{d}$

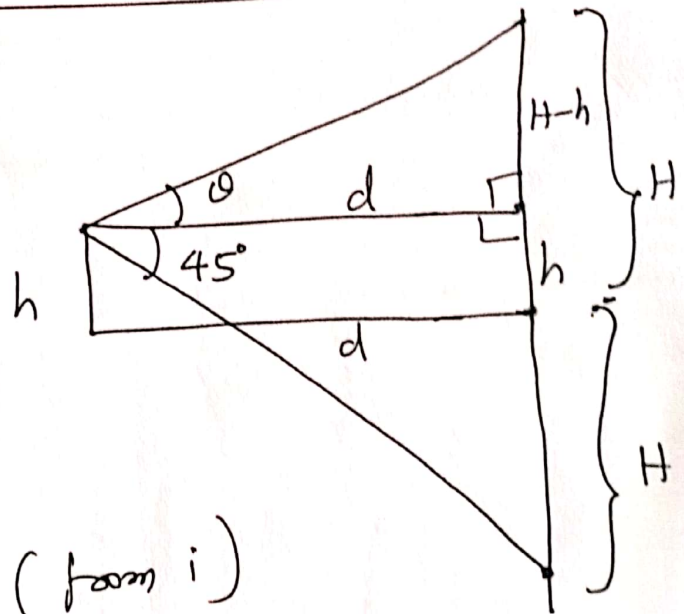
$$\Rightarrow \tan \theta = \frac{H-h}{H+h} \quad (\text{from i})$$

$$\Rightarrow H = h \left(\frac{1 + \tan \theta}{1 - \tan \theta} \right)$$

$$\Rightarrow = h \left(\frac{\cot \theta + 1}{\cot \theta - 1} \right) = h \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right)$$

$$\therefore H = h \cot(45^\circ - \theta)$$

Ans. B, D

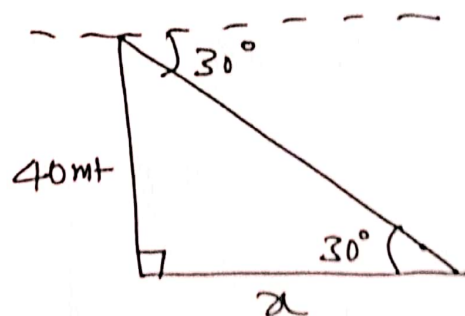


04. Statement I:

$$\tan 30^\circ = \frac{40}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{x}$$

$$\Rightarrow x = 40\sqrt{3} \text{ (true)}$$



Statement II: Conceptual (true)

Ans. A

05 Statement I: Conceptual (False) (21)
Statement II: Conceptual (False) Ans. B

06 Assertion: Conceptual (True)
Reason: Conceptual (True) Ans. A

07 Assertion: Conceptual (False) Ans. C
Reason: Conceptual (True)

08 We have $\frac{a}{b} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha}$

Given $\alpha + \beta = 105^\circ$

Let $\alpha = 60^\circ$ and $\beta = 45^\circ$

$$\therefore \frac{a}{b} = \frac{\cos 60^\circ - \cos 45^\circ}{\sin 45^\circ - \sin 60^\circ} = \frac{\left(\frac{1}{2}\right) - \frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right) - \frac{\sqrt{3}}{2}}$$

$$\Rightarrow \frac{a}{b} = \frac{\sqrt{2} - 1}{\sqrt{3} - \sqrt{2}}$$

Ans: A

09 Given $\frac{a}{b} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha} = \frac{-2 \sin \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right)}{2 \cos \left(\frac{\beta + \alpha}{2}\right) \sin \left(\frac{\beta - \alpha}{2}\right)}$

$$= \tan \left(\frac{\alpha + \beta}{2}\right)$$

$$= \tan \left(\frac{30^\circ}{2}\right)$$

$$= \tan 15^\circ$$

$$\therefore \frac{a}{b} = 2 - \sqrt{3}$$

$$\Rightarrow \frac{b}{a} = \frac{1}{2 - \sqrt{3}} = 2 + \sqrt{3}$$

$$\Rightarrow \frac{b}{a} + 1 = 2 + \sqrt{3} + 1$$

(28)

$$\Rightarrow a + b = a(3 + \sqrt{3})$$

Ans: B

10.

$$\frac{a}{b} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha}$$

$$\Rightarrow \frac{a}{b} = \tan\left(\frac{\alpha + \beta}{2}\right) = \tan\left(\frac{0^\circ}{2}\right) = 0$$

$$\Rightarrow a = 0$$

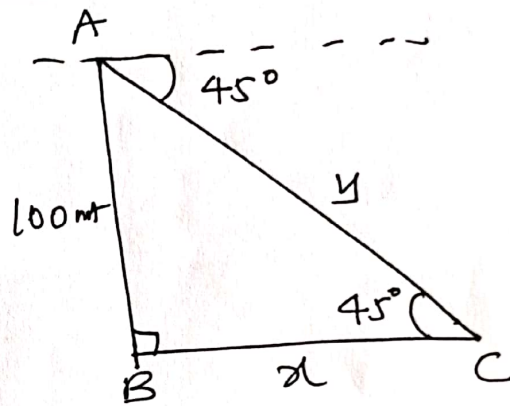
Ans: C

11

$$\tan 45^\circ = \frac{100}{x}$$

$$\Rightarrow 1 = \frac{100}{x}$$

$$\Rightarrow x = 100 \text{ mt}$$



Ans: B

12

$$\sin 45^\circ = \frac{100}{y}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{100}{y}$$

$$\begin{aligned} \Rightarrow y &= 100\sqrt{2} \\ &= 100(1.414) \\ &= 141.4 \text{ mt} \end{aligned}$$

Ans: B

13

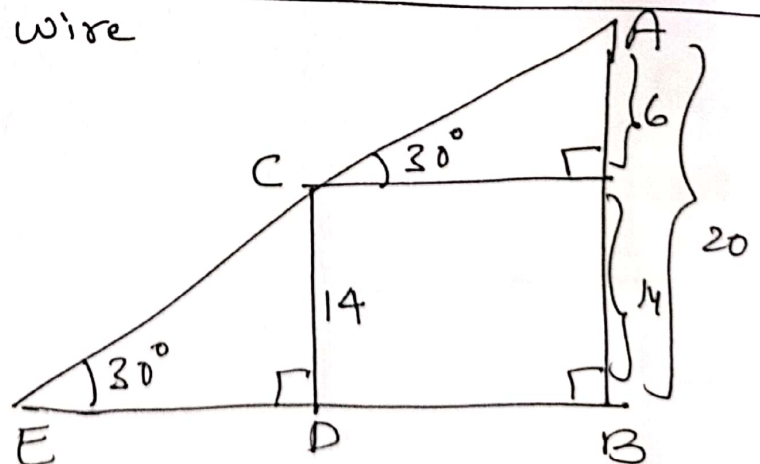
AC $\rightarrow$ length of the wire

AB, CD $\rightarrow$ poles

$$\sin 30^\circ = \frac{6}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{6}{AC}$$

$$\Rightarrow AC = 12$$



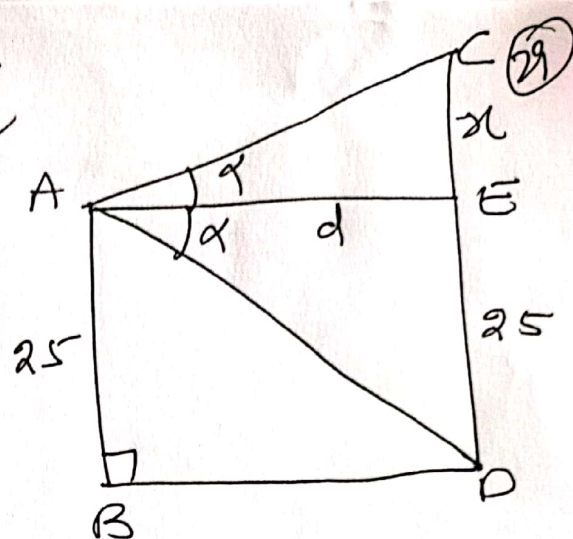
Ans: 12

14

AB → Height of the cliff
 CD → Height of the tower
 $\tan \alpha = \frac{25}{d} = \frac{x}{d}$

$$\Rightarrow x = 25$$

Height of the tower
 $= 25 + 25$
 $= 50 \text{ mt}$



Ans: 50

15 a) AB → tree
 BC → width of the river

$$\tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

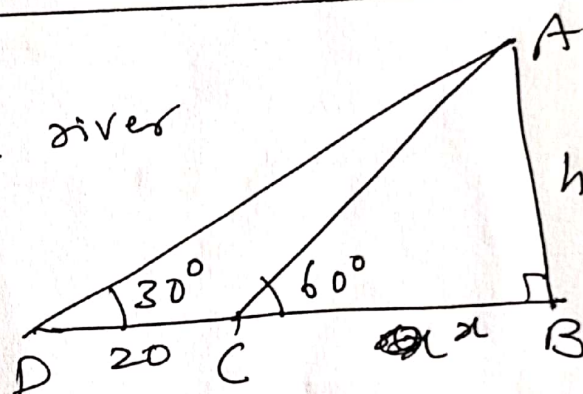
$$\Rightarrow x = \frac{h}{\sqrt{3}} \quad h = \sqrt{3}x$$

$$\text{Also, } \tan 30^\circ = \frac{h}{20+x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$$

$$\Rightarrow 20+x = \sqrt{3}h = \sqrt{3}(\sqrt{3}x) = 3x$$

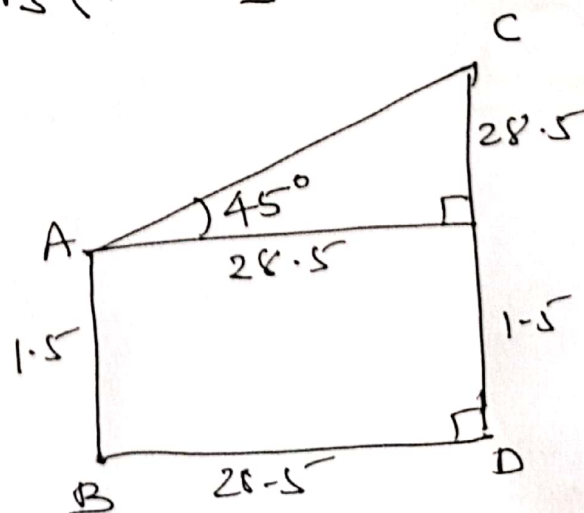
$$\Rightarrow x = 10 \text{ (s)}$$



b) AB → observer
 CD → tower

$$\tan 45^\circ = \frac{CE}{AE}$$

$$\Rightarrow 1 = \frac{CE}{28.5}$$



$$CE = 28.5$$

(30)

∴ The height of the observer = $28.5 + 1.5$
 $= 30 \text{ mt (P)}$

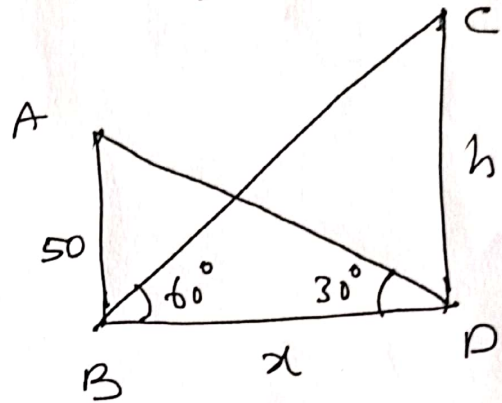
c) AB → Tower
 CD → hill

$$\tan 30^\circ = \frac{50}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{x}$$

$$\Rightarrow x = 50\sqrt{3}$$

$$\text{Again, } \tan 60^\circ = \frac{h}{x}$$



$$\Rightarrow \sqrt{3} = \frac{h}{50\sqrt{3}}$$

$$\Rightarrow h = 150 \text{ mt (x)}$$

d) AB → tower

$$\tan 45^\circ = \frac{100}{a}$$

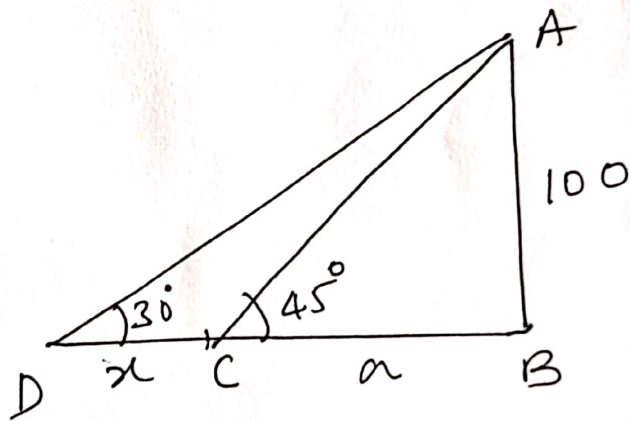
$$\Rightarrow a = 100$$

$$\tan 30^\circ = \frac{100}{x+a}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x+100}$$

$$x + 100 = 100\sqrt{3}$$

$$\Rightarrow x = 100(\sqrt{3} - 1) \text{ mt (x)}$$



Ans: S, P, x, —

16

a) Conceptual (x)

b) Conceptual (P)

c) Conceptual (q)

d) Conceptual (r, s)

Ans: x, P, q, (r, s)

ADDITIONAL PRACTICE QUESTIONS

31

Q.

~~Qm~~

$$\sin 60^\circ = \frac{x}{15-x}$$

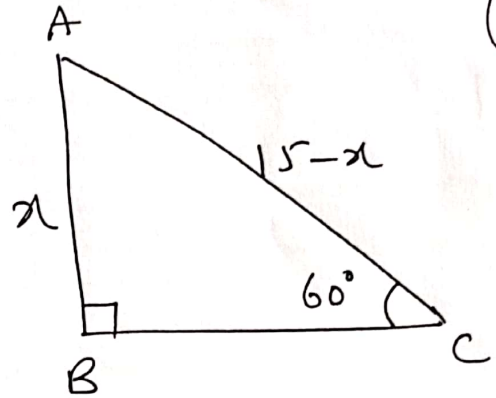
$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{x}{15-x}$$

$$\Rightarrow 15\sqrt{3} - \sqrt{3}x = 2x$$

$$\Rightarrow x(2 + \sqrt{3}) = 15\sqrt{3}$$

$$\Rightarrow x = \frac{15\sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = 15\sqrt{3}(2 - \sqrt{3})$$

Ans. B



02 Given $\alpha + \beta = 90^\circ$

$$\Rightarrow \tan \alpha \cdot \tan \beta = 1$$

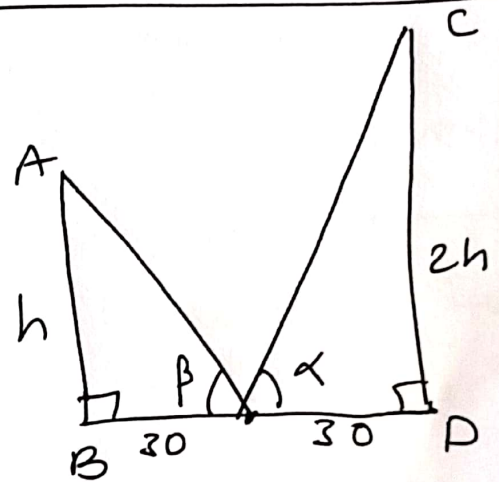
$$\Rightarrow \frac{2h}{30} \cdot \frac{h}{30} = 1$$

$$\Rightarrow h^2 = 450$$

$$\Rightarrow h = \sqrt{450}$$

$$\Rightarrow h = 21.21$$

$$\Rightarrow 2h = 42.42$$



Ans. A

03 PQ = width of the river

AB → tower

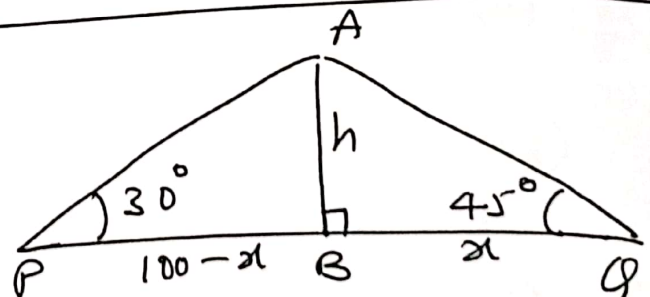
$$\tan 45^\circ = \frac{h}{x} \Rightarrow h = x$$

$$\tan 30^\circ = \frac{h}{100-x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100-h}$$

$$\Rightarrow 100 - h = \sqrt{3}h$$

$$\Rightarrow h(\sqrt{3} + 1) = 100$$



$$\Rightarrow h = \frac{100}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$h = 50(\sqrt{3} - 1) = 50 \times (0.732)$$

$$= 36.5 \text{ m}$$

$$\Rightarrow \text{tower height} \leftarrow$$

Ans. D