

PROPERTIES OF TRIANGLES

Class: IX, Advanced.

MATHEMATICS

SOLUTIONS

①

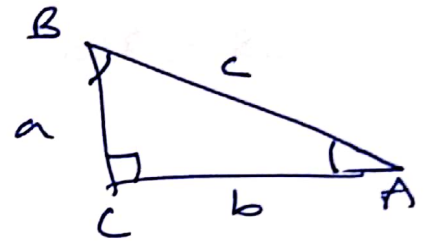
TEACHING TASK

01. $A : B : C = 2 : 3 : 5$

$$\Rightarrow A = \frac{2}{10} \times 180^\circ = 36^\circ, B = \frac{3}{10} \times 180^\circ = 54^\circ$$

$$C = \frac{5}{10} \times 180^\circ = 90^\circ$$

greatest side : smallest side



$$= c : a$$
$$= \frac{c}{a} = \frac{2R \sin C}{2R \sin A} = \frac{\sin 90^\circ}{\sin 36^\circ} = \left(\frac{1}{\frac{\sqrt{10-2\sqrt{5}}}{4}} \right)$$

$$: 4 : \sqrt{10-2\sqrt{5}}$$

Ans. B

02

$$A + C = 2B$$

$$\Rightarrow A + B + C = 3B$$

$$\Rightarrow 180^\circ = 3B$$

$$\Rightarrow B = 60^\circ$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$$

$$\Rightarrow \cos 60^\circ = \frac{c^2 + a^2 - b^2}{2ac}$$

$$\Rightarrow \frac{1}{2} = \frac{c^2 + a^2 - b^2}{2ac}$$

$$ac = c^2 + a^2 - b^2$$

$$b^2 = a^2 - ac + c^2$$

$$\text{Now } \frac{a+c}{\sqrt{a^2 - ac + c^2}} = \frac{a+c}{b} = \frac{2R \sin A + 2R \sin C}{2R \sin B} = \frac{\sin A + \sin C}{\sin B}$$

$$= \frac{2 \sin\left(\frac{A+C}{2}\right) \cos\left(\frac{A-C}{2}\right)}{\sin B} = \frac{2 \sin\left(\frac{2B}{2}\right) \cos\left(\frac{A-C}{2}\right)}{\sin B}$$

$$= \frac{2 \sin B \cos\left(\frac{A-C}{2}\right)}{\sin B} \quad \text{Ans: A}$$



$$03. c^4 - 2c^2(a^2 + b^2) + a^4 + a^2b^2 + b^4 = 0 \quad (2)$$

$$\Rightarrow (a^2)^2 + (b^2)^2 + (-c^2)^2 + a^2b^2 + 2a^2(-c^2) + 2b^2(-c^2) = 0$$

Adding a^2b^2 on b/s to make it perfect square

$$\Rightarrow (a^2 + b^2 - c^2)^2 = (ab)^2$$

$$\Rightarrow a^2 + b^2 - c^2 = ab$$

$$\text{Now } \cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{ab}{2ab} = \frac{1}{2} \Rightarrow C = 60^\circ$$

$$04. \text{ Given } x^2 - 2\sqrt{3}x + 2 = 0.$$

Let a & b be the roots

$$\text{We have } a+b = 2\sqrt{3}; \quad ab = 2, \quad \angle C = \frac{\pi}{3}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow \cos 60^\circ = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \frac{1}{2} = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow ab = a^2 + b^2 - c^2$$

$$\Rightarrow a^2 + b^2 - ab = c^2$$

$$\Rightarrow (a+b)^2 - 3ab = c^2$$

$$\Rightarrow (2\sqrt{3})^2 - 3(2) = c^2$$

$$\Rightarrow c^2 = 6 \Rightarrow c = \sqrt{6}$$

$$\text{Perimeter} = a+b+c = 2\sqrt{3} + \sqrt{6} \quad \text{Ans: A}$$

$$05. \text{ In } \triangle ABC, (a+b+c)(b+c-a) = \pi bc$$

$$\Rightarrow (2s)(2s-2a) = \pi bc$$

$$\Rightarrow \frac{s(s-a)}{bc} = \frac{\pi}{4}$$

$$\Rightarrow \cos^2 \frac{A}{2} = \frac{\pi}{4}$$

$$\text{We know } 0 \leq \cos^2 \frac{A}{2} \leq 1$$

$$0 \leq \frac{\pi}{4} \leq 1$$

$$\Rightarrow 0 \leq \pi \leq 4$$

Ans: C

$$06 \quad \tan \frac{A}{2} \cdot \tan \frac{B}{2} + \tan \frac{B}{2} \cdot \tan \frac{C}{2} = \frac{2}{3}$$

(3)

$$\Rightarrow \frac{\Delta}{s(s-a)} \cdot \frac{\Delta}{s(s-b)} + \frac{\Delta}{s(s-b)} \cdot \frac{\Delta}{s(s-c)} = \frac{2}{3}$$

$$\Rightarrow \frac{\Delta^2}{s^2(s-b)} \left[\frac{1}{s-a} + \frac{1}{s-c} \right] = \frac{2}{3}$$

$$\Rightarrow \frac{\Delta^2}{s^2(s-b)} \left(\frac{2s-a-c}{(s-a)(s-c)} \right) = \frac{2}{3}$$

$$\Rightarrow \frac{\Delta^2}{s^2(s-b)} \times \frac{b}{(s-a)(s-c)} = \frac{2}{3}$$

$$\Rightarrow \frac{b}{s} = \frac{2}{3} \quad \rightarrow 2b = 2s$$

$$\Rightarrow 3b = a+b+c$$

$$\Rightarrow a+c = 2b$$

Ans: B

07 This problem can be done in shortcut way.

Let us consider an equilateral Δ .

with sides $a=b=c=1$ unit.

Now, altitude $= \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2}$; Area $= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4}$.

$$\text{Now } \frac{1}{a} + \frac{1}{b} + \frac{1}{c} - \frac{2ab}{(a+b+c)\Delta} \cdot \cos^2 \frac{C}{2}$$

$$= \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} - \frac{2 \cdot 1 \cdot 1}{(1+1+1) \times \frac{\sqrt{3}}{4}} \times \cos^2 30^\circ$$

$$= \frac{2}{\sqrt{3}} - \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$= \frac{2}{\sqrt{3}} - \frac{2}{\sqrt{3}} = 0$$

Ans: A

$$08 \quad \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = \frac{2R \sin A}{2R \cos A} + \frac{2R \sin B}{2R \cos B} + \frac{2R \sin C}{2R \cos C}$$

$$= \tan A + \tan B + \tan C$$

$$= \tan A \cdot \tan B \cdot \tan C \quad (\because \hat{A} + \hat{B} + \hat{C} = \pi)$$

$$= \frac{a}{x} \cdot \frac{b}{y} \cdot \frac{c}{z}$$

Ans: A



100 We have $a^2 = b^2 + c^2 - 2bc \cos A$

(4)

$$\Rightarrow c^2 - 2bc \cos A + b^2 - a^2 = 0$$

This is a Q.E. in C

Let c_1 & c_2 be the roots.

$$c_1 + c_2 = 2b \cos A; \quad c_1 \cdot c_2 = b^2 - a^2$$

$$\begin{aligned} \text{Now, } (c_1 - c_2)^2 &= (c_1 + c_2)^2 - 4c_1 c_2 \\ &= 4b^2 \cos^2 A - 4(b^2 - a^2) \\ &= 4a^2 - 4b^2(1 - \cos^2 A) \\ &= 4a^2 - 4b^2 \sin^2 A \\ &= 4a^2 - \left(\frac{c_1 + c_2}{\cos A}\right)^2 \cdot \sin^2 A \\ &= 4a^2 - (c_1 + c_2)^2 \cdot \tan^2 A \end{aligned}$$

$$\therefore c_1 - c_2 = \sqrt{4a^2 - (c_1 + c_2)^2 \cdot \tan^2 A} \quad \text{Ans: A}$$

11) $c^2 x^2 - c(a+b)x + ab = 0$.

$\sin A$ and $\sin B$ are the roots.

$$\begin{aligned} \therefore \sin A + \sin B &= \frac{a+b}{c} \\ \Rightarrow \frac{a}{2R} + \frac{b}{2R} &= \frac{a+b}{c} \end{aligned} \quad \left\{ \begin{aligned} \sin A - \sin B &= \frac{ab}{c^2} \\ \frac{a}{2R} - \frac{b}{2R} &= \frac{ab}{c^2} \end{aligned} \right.$$

$$\Rightarrow c = 2R$$

$$\Rightarrow 2R \sin C = 2R \Rightarrow \sin C = 1$$

$$\Rightarrow C = \frac{\pi}{2}$$

$$\text{Now, } A + B + C = \pi$$

$$\Rightarrow A + B = \pi - C = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\text{Now } \sin A + \sin B = \frac{a+b}{c} \Rightarrow \sin A + \cos A = \frac{a+b}{c}$$

Ans: B, D

12 In $\triangle ABC$, $\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ (5)

$$\Rightarrow \sqrt{\frac{b+c}{2c}} = \sqrt{\frac{s(s-a)}{bc}}$$

$$\Rightarrow \frac{b+c}{2c} = \frac{s(s-a)}{bc}$$

$$\Rightarrow as(s-a) = b^2 + bc$$

$$\Rightarrow (a+b+c)\left(\frac{a+b+c}{2} - a\right) = b^2 + bc$$

$$\Rightarrow a^2 + b^2 = c^2$$



$$\text{Area} = \frac{1}{2} ab$$

$$\text{Circum radius} = \frac{1}{2} c$$

Ans: A, B

13 $\frac{2a}{b+c-a} = \frac{2a}{a+b+c-2a} = \frac{2a}{2s-2a} = \frac{a}{s-a}$

Similarly $\frac{2b}{c+a-b} = \frac{b}{s-b}$, $\frac{2c}{a+b-c} = \frac{c}{s-c}$

Now A.M. $\geq$ H.M.

$$\frac{\frac{a}{s-a} + \frac{b}{s-b} + \frac{c}{s-c}}{3} \geq \frac{3}{\frac{s-a}{a} + \frac{s-b}{b} + \frac{s-c}{c}}$$

$\Rightarrow$

13

Statement I

(6)

$$\text{Let } A = \frac{a}{b+c-a} + \frac{b}{c+a-b} + \frac{c}{a+b-c}$$

$$\Rightarrow 2A = \frac{2a}{b+c-a} + \frac{2b}{c+a-b} + \frac{2c}{a+b-c}$$

$$\Rightarrow 2A = \left(\frac{2a}{b+c-a} + 1 \right) + \left(\frac{2b}{c+a-b} + 1 \right) + \left(\frac{2c}{a+b-c} + 1 \right) - 3$$

$$\Rightarrow 2A = (a+b+c) \left(\frac{1}{b+c-a} + \frac{1}{c+a-b} + \frac{1}{a+b-c} \right) - 3$$

Now $AM \geq HM$.

$$\left(\frac{\frac{1}{b+c-a} + \frac{1}{c+a-b} + \frac{1}{a+b-c}}{3} \right) \geq \frac{3}{\left(\frac{b+c-a}{b+c-a} \right) + \left(\frac{c+a-b}{c+a-b} \right) + \left(\frac{a+b-c}{a+b-c} \right)}$$

$$\frac{1}{b+c-a} + \frac{1}{c+a-b} + \frac{1}{a+b-c} \geq \frac{9}{a+b+c}$$

$$(a+b+c) \left(\frac{1}{b+c-a} + \frac{1}{c+a-b} + \frac{1}{a+b-c} \right) \geq 9$$

$$\Rightarrow 2A + 3 \geq 9$$

$$\Rightarrow A \geq 3$$

$\therefore$ The minimum value = 3 (Statement I is wrong)

Statement II: (Conceptual) True

Ans: D

14

$$2\Delta = ax = by = cz$$

$$\therefore \frac{b^2 x a + c^2 y b + a^2 z c}{abc} = \frac{a^2 + b^2 + c^2}{K}$$

$$\Rightarrow \frac{b^2 (2\Delta) + c^2 (2\Delta) + a^2 (2\Delta)}{abc} = \frac{a^2 + b^2 + c^2}{K}$$

$$\Rightarrow \frac{2\Delta}{abc} = \frac{1}{K} \Rightarrow K = \frac{abc}{2\Delta} = 2R \quad \text{Ans: C}$$



$$15 \quad \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{a^2}{4\Delta^2} + \frac{b^2}{4\Delta^2} + \frac{c^2}{4\Delta^2} \quad (7)$$

$$= \frac{a^2 + b^2 + c^2}{4\Delta^2}$$

$$\text{Now } \cot A + \cot B + \cot C = \frac{R}{abc}$$

$$b^2 + c^2 - a^2 + c^2 + a^2 - b^2 + a^2 + b^2 - c^2$$

$$= \frac{R}{abc} (a^2 + b^2 + c^2)$$

$$= \frac{R}{abc} \left(\frac{4\Delta^2}{x^2} + \frac{4\Delta^2}{y^2} + \frac{4\Delta^2}{z^2} \right)$$

$$= \frac{4\Delta^2 R}{abc} \left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} \right)$$

$$= \frac{4\Delta R}{abc} \cdot \Delta \left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} \right)$$

$$= \Delta \left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} \right)$$

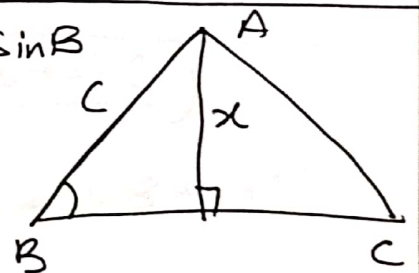
$$\therefore R = \Delta$$

$$16 \quad \text{In } \triangle ABC, \sin B = \frac{x}{c} \Rightarrow x = c \sin B$$

$$\sin C = \frac{x}{b} \Rightarrow x = b \sin C$$

$$\text{Hence } \frac{c \sin B + b \sin C}{x} = \frac{x + x}{x} = 2$$

$$\text{Therefore Given problem} = 2 + 2 + 2 = 6$$



Ans: D

17 let a, b, c be the sides of a triangle (8)

$$\therefore s = \frac{a+b+c}{2}$$

Also, $s, s-a, s-b, s-c$ will be positive

$$AM > GM$$

$$\frac{s+(s-a)+(s-b)+(s-c)}{4} \geq [s(s-a)(s-b)(s-c)]^{\frac{1}{4}}$$

$$\Rightarrow \frac{4s-(a+b+c)}{4} \geq [\Delta^2]^{\frac{1}{4}}$$

$$\Rightarrow \frac{2s}{4} \geq \Delta^{\frac{1}{2}}$$

$$\Rightarrow \frac{s}{2} \geq \Delta^{\frac{1}{2}} \Rightarrow \frac{s^2}{4} \geq \Delta$$

Ans: C

18 Given $a+b+c = 6 \left(\frac{\sin A + \sin B + \sin C}{3} \right)$

~~$\frac{a}{\sin A} = 2(\sin A + \sin B + \sin C)$~~

~~$\therefore K = 2$~~

Now

~~$\frac{a}{\sin A} = 2$~~

~~$\Rightarrow \sin A =$~~

18 Given $a+b+c = 6 \left(\frac{\sin A + \sin B + \sin C}{3} \right)$

$$\Rightarrow 2R(\sin A + \sin B + \sin C) = 2(\sin A + \sin B + \sin C)$$

$$\Rightarrow R = 1$$

Now $2R \sin A = a$

$$\Rightarrow 2 \times 1 \times \sin A = 1$$

$$\Rightarrow \sin A = \frac{1}{2} \Rightarrow A = \frac{\pi}{6}$$

Ans: A

19

$$A:B:C = 2:3:7$$

(9)

$$\therefore A = \frac{2}{12} \times 180 = 30^\circ$$

$$B = \frac{3}{12} \times 180 = 45^\circ$$

$$C = \frac{7}{12} \times 180 = 105^\circ$$

$$\text{Now } \sin 30^\circ : \sin 45^\circ : \sin 105^\circ$$

$$= \frac{1}{2} : \frac{1}{\sqrt{2}} : \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$= \sqrt{2} : 2 : (\sqrt{3}+1)$$

Ans: A

20

$$a) a=5, A=30$$

$$\text{We know, } a = 2R \sin A$$

$$\Rightarrow 5 = 2R \sin 30^\circ$$

$$\Rightarrow 5 = 2R \times \frac{1}{2}$$

$$\Rightarrow R = 5$$

Ans: 5

$$b) a = \sqrt{3}+1, B = 30^\circ, C = 45^\circ \Rightarrow A = 105^\circ$$

$$a = 2R \sin A$$

$$\Rightarrow \sqrt{3}+1 = 2R \sin 105^\circ$$

$$\Rightarrow \sqrt{3}+1 = 2R \left(\frac{\sqrt{3}+1}{2\sqrt{2}} \right)$$

$$\Rightarrow R = \sqrt{2}$$

$$\text{Now } c = 2R \sin C$$

$$= 2\sqrt{2} \sin 45^\circ$$

$$= 2\sqrt{2} \times \frac{1}{\sqrt{2}}$$

Ans: 2



21 Let x, y be any two positive numbers (10)

$\therefore x, a, y$ are in A.P.

$$\Rightarrow a = \frac{x+y}{2} \rightarrow (1)$$

Also x, b, c, y are in G.P.

$$\text{so } b^2 = cx, c^2 = by \rightarrow (2)$$

$$\begin{aligned} \text{Now } \frac{b^3 + c^3}{abc} &= \frac{\sin^3 B + \sin^3 C}{\sin A \cdot \sin B \cdot \sin C} \\ &= \frac{b^3 + c^3}{abc} = \frac{bc(x+y)}{\left(\frac{bc(x+y)}{2}\right)} = 2 \end{aligned}$$

$$22) a) (a+b+c)(b+c-a) = \lambda bc$$

(11)

$$\Rightarrow (2s) (2s-2a) = \lambda bc$$

$$\Rightarrow 4 \left(\frac{s(s-a)}{bc} \right) = \lambda$$

$$\Rightarrow 4 \cos^2 \frac{A}{2} = \lambda \Rightarrow \cos^2 \frac{A}{2} = \frac{\lambda}{4}$$

$$\text{We know } 0 \leq \cos^2 \frac{A}{2} \leq 1$$

$$\Rightarrow 0 \leq \frac{\lambda}{4} \leq 1 \Rightarrow 0 \leq \lambda \leq 4.$$

Greatest value of $\lambda = 4$.

$$b) \text{ In } \triangle ABC, \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

By Cauchy's Inequality.

$$(t_1 + t_2 + t_3) \leq \sqrt{t_1^2 + t_2^2 + t_3^2} (\sqrt{3})$$

$$\Rightarrow K < 27$$

$$\text{By } Gm \leq Am \Rightarrow 9 \cdot 3^{1/3} \leq K.$$

$$c) (OR) \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

$$Am \geq Gm$$

$$\Rightarrow \frac{\tan A + \tan B + \tan C}{3} \geq \left(\tan A \cdot \tan B \cdot \tan C \right)^{1/3}$$

$$\Rightarrow \frac{K}{3} \geq (9)^{1/3} \Rightarrow K \geq 3 \cdot 9^{2/3}$$

$$K \geq 9 \cdot 3^{1/3}$$

$$c) \text{ In } \triangle OBL, \cos A = \frac{OL}{OB} = \frac{x}{R}$$

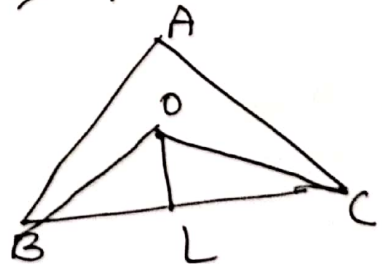
$$\Rightarrow R \cos A = x$$

$$\Rightarrow R \cos A = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$\Rightarrow \cos A = 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$\Rightarrow \cos A = \cos A + \cos B + \cos C - 1$$

$$\Rightarrow \cos A + \cos B = 1$$



d) Given $a=5, b=4, \cos(A-B) = \frac{3}{5}$

(12)

$$\tan\left(\frac{A-B}{2}\right) = \sqrt{\frac{1-\cos(A-B)}{1+\cos(A-B)}} = \sqrt{\frac{1}{63}}$$

$$\Rightarrow \frac{a-b}{a+b} \cot \frac{C}{2} = \frac{1}{\sqrt{63}} \Rightarrow \frac{5-4}{5+4} \cot \frac{C}{2} = \frac{1}{\sqrt{63}}$$

$$\Rightarrow \cot \frac{C}{2} = \frac{\sqrt{7}}{3}$$

$$\text{Now } \cos C = \frac{1 - \tan^2 \frac{C}{2}}{1 + \tan^2 \frac{C}{2}} = \frac{1 - \frac{7}{9}}{1 + \frac{7}{9}} = \frac{1}{8}$$

$$\text{Now, } c^2 = a^2 + b^2 - 2ab \cos C$$

$$\Rightarrow c^2 = 25 + 16 - 40 \times \frac{1}{8} = 36 \Rightarrow c = 6$$

23 a) $A=60^\circ \Rightarrow B+C=120^\circ \Rightarrow \frac{B}{2} + \frac{C}{2} = 60^\circ$

$$\text{Now } \frac{b}{c+a} + \frac{c}{a+b} = \frac{\sin B}{\sin A + \sin C} \neq \frac{\sin C}{\sin A + \sin B}$$

$$= \frac{\sin B}{2 \sin\left(\frac{A+C}{2}\right) \cos\left(\frac{A-C}{2}\right)} + \frac{\sin C}{2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}$$

$$\Rightarrow \frac{\sin \frac{B}{2}}{\cos\left(\frac{A-C}{2}\right)} \neq \frac{\sin \frac{C}{2}}{\cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin \frac{B}{2}}{\cos\left(30^\circ - \frac{C}{2}\right)} + \frac{\sin \frac{C}{2}}{\cos\left(30^\circ - \frac{B}{2}\right)}$$

$$= \frac{\sin B/2}{\cos\left(30^\circ - \frac{B}{2}\right)} + \frac{\sin \frac{C}{2}}{\cos\left(30^\circ - \frac{B}{2}\right)}$$

$$= \frac{2 \sin 30^\circ \cos\left(\frac{B-C}{4}\right)}{\cos\left(30^\circ - \frac{B}{2}\right)} =$$

$$= 2 \times \frac{1}{2} \times \frac{\cos\left(\frac{B}{2} - 30^\circ\right)}{\cos\left(30^\circ - \frac{B}{2}\right)}$$

$$= 1$$

$$b) c^2 = a^2 + b^2 - 2ab \cos C, \angle C = 60^\circ \quad (13)$$

$$\Rightarrow c^2 = a^2 + b^2 - 2ab \times \frac{1}{2}$$

$$\Rightarrow c^2 = a^2 + b^2 - ab \rightarrow (1)$$

$$\Rightarrow \frac{1}{a+c} + \frac{1}{b+c} = \frac{a+b+2c}{a+b+2c(a+c)(b+c)}$$

$$\Rightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$$

$$\Rightarrow b(b+c) + a(a+c) = (a+c)(b+c)$$

$\div$ by $(a+c)(b+c)$ and add 2 on b/s

$$\Rightarrow 1 + \frac{b}{a+c} + 1 + \frac{a}{b+c} = 3$$

$$\Rightarrow \frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$$

$$c) b^2 = c^2 + a^2 - 2ac \cos B \quad \text{Since } B = 60^\circ$$

$$\Rightarrow b^2 = c^2 + a^2 - ac$$

$$\Rightarrow \text{Now } \frac{1}{a+b} + \frac{1}{a+c} = \frac{a+c+a+b}{(a+b)(a+c)} = \frac{2a+b+c}{a^2+ac+ba+bc}$$

$$\Rightarrow b^2 + ac = c^2 + a^2$$

$$\Rightarrow b^2 + ac + bc + ab = c^2 + cb + a^2 + ab$$

$$\Rightarrow b(b+c) + a(c+b) = c(c+b) + a(a+b)$$

$\div$ by $(b+c)(a+b)$ and add 2 on b/s

$$\Rightarrow \frac{1}{a+b} + \frac{1}{a+c} = \frac{3}{a+b+c}$$

$$d) c = 60^\circ \Rightarrow c^2 = a^2 + b^2 - 2ab \cos C \quad (14)$$

$$\Rightarrow c^2 = a^2 + b^2 - ab$$

$$\Rightarrow ab = a^2 + b^2 - c^2 \rightarrow (1)$$

$$\text{Also } c^2 - a^2 = b^2 - ab \quad \left| \begin{array}{l} c^2 = a^2 + b^2 - a^2 - b^2 + c^2 \\ c^2 - a^2 = c^2 - a^2 \end{array} \right.$$

$$\Rightarrow c^2 - a^2 = b(b-a)$$

$$\text{Now } \frac{b}{c^2 - a^2} + \frac{a}{c^2 - b^2} = \frac{b}{b(b-a)} + \frac{a}{a(a-b)}$$

$$= \frac{1}{b-a} + \frac{1}{-(b-a)} = 0$$

$$24 a) (a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2}$$

$$= (a^2 - 2ab + b^2) \cos^2 \frac{C}{2} + (a^2 + 2ab + b^2) \sin^2 \frac{C}{2}$$

$$= (a^2 + b^2) \left(\cos^2 \frac{C}{2} + \sin^2 \frac{C}{2} \right) - 2ab \left(\cos^2 \frac{C}{2} - \sin^2 \frac{C}{2} \right)$$

$$= a^2 + b^2 - 2ab \cos C = c^2$$

$$b) \frac{b^2 - c^2}{a^2} \sin^2 A + \frac{c^2 - a^2}{b^2} \sin^2 B$$

$$= \frac{b^2 - c^2}{a^2} \times \frac{a^2}{4R^2} + \frac{c^2 - a^2}{b^2} \times \frac{b^2}{4R^2}$$

$$= \frac{b^2 - c^2 + c^2 - a^2}{4R^2} = \frac{b^2 - a^2}{4R^2}$$

$$c) \text{ let } a=b=c=1 \text{ \& } A=B=C=60^\circ$$

$$\frac{b \cos A + c \cos B + a \cos C}{\cot A + \cot B + \cot C}$$

$$= \frac{1 \cdot 1 \cdot \frac{1}{2} + 1 \cdot 1 \cdot \frac{1}{2} + 1 \cdot 1 \cdot \frac{1}{2}}{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{\left(\frac{3}{2} \right)}{\left(\frac{3}{\sqrt{3}} \right)} = \frac{\sqrt{3}}{2}$$

$$= 2 \times \frac{\sqrt{3}}{4}$$

$$= 2 \times \frac{\sqrt{3}}{4}$$

d) Let $a=b=c=1$ & $A=B=C=60^\circ$

(15)

$$\frac{\cos C + \cos A}{c+a} + \frac{\cos B}{b}$$

$$= \frac{\frac{1}{2} + \frac{1}{2}}{1+1} + \frac{\frac{1}{2}}{1} \Rightarrow \frac{1}{2} + \frac{1}{2} = 1 \Rightarrow \frac{1}{b}$$

Ans: P, S, r, v

LEARNERS TASK

01. $a=3 \Rightarrow a=2R \sin A$ $\Rightarrow R = \frac{1}{\sqrt{3}}$
 $\Rightarrow 1=2R \sin 60^\circ$

Ans: A

02. $a=2R \sin A$
 $\Rightarrow 10=2 \times 5 \times \sin A \Rightarrow \sin A=1 \Rightarrow A=90^\circ$

Ans: D

03. $A=30^\circ, C=90^\circ \Rightarrow B=60^\circ$
 $c=2R \sin C \Rightarrow 7\sqrt{3}=2R \sin 90^\circ$
 $\Rightarrow R = \frac{7\sqrt{3}}{2}$

$a=2R \sin A$
 $\Rightarrow a=2 \times \frac{7\sqrt{3}}{2} \times \sin 30^\circ = \frac{2 \times 7\sqrt{3}}{2} \times \frac{1}{2} = \frac{7\sqrt{3}}{2}$

Ans: C

04. $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
 $\Rightarrow \frac{4}{5} = \frac{20^2 + 21^2 - c^2}{2 \times 20 \times 21}$

$\Rightarrow c=13$

Ans: B

$$05 \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3^2 + 4^2 - 2^2}{2 \times 3 \times 4} = \frac{7}{8} \quad (16)$$

Ans: A

$$06 \quad \tan \frac{A}{2} = \frac{(s-b)(s-c)}{\Delta} \quad (R) \quad \Delta^2 = s(s-a)(s-b)(s-c)$$

$$= \frac{\Delta}{s(s-a)}$$

$$= \frac{30}{15(15-5)} = \frac{1}{5}$$

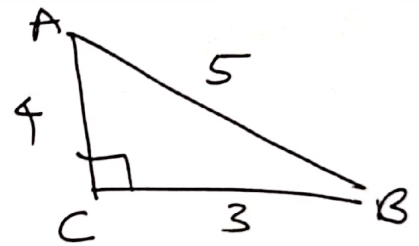
Ans: D

$$07 \quad \cot \frac{B}{2} = \frac{s(s-b)}{\Delta}$$

$$= \frac{66(66-60)}{330} = \frac{6}{5}$$

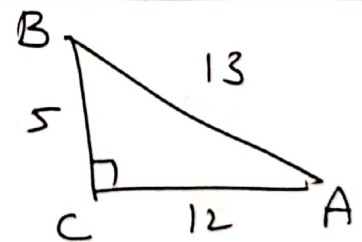
Ans: B

$$08 \quad C = 90^\circ \Rightarrow \cos \frac{C}{2} = \cos 45^\circ = \frac{1}{\sqrt{2}}$$



Ans: D

$$09 \quad C = 90^\circ \Rightarrow \sin \frac{C}{2} = \sin 45^\circ = \frac{1}{\sqrt{2}}$$



Ans: A

$$10 \quad \text{Let } a=b=c=1, \quad A=B=C=60^\circ$$

$$abc \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2} = 1 \cdot 1 \cdot 1 \cdot \sin 30^\circ \cdot \sin 30^\circ \cdot \sin 30^\circ = \frac{1}{8}$$

$$\text{Now } \frac{\Delta^2}{s} = \frac{\left(\frac{\sqrt{3}}{4}\right)^2}{\left(\frac{3}{2}\right)} = \frac{3}{16} \times \frac{2}{3} = \frac{1}{8}$$

$$\Delta = \frac{\sqrt{3}}{4}$$

$$s = \frac{3}{2}$$

Ans: B

JEE MAIN LEVEL

(17)

01. Let $A=B=C=60^\circ$ and $a=b=c=1$

$$\sum \frac{\sin(B-C)}{bc} = \sum \frac{\sin(60^\circ-60^\circ)}{1 \cdot 1} = \sum 0 = 3 \times 0 = 0$$

Ans: C

02. Let $A=B=C=60^\circ$ and $a=b=c=1$

i.e. Consider an equilateral triangle.

$$b^2 \sin 2C + c^2 \sin 2B \\ = 1^2 \sin 120^\circ + 1^2 \sin 120^\circ = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$\left. \begin{aligned} \text{opt: } B &= 2\Delta = 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \\ C &= 3\Delta = 3 \times \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{4} \end{aligned} \right\} \begin{aligned} \text{opt: } A &= 4\Delta \\ &= 4 \times \frac{\sqrt{3}}{4} = \sqrt{3} \end{aligned}$$

Ans: A

03 $a \cos A = b \cos B$

$$\Rightarrow 2R \sin A \cos A = 2R \sin B \cos B$$

$$\Rightarrow \sin 2A = \sin 2B$$

$$\Rightarrow A = B \quad (\text{or}) \quad \sin 2A = \sin (180^\circ - 2B)$$

$$\Rightarrow 2A = 180^\circ - 2B$$

$$\Rightarrow A + B = 90^\circ \Rightarrow C = 90^\circ$$

Ans: C

$\therefore$ Right angled isosceles

04 $\frac{b}{c+a} + \frac{c}{a+b} - 1 = 0$

$$\Rightarrow b^2 + c^2 - a^2 - bc = 0$$

$$\Rightarrow \frac{b^2 + c^2 - a^2}{bc} = 0$$

$$\cos A = 0$$

$$A = 90^\circ$$

Ans: D

05 Let $a = \sqrt{2}$, $b = \sqrt{6}$, $c = \sqrt{8}$

$$a^2 = 2, b^2 = 6, c^2 = 8$$

$$a^2 + b^2 = c^2 \Rightarrow \angle C = 90^\circ$$

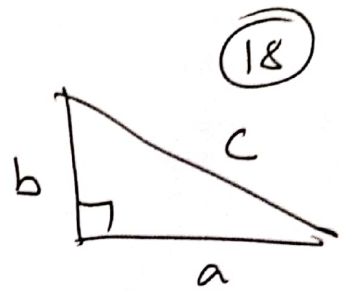
By trial and Error method.

Opt: C, 30° , 60° , 90° is Correct Answer

Since 45° , 45° , 90° Can not be the answer

$$\text{Since } a \neq b$$

Ans: C



(18)

06 $a \cos B + b \cos A + c \cos C$

$$= (2R)^2 [a \cos B \cos C + b \cos A \cos C + c \cos A \cos B]$$

$$= 4R^2 [\cos C (a \cos B + b \cos A) + c \cos A \cos B]$$

$$= 4R^2 [c \cos C + c \cos A \cos B]$$

$$= 4R^2 \cdot c [-\cos(A+B) + \cos A \cos B]$$

$$= 4R^2 \cdot c [\sin A \cdot \sin B]$$

$$= 4R^2 \cdot 2R \sin C \cdot \sin A \cdot \sin B$$

$$= (2R \sin A) (2R \sin B) (2R \sin C) = abc$$

Ans: B

07 $AD = 4$, $BD = DC$

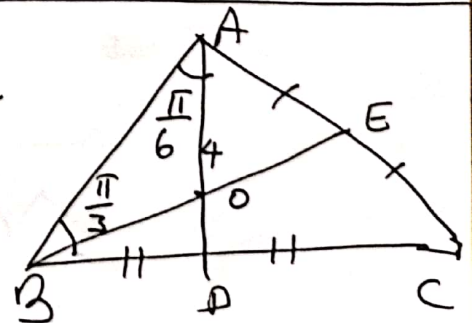
Since Centroid O divides AD in the ratio 2:1

$$AO = \frac{8}{3}, OD = \frac{4}{3}$$

$$\Rightarrow BO = AO \cot \frac{\pi}{3}$$

$$\Rightarrow BO = \frac{8}{3\sqrt{3}}$$

$$A \Delta ADB = \frac{1}{2} \times 4 \times \frac{8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}}$$



$$A \Delta ABC = 2 \times A \Delta ADB$$

$$= \frac{32}{3\sqrt{3}}$$

Ans: C



$$08 \quad (a+b+c)(b+c-a) = 3bc$$

(19)

$$\Rightarrow (2s)(2s-2a) = 3bc$$

$$\Rightarrow \frac{s(s-a)}{bc} = \frac{3}{4}$$

$$\Rightarrow \cos^2 \frac{A}{2} = \frac{3}{4} \Rightarrow \cos \frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{A}{2} = 30^\circ \Rightarrow A = 60^\circ$$

Ans: A

$$09 \quad 1 - \tan \frac{A}{2} \cdot \tan \frac{B}{2}$$

$$= 1 - \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \times \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$= 1 - \frac{s-c}{s} = \frac{c}{s} = \frac{2s}{a+b+c}$$

Ans: C

$$10 \quad a, b, c \text{ AP} \Rightarrow 2b = a+c$$

$$\frac{2 \sin \frac{A}{2} \sin \frac{C}{2}}{\sin \frac{B}{2}} = \frac{2 \sqrt{\frac{(s-b)(s-c)}{bc}} \cdot \sqrt{\frac{(s-a)(s-b)}{ab}}}{\sqrt{\frac{(s-a)(s-c)}{ac}}}$$

$$= 2 \left(\frac{s-b}{b} \right)$$

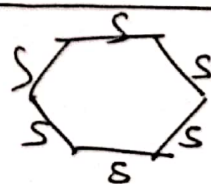
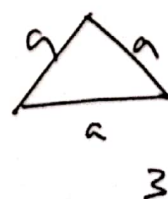
$$= \frac{2s-2b}{b} = \frac{a+b+c-a-c}{b} = \frac{b}{b} = 1$$

$$11. \quad 3a = 6s \Rightarrow a = 2s$$

$$\frac{\sqrt{3}}{4} a^2 = 6 \times \frac{\sqrt{3}}{4} s^2$$

$$\Rightarrow 4s^2 = 6s^2$$

$$\Rightarrow 2:3$$



Ans: C