FOUNDATION

Class: IX, MATHEMATICS

4. LIMITS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Teaching Task (JEE MAINS)

01. $\lim_{x\to 0} \log\left|\frac{\log(1+x)}{x}\right|$

$= \lim_{x\to 0} \log\left|\frac{x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots}{x}\right|$

$= \lim_{x\to 0} \log\left|1-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+\cdots\right|$

$= \log(1) = 0$

Ans. A

02 $\lim_{x\to 0} \frac{e^x+\sin x-1}{\log(1+x)}$ $\left(\frac{0}{0}\right)$

By L-Hospital Rule

$= \lim_{x\to 0} \frac{e^x+\cos x}{\left(\frac{1}{1+x}\right)} = \frac{1+1}{1} = 2$

Ans. D

03 $\lim_{x\to 0} \frac{10^x-2^x-5^x+1}{x\tan x}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{2^x.5^x-2^x-5^x+1}{x\tan x}$

$= \lim_{x\to 0} \frac{(2^x-1)(5^x-1)}{x\tan x}$

$= \lim_{x\to 0} \frac{\left(\frac{2^x-1}{x}\right).\left(\frac{5^x-1}{x}\right)}{\left(\frac{\tan x}{x}\right)}$

$= \log 2 . \log 5$

Ans. C

04. $\lim_{x\to\infty} \frac{x-\log x}{x+\log x}$

$= \lim_{x\to\infty} \frac{1-\frac{\log x}{x}}{1+\frac{1}{x}} = \frac{1-0}{1+0} = 1$ Ans. A

05 $\lim_{x\to\infty} \frac{a^x - a^{-x}}{a^x + a^{-x}}$

$= \lim_{x\to\infty} \frac{\cancel{a^x}(1-a^{-2x})}{\cancel{a^x}(1+a^{-2x})}$

$= \lim_{x\to\infty} \frac{1-\frac{1}{a^{2x}}}{1+\frac{1}{a^{2x}}}$

$= \frac{1-0}{1+0} = 1$ Ans. A

06 $\lim_{x\to\infty} \frac{(x+1)^{10}+(x+2)^{10}+ \cdots +(x+100)^{10}}{x^{10}+10^{10}}$

$= \lim_{x\to\infty} \frac{\cancel{x^{10}}\left(\left(1+\frac{1}{10}\right)^{10}+\left(1+\frac{2}{10}\right)^{10}+\cdots+\left(1+\frac{100}{x}\right)^{10}\right]}{\cancel{x^{10}}\left(1+\frac{10^{10}}{x^{10}}\right)}$

$= \frac{(1+0)+(1+0)+\cdots+(1+0)}{(1+0)}$ (110 times)

$= 1\times 100 = 100$ Ans: B

07. $\lim_{n\to\infty} \frac{3\cdot 2^{n+1} - 4\cdot 5^{n+1}}{5\cdot 2^n + 7\cdot 5^n}$

$= \lim_{n\to\infty} \frac{3\cdot 2^n\cdot 2^1 - 4\cdot 5^n\cdot 5^1}{5\cdot 2^n + 7\cdot 5^n}$

$= \lim_{n\to\infty} \frac{6\cdot 2^n - 20\cdot 5^n}{5\cdot 2^n + 7\cdot 5^n}$

$= \lim_{n\to\infty} \frac{5^n\left[6\left(\frac{2}{5}\right)^n - 20\right]}{5^n\left[5\left(\frac{2}{3}\right)^n + 7\right]}$

$= \frac{6\cdot 0 - 20}{5\cdot 0 + 7}$

$= \frac{-20}{7}$ Ans. A

08 $$\lim_{x\to\infty} x\left[\log(1+x)-\log x\right]$$

$$=\lim_{x\to\infty} x\left(\log\left(\frac{1+x}{x}\right)\right)$$

$$=\lim_{x\to\infty} \frac{\log\left(1+\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

let $\frac{1}{x}=y$

$x\to\infty,\ y\to 0$

$$\lim_{y\to 0}\frac{\log(1+y)}{y}=1$$

Ans: C

09. $$\lim_{x\to\infty}\frac{a^{1/x}-b^{1/x}}{\left(\frac{1}{x}\right)}$$

let $\frac{1}{x}=y \Rightarrow x\to\infty,\ \frac{1}{x}=y\to 0$

$$\lim_{y\to 0}\frac{a^y-b^y}{y}=\log_e\left(\frac{a}{b}\right)$$

Ans: B

10 $$\lim_{x\to 1}\left(\log_3 3x\right)^{\log_x 3} \quad (1^\infty)$$

$$= e^{\lim_{x\to 1}\log_x 3\left(\log_3 3x-1\right)}$$

$$= e^{\lim_{x\to 1}\log_x 3\left(\log_3 3+\log_3 x-1\right)}$$

$$= e^{\lim_{x\to 1}\log_x 3\left(\cancel{1}+\log_3 x\cancel{-1}\right)}$$

$$= e^{\lim_{x\to 1}\log_x 3\cdot\log_3 x}$$

$$= e^{1}$$

$$= e$$

$$= e$$

NOTE

$$\lim_{x\to a}\left[f(x)\right]^{g(x)}=1^\infty$$

$$\Rightarrow e^{\lim_{x\to 0} g(x)\left[f(x)-1\right]}$$

Ans: D

011. $\lim_{x \to 0} \dfrac{\log(1+x)}{3^x - 1}$

$= \lim_{x \to 0} \dfrac{\left(\frac{1}{1+x}\right)}{3^x \cdot \log 3}$

$= \dfrac{1}{3^0 \cdot \log 3} = \dfrac{1}{\log_e 3} = \log_3 e$

Ans: D

12 $\lim_{x \to 0} \dfrac{e^{\alpha x} - e^{\beta x}}{\sin\alpha x - \sin\beta x}$ $\left(\frac{0}{0}\right)$

$= \lim_{x \to 0} \dfrac{e^{\alpha x} \cdot \alpha - e^{\beta x} \cdot \beta}{\cos\alpha x \cdot \alpha - \cos\beta x \cdot \beta} = \dfrac{\alpha - \beta}{\alpha - \beta} = 1$

A) 1 ✓ B) $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$ ✓ C) $\lim_{x \to 0} \cos x = 1$ ✓

Ans: A, B, C

13 ~~statement I~~ $\cancel{\lim_{x \to 0} \dfrac{1 - \cos x}{\log(1+x) \cdot x}}$ $\left(\frac{0}{0}\right)$

$\cancel{= \lim_{x \to 0} \dfrac{\sin x}{\left(\frac{1}{1+x}\right)} = \dfrac{\sin 0}{\left(\frac{1}{1+0}\right)} = 0}$

13. Statement I: $\lim_{x \to 0} \dfrac{1 - \cos x}{x \cdot \log(1+x)}$

$= \lim_{x \to 0} \dfrac{2\sin^2\left(\frac{x}{2}\right)}{x \cdot \log(1+x)}$

$= \lim_{x \to 0} \dfrac{\frac{2\sin^2\left(\frac{x}{2}\right)}{x^2}}{\frac{\log(1+x)}{x}}$

$= 2 \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{2}$ (True)

Statement II: Conceptual (True)

Ans. A

14. Statement I: $\lim_{x\to 0} \frac{e^{2x}-1}{3x}$

$= \frac{2}{3} \cdot \lim_{x\to 0} \frac{e^{2x}-1}{2x} = \frac{2}{3}\times 1 = \frac{2}{3}$

(True)

Ans..A

Statement II: Conceptual (True)

15 Assertion: $\lim_{x\to 0} \frac{e^{x}+e^{-x}-2}{x^2}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{e^{x}-e^{-x}}{2x}$

$= \lim_{x\to 0} \frac{e^{x}+e^{-x}}{2} = \frac{1+1}{2} = 1$ (True)

Ans: A

Reason: Conceptual (True)

16. Assertion: $\lim_{x\to 0} \frac{e^{x}-1-x}{x^2}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{e^{x}-1}{2x} = \frac{1}{2}$ (true)

Ans: A

Reason: Conceptual (True)

17. $\lim_{x\to 0} \frac{x\cdot 2^{3x}-x}{1-\cos(3x)} = \frac{2a}{m^2}\cdot \log a$

$= \frac{2\cdot 3}{3^2}\cdot \log 2 = \frac{2}{3}\log 2$

Ans..A

18 $\lim_{x\to 0} \frac{x\cdot 2^{4x}-x}{1-\cos 4x} = \frac{2\cdot 4}{4^2}\cdot \log 2 = \frac{1}{2}\log 2$

$= \log\sqrt{2}$ Ans. D

⑥

19. $\lim_{x \to 0} \dfrac{e^{\sin x}-1}{\sin x}$ here $\lim_{x\to 0} \sin x = 0$

$= \log_e(\cancel{1+x}) = \log_e 0 \rightarrow$ not defined

Ans: D

20 $\lim_{x \to \frac{\pi}{2}} \dfrac{e^{\cos x}-1}{\cos x}$ here $\lim_{x\to \frac{\pi}{2}} \cos x = 0$

$= \log_e\left(\frac{\pi}{2}\right)$

Ans: B

21. $\lim_{x \to 0} \dfrac{e^{3x}-1-3x}{x^2}$ $\left(\frac{0}{0}\right)$

$= \lim_{x \to 0} \dfrac{e^{3x}.3-3}{2x}$

$= \dfrac{3}{2} . \lim_{x \to 0} \dfrac{e^{3x}-1}{x}$

$= \dfrac{3}{2} \times 3$

$= \dfrac{9}{2} = 4.5$

Integral value $= 4$

Ans: 4

22 $\lim_{x \to 0} \dfrac{e^{2x}+e^{-2x}-2}{x^2}$

$= \lim_{x \to 0} \dfrac{e^{2x}.2+e^{-2x}.(-2)}{2x}$

$= \lim_{x \to 0} \dfrac{e^{2x}-e^{-2x}}{x}$

$= \lim_{x \to 0} \dfrac{e^{2x}.2+e^{-2x}.2}{1}$

$= 2+2$

$= 4$

Ans. 4

23 a) $\lim_{x \to 0} \dfrac{e^{x}-1}{x} = 1$ (s)

b) $\log_b a$ (r)

b) e (q)

d) $\log_e a$ (p)

Ans: s, r, q, p

24 a) 2 (e) c) $\log_e a^2$ (p)

b) e^2 (t) d) 1 (q) Ans: e, t, p, q

LEARNERS TASK (CUQ)

01 Conceptual Ans. A

02 $\lim_{x\to a} \frac{\log[1+(x-a)]}{(x-a)}$

Let $x-a=y \Rightarrow x=a+y$

As $x\to a$, $y\to 0$

$\therefore \lim_{y\to 0} \frac{\log(1+y)}{y}=1$ Ans. B

03 $\lim_{h\to 0} \frac{\log_{10}(1+h)}{h}$

$= \lim_{h\to 0} \frac{\log_e(1+h) \times \log_{10} e}{h} = 1\times \log_{10} e$ Ans. D

04 Conceptual Ans. A

05 $\lim_{x\to 0} \frac{a^x.b^x - a^x - b^x + 1}{x^2}$

$= \lim_{x\to 0} \frac{(a^x-1)(b^x-1)}{x^2}$

$= \lim_{x\to 0} \frac{a^x-1}{x} . \lim_{x\to 0} \frac{b^x-1}{x}$

$= \log_e a . \log_e b$

Ans. C

06 Conceptual Ans. A

07 Conceptual Ans B

08 Conceptual Ans. D

09. e^2 Ans. D

10. $\lim_{x\to 0} \dfrac{\log(1+ax) - \log(1+bx)}{x}$

$= \lim_{x\to 0} \dfrac{\frac{1}{1+ax}\cdot a - \frac{1}{1+bx}\cdot b}{1} = a-b$ Ans. B

JEEMAINS LEVEL

01. $\lim_{x\to 0} \dfrac{4^x - 9^x}{x(4^x+9^x)}$

$= \lim_{x\to 0} \dfrac{(4^x-1)-(9^x-1)}{x} \cdot \lim_{x\to 0} \dfrac{1}{4^x+9^x}$

$= \log\left(\frac{4}{9}\right) \times \frac{1}{2} = \log\sqrt{\frac{4}{9}} = \log\left(\frac{2}{3}\right)$ Ans: B

02. $\lim_{x\to\infty} 5^x \cdot \sin\left(\frac{a}{5^x}\right)$

$= \lim_{x\to\infty} \dfrac{\sin\left(\frac{a}{5^x}\right)}{\left(\frac{1}{5^x}\right)}$

Let $\frac{1}{5^x} = y$

As $x\to\infty$, $y\to 0$

$\therefore \lim_{y\to 0} \dfrac{\sin ay}{y} = a$

Ans. B

03. $\lim_{n\to\infty} \dfrac{2^{\frac{1}{n}}-1}{2^{1/n}+1}$

Let $\frac{1}{n} = y$

As $n\to\infty$, $y\to 0$

$\lim_{y\to 0} \dfrac{2^y-1}{2^y+1} = \dfrac{2^0-1}{2^0+1} = \dfrac{0}{2} = 0$

Ans. D

(9)

04 $\lim\limits_{x \to 0} \dfrac{\log(1+x) - x}{x^2} \quad \left(\frac{0}{0}\right)$

$= \lim\limits_{x \to 0} \dfrac{\frac{1}{1+x} - 1}{2x} \quad \left(\frac{0}{0}\right)$

$= \lim\limits_{x \to 0} \dfrac{\frac{-1}{(1+x)^2}}{2} = \frac{-1}{2}$

Ans.. B

05 $\lim\limits_{x \to a} \dfrac{\log(x-a)}{\log(e^x - e^a)}$

$= \lim\limits_{x \to 0} \dfrac{\frac{1}{x-a}}{\frac{1}{e^x - e^a}(e^x - 0)}$

$= \lim\limits_{x \to 0} \dfrac{e^x - e^a}{e^x(x-a)}$

$= \lim\limits_{x \to 0} \dfrac{e^a(e^{x-a} - 1)}{e^x (x-a)}$

$= \dfrac{e^a}{e^a} \times 1 = 1$

Ans: A

06 $\lim\limits_{x \to \infty} \dfrac{a^{\sqrt{x}} - a^{\frac{1}{\sqrt{x}}}}{a^{\sqrt{x}} + a^{1/\sqrt{x}}}$

$= \lim\limits_{x \to \infty} \dfrac{a^{\sqrt{x}}\left(1 - a^{\frac{1}{\sqrt{x}} - \sqrt{x}}\right)}{a^{\sqrt{x}}\left(1 + a^{\frac{1}{\sqrt{x}} - \sqrt{x}}\right)}$

$= \lim\limits_{x \to \infty} \dfrac{1 - a^{-\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)}}{1 + a^{-\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)}}$

We know $x \to \infty$, $\sqrt{x} - \frac{1}{\sqrt{x}} \to \infty$

$= \dfrac{1-0}{1+0}$

$= 1$

Ans. A

07. $$\lim_{x\to 0} \frac{2^x - 1}{(1+x)^{1/2} - 1} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{2^x . \log 2}{\frac{1}{2}(1+x)^{-\frac{1}{2}}} = 2.\log 2 = \log 4$$

Ans.. B

08 $$\lim_{x\to 0} \frac{10^x - 2^x - 5^x + 1}{x^2}$$

$$= \lim_{x\to 0} \frac{2^x . 5^x - 2^x - 5^x + 1}{x^2}$$

$$= \lim_{x\to 0} \frac{(5^x - 1)}{x} \frac{(2^x - 1)}{x} = (\log 5).(\log 2)$$

Ans. C

09. $$\lim_{x\to 0} \frac{(729)^x - (293)^x - (81)^x + 9^x + 3^x - 1}{x^3}$$

$$= \lim_{x\to 0} \frac{3^{6x} - 3^{5x} - 3^{4x} + 3^{2x} + 3^x - 1}{x^3}$$

$$= \lim_{x\to 0} \frac{3^{5x}(3^x - 1) - 3^{2x}(3^{2x} - 1) + 1(3^x - 1)}{x^3}$$

$$\boxed{\cancel{= \lim_{x\to 0} (3^x - 1)\left[3^{5x} - 3^{2x}\right]}}$$

$$= \lim_{x\to 0} \frac{3^{5x}(3^x - 1) - 3^{2x}(3^x + 1)(3^x - 1) + 1(3^x - 1)}{x^3}$$

$$= \lim_{x\to 0} \frac{(3^x - 1)\left[3^{5x} - 3^{2x}(3^x + 1) + 1\right]}{x^3}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)\left[3^{5x} - 3^{3x} - 3^{2x} + 1\right]}{x^3}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)\left[3^{3x}(3^{2x} - 1) - (3^{2x} - 1)\right]}{x^3}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)(3^{2x} - 1)(3^{3x} - 1)}{x^3}$$

$$= \lim_{x \to 0} \frac{3^x - 1}{x} \cdot \lim_{x \to 0} \frac{3^{2x} - 1}{x} \lim_{x \to 0} \frac{3^{3x} - 1}{x}$$

$= \log 3 \times$ [illegible] $\cdot \log 3 \times 2 \cdot \log 3 \times 3$

$= 6(\log 3)^3$

Ans: B

---

10. $\lim_{x \to 0} \dfrac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$

$$= \lim_{x \to 0} \frac{a^{\sin x}\left(a^{\tan x - \sin x} - 1\right)}{(\tan x - \sin x)}$$

$= a^{\sin 0} \cdot \log a$

$= \log a$

Ans.. A

JEE ADVANCED

---

1) $\lim_{x \to 0} \dfrac{e^{x^2} - \cos x}{x^2}$

$$= \lim_{x \to 0} \frac{(e^{x^2} - 1) - (\cos x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$= 1 + \lim_{x \to 0} \dfrac{\sin x}{2x}$

$= 1 + \frac{1}{2}$

$= \frac{3}{2}$

Also, A, B are also correct

Ans: A, B, C

12. $\lim_{x \to \infty} 2^{-x} \cdot \sin(2^x)$

$= \lim_{x \to \infty} \frac{\sin(2^x)}{2^x}$

We know, $-1 \le \sin(2^x) \le 1$

$-\frac{1}{2^x} \le \frac{\sin(2^x)}{2^x} \le \frac{1}{2^x}$

$\therefore \lim_{x \to \infty} \left(-\frac{1}{2^x}\right) = \lim_{x \to \infty} \left(\frac{1}{2^x}\right) = 0$

$\therefore \lim_{x \to \infty} 2^{-x} \cdot \sin(2^x) = 0$

Also $\lim_{x \to 0} \sin x = \sin 0 = 0$ Ans: A, B

---

13. Statement I: Conceptual (True)

Statement II: Conceptual (True) Ans: A

---

14. Statement I: $\lim_{x \to 1} \frac{x^3 - x^2 \log x + \log x - 1}{x^2 - 1}$

$= \lim_{x \to 1} \frac{(x^3 - 1) - \log x (x^2 - 1)}{x^2 - 1}$

$= \lim_{x \to 1} \frac{(x-1)(x^2+x+1) - \log(x) \cdot (x+1)(x-1)}{(x+1)(x-1)}$

$= \lim_{x \to 1} \frac{(x^2+x+1) - \log x \cdot (x+1)}{x+1}$

$= \frac{3}{2}$ (True)

Statement I: $\lim_{x \to e} \log_e x = \log_e e = 1$ (False)

Ans: C

15. Assertion: 3 (True)
Reason: Conceptual (True) Ans.. A

16. Assertion: 2 (True)
Reason: Conceptual (True) Ans.. A

17. $\lim_{x \to 0} \frac{e^{3x}-1}{x} = 3$ Ans: C

18. $\lim_{x \to 0} \frac{e^{\frac{x}{2}}-1}{x} = \frac{1}{2}$ Ans.. A

19. $\log_e\left(\frac{4}{2}\right) = \log_e 2$ Ans: B

20. $\log_e\left(\frac{2}{3}\right)$ Ans: A

21. 5 Ans: 5

22. $\lim_{x \to 0} \frac{e^{2x}-e^{x}}{x} \quad \left(\frac{0}{0}\right)$

$= \lim_{x \to 0} \frac{e^{2x}\cdot 2 - e^{x}}{1} = 2-1=1$ Ans: 1

23. a) e (p)

b) $\left(\frac{4}{2}\right)^3 = 2^3 = 8$ (e)

c) $\lim_{x \to \infty} \frac{\log_e x}{x} = \lim_{x \to \infty} \frac{\left(\frac{1}{x}\right)}{1}$

$= 0$ (r)

d) $\lim_{x \to 0} \frac{\sin x + \log(1-x)}{x^2}$

$= \lim_{x \to 0} \frac{\cos x - \frac{1}{1-x}(-1)}{2x}$

d) $= \lim_{x \to 0} \frac{-\sin x - \frac{1}{(1-x)^2}}{2}$

$= -\frac{1}{2}$ (s)

Ans: ~~p~~, e, r, s

24 a) $\underset{x\to 0}{\mathrm{Lt}} \dfrac{e^x - 1 - x}{x^2}$

$= \underset{x\to 0}{\mathrm{Lt}} \dfrac{e^x - 1}{2x} = \dfrac{1}{2}$ (e)

b) $\underset{x\to 0}{\mathrm{Lt}} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$ $\left(\frac{0}{0}\right)$

$= \underset{x\to 0}{\mathrm{Lt}} \dfrac{e^x + e^{-x} - 2}{1 - \cos x}$ $\left(\frac{0}{0}\right)$

$= \underset{x\to 0}{\mathrm{Lt}} \dfrac{e^x - e^{-x}}{\sin x}$ $\left(\frac{0}{0}\right)$

$= \underset{x\to 0}{\mathrm{Lt}} \dfrac{e^x + e^{-x}}{\cos x} = \dfrac{1+1}{1} = 2$ (t)

c) $\underset{x\to \infty}{\mathrm{Lt}} 2^{x-1} \cdot \mathrm{Tan}\left(\dfrac{a}{2^x}\right)$

$= \underset{x\to \infty}{\mathrm{Lt}} \dfrac{2^x}{2} \cdot \mathrm{Tan}\left(\dfrac{a}{2^x}\right)$

$= \dfrac{1}{2} \underset{x\to \infty}{\mathrm{Lt}} \dfrac{\mathrm{Tan}\left(\frac{a}{2^x}\right)}{\left(\frac{1}{2^x}\right)} = \dfrac{a}{2} \times 1 = \dfrac{a}{2}$ (s)

An: e, t, s, r

d) $\underset{x\to 0}{\mathrm{Lt}} e^x = e^0 = 1$ (r)

$\Longrightarrow$ THE END $\Longleftarrow$