

Dot ProductExample

①

Given $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + \hat{j} - 2\hat{k}$

$$\therefore \vec{A} \cdot \vec{B} = (\hat{i} + 2\hat{j} - \hat{k}) \cdot (-\hat{i} + \hat{j} - 2\hat{k})$$

$$= (1 \times (-1)) + 2(1) + (-1)(-2)$$

$$= -1 + 2 + 2 = 3$$

$$|\vec{A}| = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6} \quad ; \quad |\vec{B}| = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$$

$$\therefore \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{3}{\sqrt{6} \sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$$\theta = 60^\circ$$

②

We know that $\vec{a} \cdot \vec{b} = ab \cos \theta$ where θ is angle between $\vec{a}$ and $\vec{b}$

From given right angle triangle $\theta = 90^\circ$

$$\therefore \vec{a} \cdot \vec{b} = ab \cos 90 = \underline{0}$$

③

let the given vector be $\vec{A} = \hat{i} + 2\hat{j}$.

let $\vec{r}$ is a vector whose magnitude is $3\sqrt{5}$

$$\vec{r} = x\hat{i} + y\hat{j}$$

if $\vec{A}$ and $\vec{r}$ are perpendicular then $\vec{A} \cdot \vec{r} = 0$

$$\Rightarrow (\hat{i} + 2\hat{j}) \cdot (x\hat{i} + y\hat{j}) = 0$$

$$\Rightarrow x + 2y = 0 \rightarrow \textcircled{1} \Rightarrow$$

$$\therefore |\vec{r}| = \sqrt{x^2 + y^2} = 3\sqrt{5}$$

By doing squaring on both sides

$$(\sqrt{x^2 + y^2})^2 = (3\sqrt{5})^2$$

$$\Rightarrow x^2 + y^2 = 9 \times 5 \Rightarrow x^2 + y^2 = 45 \rightarrow \textcircled{2}$$

$$\text{From } \textcircled{1} \quad x = -2y \quad \therefore (-2y)^2 + y^2 = 45$$

$$\Rightarrow 5y^2 = 45 \Rightarrow y = 3.$$

$$\text{Sub } y \text{ in } x = -2y \Rightarrow x = -2 \times 3 = -6$$

$$\therefore \vec{r} = -6\hat{i} + 3\hat{j} \text{ is the vector perpendicular to } \vec{A}$$

option B correct

④

Given $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 2\hat{j} + 3\hat{k}$

$$\vec{A} \cdot \vec{B} = (2\hat{i} + 3\hat{j}) \cdot (2\hat{j} + 3\hat{k})$$

$$\Rightarrow (2 \times 3) = 6.$$

$$|\vec{A}| = \sqrt{2^2 + 3^2} \\ = \sqrt{4 + 9} = \sqrt{13}.$$

$$\therefore \text{Component of } \vec{B} \text{ along } \vec{A} = \frac{(\vec{A} \cdot \vec{B})}{|\vec{A}|}$$

$$= \frac{6}{\sqrt{13}}$$

⑤

$$\vec{A} = a\hat{i} + a\hat{j} + 3\hat{k} \quad ; \quad \vec{B} = a\hat{i} - 2\hat{j} - \hat{k}$$

Given $\vec{A}$ and $\vec{B}$ are perpendicular to each other

$$\therefore \vec{A} \cdot \vec{B} = 0 \Rightarrow (a\hat{i} + a\hat{j} + 3\hat{k}) \cdot (a\hat{i} - 2\hat{j} - \hat{k}) = 0$$

$$\Rightarrow a^2 - 2a - 3 = 0$$

$$\Rightarrow a^2 - 3a + a - 3 = 0$$

$$\Rightarrow a(a-3) + 1(a-3) = 0$$

$$\Rightarrow (a-3)(a+1) = 0 \quad \text{ie } a = 3 \text{ or } -1$$

In question asking only +ve value of a ie 3.

⑥

Given $\vec{A} = 5\hat{i} - 2\hat{j} + 3\hat{k}$; $\vec{B} = 2\hat{i} + \hat{j} + 2\hat{k}$

$$|\vec{A}| = \sqrt{5^2 + (-2)^2 + 3^2}$$

$$= \sqrt{25 + 4 + 9}$$

$$= \sqrt{38}$$

or $\vec{A} \cdot \vec{B} = (5\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} + \hat{j} + 2\hat{k})$

$$= 5 \times 2 + (-2) \times 1 + 3 \times 2$$

$$= 10 - 2 + 6 = 14$$

$\therefore$ component of $\vec{B}$ along $\vec{A} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|} = \frac{14}{\sqrt{38}}$

⑦

Given $\vec{A} = a\hat{i} + \hat{j} - 2\hat{k}$; $\vec{B} = a\hat{i} - a\hat{j} + \hat{k}$

$\vec{A}$ and $\vec{B}$ are perpendicular to each other.

$\therefore \vec{A} \cdot \vec{B} = 0 \Rightarrow (a\hat{i} + \hat{j} - 2\hat{k}) \cdot (a\hat{i} - a\hat{j} + \hat{k}) = 0$

$$\Rightarrow a^2 - a - 2 = 0$$

$$\Rightarrow a^2 - 2a + a - 2 = 0$$

$$\Rightarrow a(a-2) + 1(a-2) = 0$$

$$\Rightarrow (a-2)(a+1) = 0 \Rightarrow a = 2 \text{ (or) } -1$$

only +ve value of a is 2.

⑧

Given $\vec{F} = 8\hat{i} + 4\hat{j}$; $\vec{S} = 3\hat{i} - 3\hat{j}$; $P = 0.6 \text{ W}$; $t = ?$

From def of $P = \frac{W}{t}$

$$\Rightarrow P = \frac{\vec{F} \cdot \vec{S}}{t}$$

$$\Rightarrow 0.6 = \frac{(8\hat{i} + 4\hat{j}) \cdot (3\hat{i} - 3\hat{j})}{t}$$

$$\Rightarrow 0.6 = \frac{24 - 12}{t}$$

$$\Rightarrow t = \frac{12}{0.6}$$

$$= \frac{120}{6}$$

$$= 20 \text{ sec}$$



(9)

Given $\vec{a}$ and $\vec{b}$ are unit vectors

$$\therefore |\vec{a}| = |\vec{b}| = 1 \quad ; \quad \theta = 60^\circ$$

$$\frac{1 + \vec{a} \cdot \vec{b}}{1 - \vec{a} \cdot \vec{b}} = \frac{1 + ab \cos 60}{1 - ab \cos 60} = \frac{1 + 1 \times 1 \times \cos 60}{1 - 1 \times 1 \times \cos 60} = \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$$

(10)

Given $\vec{A} + \vec{B} = \vec{C}$

$\therefore$ By doing dot Product with $\vec{B}$ on both sides

$$\Rightarrow (\vec{A} + \vec{B}) \cdot \vec{B} = \vec{C} \cdot \vec{B}$$

$$\Rightarrow \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{B} = \vec{C} \cdot \vec{B}$$

$$\Rightarrow \vec{A} \cdot \vec{B} + B^2 = \vec{C} \cdot \vec{B}$$

$$\Rightarrow B^2 = \vec{C} \cdot \vec{B} - \vec{A} \cdot \vec{B} \Rightarrow B = \sqrt{\vec{C} \cdot \vec{B} - \vec{A} \cdot \vec{B}}$$

Advanced

(11)

Given $\vec{a} = m\vec{b} + \vec{c}$

By doing dot Product with $\vec{b}$ on both sides we get

$$\Rightarrow \vec{a} \cdot \vec{b} = (m\vec{b} + \vec{c}) \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = m \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b} = m b^2$$

$$\Rightarrow m = \frac{\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b}}{b^2}$$

②

$$\vec{v} = 2\hat{i} + c\hat{j} \quad ; \quad \alpha = 3\hat{i} + 4\hat{j} \quad \text{Given } \vec{v} \perp \alpha$$

$$\therefore \vec{v} \cdot \alpha = 0$$

$$\Rightarrow (2\hat{i} + c\hat{j}) \cdot (3\hat{i} + 4\hat{j}) = 0$$

$$\Rightarrow 2 \times 3 + c \times 4 = 0 \Rightarrow 4c = -6 \Rightarrow c = \frac{-6}{4} = -\frac{3}{2}$$

$$\Rightarrow c = -1.5$$

③

⑤ Given $\alpha = m b$ By doing dot Product on both

side $\alpha \cdot b = m b \cdot b$

$$\Rightarrow \alpha \cdot b = m b^2 \Rightarrow m = \frac{\alpha \cdot b}{b^2}$$

④

Given

$$\vec{A} = 5\hat{i} + 6\hat{j} + 3\hat{k} \quad \text{and} \quad \vec{B} = 6\hat{i} - 2\hat{j} - 6\hat{k}$$

If $\vec{A}$ and $\vec{B}$ are perpendicular.

$$\therefore \vec{A} \cdot \vec{B} = (5\hat{i} + 6\hat{j} + 3\hat{k}) \cdot (6\hat{i} - 2\hat{j} - 6\hat{k})$$

$$= (5 \times 6 + 6 \times (-2) + 3 \times (-6))$$

$$= 30 - 12 - 18 = 0$$

$\therefore \vec{A}$ & $\vec{B}$ are perpendicular to each other.

Since dot Product obeys commutative law $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

⑥

Given $\theta = 60^\circ$

$$\vec{a} \cdot \vec{b} = ab \cos \theta = ab \cos 60 = \frac{ab}{2}$$

⑦

Component of $\vec{b}$ along $\vec{a}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{ab \cos \theta}{|\vec{a}|}$

$$= \hat{a} b \cos 60$$

$$= \frac{ab}{2} \quad \left[\hat{a} = \frac{\vec{a}}{|\vec{a}|} \text{ unit vector} \right]$$

⑧

$$F = 1200 \text{ N} \quad S = 10 \text{ m} \quad \theta = 60^\circ$$

$$W = F S \cos \theta$$

$$= 1200 \times 10 \times \cos 60$$

$$= 12000 \times \frac{1}{2}$$

$$= 6000 = 6 \times 10^3 \text{ J}$$

⑨

$$\text{Let } \vec{A} = 4\hat{i} - 3\hat{j} + 3\hat{k} \text{ and } \vec{B} = 3\hat{i} + 8\hat{j} + 4\hat{k}$$

$\therefore$ Dot Product of $\vec{A}$ and $\vec{B} = \vec{A} \cdot \vec{B}$

$$= (4\hat{i} - 3\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 8\hat{j} + 4\hat{k})$$

$$= (4 \times 3 + (-3) \times 8) + 3 \times 4$$

$$= 12 - 24 + 12$$

$$= 0$$

(10)

(a)

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

$$\Rightarrow \sqrt{A^2 + B^2 + 2AB \cos \theta} = \sqrt{A^2 + B^2 - 2AB \cos \theta} \quad \text{S.O.B.S}$$

$$\Rightarrow A^2 + B^2 + 2AB \cos \theta = A^2 + B^2 - 2AB \cos \theta$$

$$\Rightarrow 2AB \cos \theta = -2AB \cos \theta$$

$$\Rightarrow 4AB \cos \theta = 0 \Rightarrow \cos \theta = 0 \Rightarrow \theta = 90^\circ$$

(b)

$$\vec{A} \times \vec{B} = \vec{A} \cdot \vec{B}$$

$$\Rightarrow AB \sin \theta = AB \cos \theta$$

$$\Rightarrow \sin \theta = \cos \theta \Rightarrow \tan \theta = 1 \Rightarrow \theta = 45^\circ$$

(c)

$$\vec{A} \cdot \vec{B} = \frac{AB}{2}$$

$$\Rightarrow AB \cos \theta = \frac{AB}{2} \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

(d)

$$\vec{A} \times \vec{B} = \frac{AB}{2}$$

$$\Rightarrow AB \sin \theta = \frac{AB}{2} \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

①

Given $\vec{A} \cdot \vec{B} = 0 \Rightarrow AB \cos \theta = 0$
 $\Rightarrow \theta = 90^\circ$

②

Let the two vectors be $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{j} + \hat{k}$
 $\therefore (\vec{A} \cdot \vec{B}) : (\hat{i} + \hat{j}) \cdot (\hat{j} + \hat{k}) = 1 \cdot 1 = 1$
 $|\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2} \quad ; \quad |\vec{B}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\therefore$ if θ is angle between the vectors

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{1}{\sqrt{2} \sqrt{2}} = \frac{1}{2}$$

$\Rightarrow \theta = 60^\circ$

③

Let $\vec{A} = 4\hat{i} - 2\hat{j} + 5\hat{k}$; $\vec{B} = 3\hat{i} + \hat{j} - 2\hat{k}$

$$\begin{aligned} \therefore \vec{A} \cdot \vec{B} &= (4\hat{i} - 2\hat{j} + 5\hat{k}) \cdot (3\hat{i} + \hat{j} - 2\hat{k}) \\ &= 4 \times 3 + (-2)(1) + (5)(-2) \\ &= 12 - 2 - 10 = 0 \end{aligned}$$

Angle between $\vec{A}$ and $\vec{B}$ is $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$

$$\Rightarrow \cos \theta = \frac{0}{|\vec{A}| |\vec{B}|} = 0$$

$\Rightarrow \theta = 90^\circ$

(4)

Let $\vec{A} = 4\hat{i} - 2\hat{j}$, $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\begin{aligned}\therefore \vec{A} \cdot \vec{B} &= (4\hat{i} - 2\hat{j}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) \\ &= (4 \cdot 1) + (-2)(2) + 0 \cdot 3 \\ &= 4 - 4 + 0 = 0\end{aligned}$$

(5)

Given $\vec{A} = \hat{i} + \hat{j}$

Angle made by the vector with x-axis is

$$\tan \theta = \frac{y\text{-component}}{x\text{-component}}$$

$$\Rightarrow \tan \theta = \frac{1}{1} = 1 \Rightarrow \theta = 45^\circ$$

(6)

Given $\vec{A} = \underset{x}{2\hat{i}} + \underset{y}{3\hat{j}}$

Angle made by the vector with y-axis

$$\tan \theta = \frac{x\text{-component}}{y\text{-component}} = \frac{2}{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2}{3}\right)$$

(7)

Given $\vec{A} = a\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} - 4\hat{k}$

If $\vec{A}$ is $\perp$ to $\vec{B}$ $\therefore \vec{A} \cdot \vec{B} = 0$

$$\Rightarrow (a\hat{i} + 2\hat{j} - 5\hat{k}) \cdot (2\hat{i} - \hat{j} - 4\hat{k}) = 0$$

$$\begin{aligned}\Rightarrow 2a - 2 + (-5)(-4) &= 0 \Rightarrow 2a - 2 + 10 = 0 \\ \Rightarrow a &= -4.\end{aligned}$$



8

Given $\vec{P} = 2\hat{i} + 4\hat{j} + \hat{k}$

and $\vec{S} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

$$\begin{aligned}\therefore W &= \vec{P} \cdot \vec{S} = (2\hat{i} + 4\hat{j} + \hat{k}) \cdot (3\hat{i} + 2\hat{j} + 5\hat{k}) \\ &= (2 \cdot 3) + (4 \cdot 2) + (1 \cdot 5) \\ &= 6 + 8 + 5 = 19\end{aligned}$$

6

9

let the vector be $\vec{A} = \sqrt{3}\hat{i} + \hat{j}$

Angle made by the vector with x-axis

$$\tan \theta = \frac{\text{y-component}}{\text{x-component}} \rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^\circ \text{ or } \frac{\pi}{6}$$

10

a

$$\vec{A} = 3\hat{i} + 4\hat{j}$$

Angle made by $\vec{A}$ with x-axis $\tan \theta = \frac{4}{3} \Rightarrow \theta = 53^\circ$

b

$$\vec{B} = 4\hat{i} + 3\hat{j}$$

Angle made by $\vec{B}$ with x-axis ; $\tan \theta = \frac{3}{4} \Rightarrow \theta = 37^\circ$

c

$$\vec{C} = \hat{i} + \hat{j}$$

Angle made by $\vec{C}$ with x-axis $\Rightarrow \tan \theta = 1 \Rightarrow \theta = 45^\circ$

$\Rightarrow$ ans: b, c, a

11

if (a) $\vec{A}$ and $\vec{B}$ are parallel $\theta = 0^\circ$

$$\vec{A} \cdot \vec{B} = AB \cos \theta = AB$$

(b) $\theta = 60^\circ$ $\vec{A} \cdot \vec{B} = AB \cos 60^\circ = \frac{AB}{2}$

(c) $\theta = 180^\circ$ $\vec{A} \cdot \vec{B} = AB \cos 180^\circ = -AB$

c, b, a.

Jee mains level

1

$$\vec{F} = 2\hat{i} - 0.75\hat{j} + 5\hat{k} \text{ N.} \quad ; k = 4 \text{ sec.}$$

$$\vec{S} = 3\hat{i} + 4\hat{j} + 5\hat{k} \text{ m.}$$

$$\text{Power} = \frac{W}{t} = \frac{\vec{F} \cdot \vec{S}}{t} = \frac{(2\hat{i} - 0.75\hat{j} + 5\hat{k}) \cdot (3\hat{i} + 4\hat{j} + 5\hat{k})}{4}$$

$$P = \frac{2 \times 3 + (-0.75)(4) + 5 \times 5}{4} = \frac{6 - 3 + 25}{4} = \frac{28}{4} = 7 \text{ W}$$

2

Given $\frac{1 + \vec{A} \cdot \vec{B}}{1 + \vec{A} \cdot \vec{B}} = \frac{1 - AB \cos \theta}{1 - AB \cos \theta}$

Given A, B are unit

vectors i.e. $|\vec{A}| = |\vec{B}| = 1$

$$= \frac{1 - 1 \times 1 \cos \theta}{1 + 1 \times 1 \cos \theta} = \frac{1 - \cos \theta}{1 + \cos \theta} = \frac{2 \sin^2 \theta / 2}{2 \cos^2 \theta / 2}$$

$$= \tan^2 \theta / 2$$

(3)

$$\vec{v} = 4\hat{i} - 2\hat{j} + \hat{k} \text{ m/s}, \quad \vec{F} = 5\hat{i} - 10\hat{j} + 6\hat{k} \text{ N}$$

$$\text{Power} = \vec{F} \cdot \vec{v}$$

$$= (5\hat{i} - 10\hat{j} + 6\hat{k}) \cdot (4\hat{i} - 2\hat{j} + \hat{k})$$

$$= (5 \cdot 4) + (-10)(-2) + 6 \cdot 1$$

$$\Rightarrow 20 + 20 + 6 = 46 \text{ W}$$

(4)

$$\text{Let } \vec{A} = -2\hat{i} + 3\hat{j} - \hat{k} \text{ and } \vec{B} = \hat{i} + 2\hat{j} + 4\hat{k}$$

$$\therefore \vec{A} \cdot \vec{B} = (-2\hat{i} + 3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} + 4\hat{k})$$

$$= (-2)(1) + 3 \times 2 + (-1)(4)$$

$$\Rightarrow -2 + 6 - 4 = 0$$

$$\text{The angle b/w two vectors is } \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\Rightarrow \cos \theta = \frac{0}{|\vec{A}| |\vec{B}|} = 0 \Rightarrow \theta = 90^\circ$$

(5)

$$\text{Given } \vec{A} = 2\hat{i} + 2\hat{j} + 3\hat{k}; \quad \vec{B} = 3\hat{i} + 6\hat{j} + n\hat{k}$$

If $\vec{A}$ and $\vec{B}$ are perpendicular to each other

$$\therefore \vec{A} \cdot \vec{B} = 0$$

$$\Rightarrow (2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 6\hat{j} + n\hat{k}) = 0$$

$$\Rightarrow 2 \times 3 + 2 \times 6 + 3n = 0$$

$$\Rightarrow 6 + 12 + 3n = 0 \Rightarrow 3n = -18$$

$$\Rightarrow n = -6$$

Given $\vec{v} = 3\hat{i} + 4\hat{j}$

$$|cu| = 7.5$$

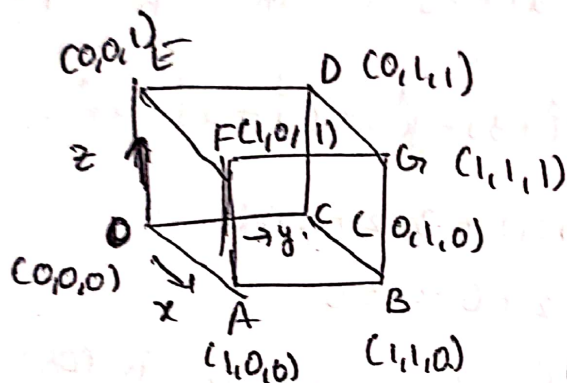
$$\Rightarrow |c| |\vec{v}| = 7.5$$

$$\Rightarrow c \sqrt{3^2 + 4^2} = 7.5$$

$$\Rightarrow c \times 5 = 7.5$$

$$\Rightarrow c = 1.5$$

(7)



Diagonals $\vec{AD}, \vec{OG}, \vec{CF}, \vec{BE}$

$$\begin{aligned} \vec{AD} &= (0-1)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} \\ &= -\hat{i} + \hat{j} + \hat{k} \end{aligned}$$

$$\begin{aligned} \vec{OG} &= (1-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} \\ &= \hat{i} + \hat{j} + \hat{k} \end{aligned}$$

$$|\vec{AD}| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|\vec{OG}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\begin{aligned} \vec{OG} \cdot \vec{AD} &= (\hat{i} + \hat{j} + \hat{k}) \cdot (-\hat{i} + \hat{j} + \hat{k}) \\ &= -1 + 1 + 1 = 1 \end{aligned}$$

$$\cos \theta = \frac{\vec{OG} \cdot \vec{AD}}{|\vec{AD}| |\vec{OG}|} = \frac{1}{\sqrt{3} \times \sqrt{3}} = \frac{1}{3}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right) \text{ [Between two diagonals]}$$

$$\vec{OA} = \text{side length} = (1-0)\hat{i} + (0-0)\hat{j} + (0-0)\hat{k} = \hat{i}$$

$$|\vec{OA}| = 1$$

7th continuation

⑧

$$\vec{OA} \cdot \vec{OG} = \hat{i} \cdot (\hat{i} + \hat{j} + \hat{k}) \\ = \hat{i} \cdot \hat{i} = 1$$

$$\therefore \cos \theta = \frac{\vec{OA} \cdot \vec{OG}}{|\vec{OA}| |\vec{OG}|} = \frac{1 \cdot 1}{1 \times \sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is the angle made by the diagonal with unit length of cube.

⑧

Given $\vec{A} = 2\hat{i} + 3\hat{j}$

Angle made by the vector with y-axis

$$\tan \theta = \frac{x\text{-component}}{y\text{-component}} = \frac{2}{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2}{3}\right)$$

⑩

Let $\vec{A} = 6\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} - \hat{k}$

$$\therefore \vec{A} \cdot \vec{B} = (6\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - 2\hat{j} - \hat{k})$$

$$= 6 \times 2 + (3)(-2) + 2(-1) = 12 - 6 - 2 = 4$$

$$|\vec{A}| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{49} = 7 \quad ; \quad |\vec{B}| = \sqrt{2^2 + (-2)^2 + (-1)^2} = \sqrt{9} = 3$$

$$\therefore \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{4}{7 \times 3} = \frac{4}{21}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

⑪

$$\begin{aligned} \text{I } \vec{p} \cdot \vec{a} &= p a \cos \theta \\ &= p a \cos 0^\circ \\ &= p a. \end{aligned} \quad \text{if } \theta = 0^\circ$$

$$\begin{aligned} \text{II } \vec{p} \cdot \vec{a} &= p a \cos 30^\circ \\ &= \frac{\sqrt{3}}{2} p a \quad \underline{\theta = 30^\circ} \end{aligned}$$

Statement II is wrong, I is correct

⑫

⑬

$$F_1 = 5\hat{i} - 3\hat{j} + 2\hat{k} \quad ; \quad r_1 = 2\hat{i} + 7\hat{j} + 4\hat{k}$$

$$r_2 = 5\hat{i} + 2\hat{j} + 8\hat{k}$$

$$\begin{aligned} \text{Displacement} &= \vec{r}_2 - \vec{r}_1 = (5\hat{i} + 2\hat{j} + 8\hat{k}) - (2\hat{i} + 7\hat{j} + 4\hat{k}) \\ &= 3\hat{i} - 5\hat{j} + 4\hat{k} \end{aligned}$$

⑬

$$\text{work done} = \vec{F} \cdot \vec{J}$$

$$= (5\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (3\hat{i} - 5\hat{j} + 4\hat{k})$$

$$= 5 \times 3 + (-3)(-5) + 2(4)$$

$$= 15 + 15 + 8$$

$$= 38 \text{ J}$$

(4)

Given Scalar Product of $(3\hat{i} + 4\hat{j} - 7\hat{k})$ & $(2\hat{i} + 4\hat{j} + p\hat{k})$ is 8.

$$\Rightarrow (3\hat{i} + 4\hat{j} - 7\hat{k}) \cdot (2\hat{i} + 4\hat{j} + p\hat{k}) = 8$$

$$\Rightarrow 3 \times 2 + 4 \times 4 - 7p = 8$$

$$\Rightarrow 6 + 16 - 7p = 8$$

$$\Rightarrow -7p = -14 \Rightarrow p = 2.$$

(15)

$$\text{let } \vec{A} = \sqrt{3} \hat{i} + \hat{j}.$$

Angle made by the vector with x-axis

$$\tan \theta = \frac{\text{y-comp}}{\text{x-comp}}$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ = \frac{\pi}{6}.$$

$$\Rightarrow \frac{\pi}{k} = \frac{\pi}{6} \Rightarrow k = 6$$

(6)

$$\text{let } \vec{A} = \hat{i} - \hat{j} - 3\hat{k} ; \vec{B} = -2\hat{i} - c\hat{j} + \hat{k}$$

$$\theta = 90^\circ$$

$$(e) \vec{A} \cdot \vec{B} = 0$$

$$\Rightarrow (\hat{i} - \hat{j} - 3\hat{k}) \cdot (-2\hat{i} - c\hat{j} + \hat{k}) = 0$$

$$\Rightarrow 1 \times (-2) + (-1) \times (-c) + (-3) \times 1 = 0$$

$$\Rightarrow -2 + c - 3 = 0$$

$$\Rightarrow c = 5$$