9th integrated WS-4 ③

Task

⑨

$m_1 \rightarrow u_1 = 40\ m/s$; $m_2 \rightarrow u_2 = 0.$

After collision $v_1 = v_2 = v_{com} = 30\ m/s$

According to law of conservation of linear momentum

$$P_{Before} = P_{After}$$

$$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow \cancel{40}\ 40m_1 + m_2 \times 0 = m_1 \times 30 + m_2 \times 30$$

$$\Rightarrow 40m_1 = 30m_1 + 30m_2 \Rightarrow 10m_1 = 30m_2$$

$$\Rightarrow m_1 = 3m_2 \Rightarrow \frac{m_1}{m_2} = \frac{3}{1}$$

⑩

$m_1 = 25\ kg \rightarrow u_1 = 40\ m/s$

$m_2 = 15\ kg \rightarrow u_2 = 0.$ After collision $v_1 = v_2 = v_{com}$

According to law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow 25 \times 40 + 15 \times 0 = \cancel{m}25v_{com} + 15v_{com}$$

$$\Rightarrow 25 \times 40 = 40v_{com} \Rightarrow v_{com} = 25\ m/s$$

WS 4 Integrated

Task

1

$M = 12\ kg$; $m_1 = 4\ kg$; $m_2 = 8\ kg$

$u = 0$; $v_1 = ?$; $v_2 = 6\ m/s$

According to law of conservation of linear momentum

$$Momentum_{Before} = Momentum_{After}$$

$$\Rightarrow Mu = m_1v_1 + m_2v_2$$

$$\Rightarrow 12 \times 0 = 4v_1 + 8 \times 6$$

$$\Rightarrow \cancel{4}v_1 = -\cancel{8}^{2} \times 6 = -12\ m/s.$$

2

$M_c = 400\ kg$; $m_t = 4000\ kg$

$u_c = 72\ kmph$; $u_t = 9\ kmph$

$v_c = -18\ kmph$; $v_t = ?$

According to law of conservation of linear momentum

$$P_{Before} = P_{After}$$

$$\Rightarrow m_c u_c + m_t u_t = m_c v_c + m_t v_t$$

$$\Rightarrow \cancel{400} \times 72 + \cancel{4000} \times 9 = \cancel{400} \times (-18) + \cancel{4000}(v_t)$$

$$\Rightarrow 72 + 90 = -18 + 10v_t$$

$$\Rightarrow 72 + 90 + 18 = 10v_t \Rightarrow v_t = \frac{180}{10} = 18\ kmph$$

(3)

$M_{sand} = m$ $m_{Bullet} = \frac{m}{20}$

$u_{sand} = 0$ $u_{Bullet} = v$

After bullet embedded in sand bag both together move with same velocity $V_{sand} = V_{Bullet} = V_{com}$

According to law of conservation of linear momentum

$$P_{Before} = P_{After}$$

$$\Rightarrow M_{sand} u_{sand} + m_{Bullet} u_{Bullet} = M_{sand} V_{sand} + M_{Bullet} V_{Bullet}$$

$$\Rightarrow m \times 0 + \frac{m}{20} v = m V_{com} + \frac{m}{20} V_{com}$$

$$\Rightarrow 0 + \frac{\cancel{m} v}{\cancel{20}} = \frac{21 \cancel{m} V_{com}}{\cancel{20}}$$

$$\Rightarrow v = 21 V_{com} \Rightarrow V_{com} = \frac{v}{21}$$

(4)

$m_{bullet} = 200\,gm = 200 \times 10^{-3} kg$ $M_{gun} = 1\,kg$

$u_{bullet} = 5\,m/s$ $V_{gun} = ?$

According to law of conservation of linear momentum

$$\Rightarrow M_{gun} V_{gun} = - m_{bullet} u_{bullet}$$

$$\Rightarrow 1 \times V_{gun} = -200 \times 10^{-3} \times 5$$

$\Rightarrow V_{gun} = -1\,m/s$ '-ve' sign shows the gun gives kick in backward direction after firing bullet.

(5)

$M_{man} = 50$ kg ; $V_{man} = ?$

$M_{stone} = 0.1$ kg ; $V_{stone} = 10$ m/s

According to law of conservation of linear momentum

$\Rightarrow P_{man} = -P_{stone}$

$\Rightarrow M_{man} \times V_{man} = -M_{stone} \times V_{stone}$

$\Rightarrow 50 \times V_{man} = -0.1 \times 10$

$\Rightarrow V_{man} = -\frac{1}{50} = -0.02$ m/s -ve sign shows the direction of motion of a man is opposite to stone.

(6)

$M_{shell} = 40$ kg $\rightarrow V_{shell} = 80$ m/s.

$m_1 = 32$ kg ; $m_2 = 8$ kg ; $V_2 = 0$; $V_1 = ?$

According to law of conservation of linear momentum

$P_{shell} = m_1 V_1 + m_2 V_2$

$\Rightarrow m_{shell} \times V_{shell} = 32 \times V_1 + 8 \times 0$

$\Rightarrow 40 \times 80 = 32 V_1 \Rightarrow 32 V_1 = 3200$

$\Rightarrow V_1 = 100$ m/s.

(7)

$m_{t_1} = 10^4\ kg \longrightarrow u_{t_1} = 1\ m/s$

$m_{t_2} = 2m_{t_1} \rightarrow u_{t_2} = -0.2\ m/s$ After collision two trucks are coupled with each other

(ie) $v_{t_1} = v_{t_2} = v_{com}$ [After collision]

According to law of conservation of linear momentum

$\Rightarrow \quad P_{Before} = P_{After}$

$\Rightarrow m_{t_1}u_{t_1} + m_{t_2}u_{t_2} = m_{t_1}v_{t_1} + m_{t_2}v_{t_2}$

$\Rightarrow 10^4 \times 1 + 2\times10^4 \times (-0.2) = m_{t_1}v_{com} + 2m_{t_1}v_{com}$

$\Rightarrow 10^4 - 0.4\times10^4 = 3m_t\, v_{com}$

$\Rightarrow \cancel{10^4}(1-0.4) = 3\times\cancel{10^4}\, v_{com} \Rightarrow 0.6 = 3v_{com} \Rightarrow v_{com} = 0.2\ m/s$

(8)

$M_{block} = 200\ kg \quad ; \quad u_{block} = 10\ m/s.$

$m_{bullet} = 10\ gm = 10^{-2}\ kg \quad ; \quad u_{bullet} = 200\ m/s.$

The force exerted by 'n' bullets = Force on gun

$\Rightarrow n \times F_{Bullets} = F_{gun/block}$

$\Rightarrow n \times \left(\frac{dp}{dt}\right)_{Bullets} = \left(\frac{dp}{dt}\right)_{gun/block}$

$\Rightarrow n \times m_{bullet} \times u_{bullet} = m_{block}\, v_{block}$

$\Rightarrow n \times 10^{-2} \times \cancel{200} = \cancel{200} \times 10$

$\Rightarrow n = 10\times10^{+2} = 1000.$

Task
Advanced

(6)

$m_1 = 1\,kg$; $m_2 = 3\,kg$; $v_2 = 2\,m/s$

$u_1 = 9\,m/s$; $u_2 = 3\,m/s$; $v_1 = ?$

According to law of conservation of linear momentum

$$P_{Before} = P_{After}$$

$$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow 1\times 9 + 3\times 3 = 1v_1 + 3\times 2$$

$$\Rightarrow 9+9 = v_1 + 6 \Rightarrow v_1 + 6 = 18 \Rightarrow v_1 = 18-6 = 12\,m/s$$

Total momentum Before collision $= m_1u_1 + m_2u_2$

$= 1\times 9 + 3\times 3$

$= 9+9 = 18\,kg\,m/s$

(7)

$m_1 = 20$; $m_2 = ?$

$u_1 = 20\,cm/s$; $u_2 = 5\,cm/s$

$v_1 = v_2 = 10\,cm/s$

According to law of conservation of linear momentum.

$$P_{Before} = P_{After}$$

$$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow 20\times 20 + m_2\times 5 = 20\times 10 + m_2\times 10$$

$$\Rightarrow 400 + 5m_2 = 200 + 10m_2$$

$$\Rightarrow 200 = 5m_2 \Rightarrow m_2 = \frac{200}{5} = 40\,gm$$

(8)

$m_1 = 2\,kg \rightarrow u_1 = \hat{i} + 2\hat{j} - 3k$ m/s

$m_2 = 3\,kg \rightarrow u_2 = 2\hat{i} + \hat{j} + \hat{k}$ m/s

After collision with both bodies moving together

$V_1 = V_2 = V_{com}$

According to law of conservation of Linear momentum.

$$\Rightarrow m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$\Rightarrow 2(\hat{i} + 2\hat{j} - 3k) + 3(2\hat{i} + \hat{j} + \hat{k}) = 2V_{com} + 3V_{com}$$

$$\Rightarrow 2\hat{i} + 4\hat{j} - 6\hat{k} + 6\hat{i} + 3\hat{j} + 3\hat{k} = 5V_{com}.$$

$$\Rightarrow 8\hat{i} + 7\hat{j} - 3\hat{k} = 5V_{com}$$

$$\Rightarrow \vec{V}_{com} = \frac{1}{5}[8\hat{i} + 7\hat{j} - 3\hat{k}]$$

L Task [SAQ's]

(1)

$m_1 = 6\,kg$; m_2

u_1 . $u_2 = 0$ Given $V_1 = V_2 = \frac{u_1}{2}$ [After collision]

According to law of conservation of Linear momentum

$$\Rightarrow m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$\Rightarrow 6u_1 + m_2 \times 0 = 6\frac{u_1}{2} + m_2\frac{u_1}{2}$$

$$\Rightarrow 6u_1 = 3u_1 + \frac{m_2u_1}{2}$$

$$\Rightarrow 6 = 3 + \frac{m_2}{2} \Rightarrow \frac{m_2}{2} = 6 - 3 = 3$$

$$\Rightarrow m_2 = 2 \times 3 = 6\,kg$$

(2)

$M_{gun} = 10\ kg$; $m_{Bullet} = 10\ gm = 10^{-2}\ kg$

$V_{recoil} = ?$ $\quad u_{Bullet} = 200\ m/s$

According to law of conservation of linear momentum

$$V_{recoil} = -\frac{m_{Bullet} \times u_{Bullet}}{M_{gun}}$$

$$= \frac{10^{-2} \times 200}{10} = 0.2\ m/s$$

(3)

$m_A = 60\ kg$; $m_B = 40 kg$

$u_A = ?$ $\quad u_B = 4.5(\hat{-j})\ m/s$ $\quad \hat{-j}$ is unit ~~velocit~~ vector along south.

According to law of conservation of linear momentum

momentum of A = − momentum of B

$\Rightarrow m_A u_A = - m_B u_B$

$\Rightarrow \cancel{60}^{3}\, u_A = - \cancel{40}_{2}\,(\cancel{4.5}_{1.5})(-\hat{j})$

$\Rightarrow u_A = 2 \times 1.5\,\hat{j} = 3\hat{j}\ m/s$ towards north

$\hat{j}$ is unit vector towards north

(4)

$m_1 = 5\ kg$; $u_1 = -10\ m/s$ (left)

$m_2 = 6\ kg$; $u_2 = 12\ m/s$ (right)

After collision both bodies move with same velocity $V_1 = V_2 = V_{com}$

According to law of conservation of linear momentum

$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$

$\Rightarrow 5 \times (-10) + 6 \times 12 = 5 V_{com} + 6 V_{com}$

$\Rightarrow -50 + 72 = 11\ V_{com}$ $\quad \Rightarrow 11 V_{com} = 22 \Rightarrow V_{com} = 2\ m/s$

(5)

$n = 5$; $m_{Bullet} = 200\ gm$; $u_{bullet} = 10\ m/s$

$m_{Block} = 3\ kg$ $u_{Block} = 0$

After collision both bullets and Block are moving together.

momentum Before collision = momentum After collision

$\Rightarrow n \times m_{Bullet} u_{Bullet} + m_{block} u_{block} = m_{Bullet} V_{com} + m_{Block} V_{com}$

$\Rightarrow 5 \times 200 \times 10^{-3} \times 10 + 3 \times 0 = 5 \times 200 \times 10^{-3} V_{com} + 3 V_{com}$

$\Rightarrow 10 = 4 V_{com} \Rightarrow V_{com} = \frac{10}{4} = 2.5\ m/s$

(6)

mass of space craft $= M$; velocity $= V$.

mass left stationary m Remaining mass $(M-m)$

$u_1 = 0$ $u_2 = ?$

According to law of conservation of linear momentum

$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$

$\Rightarrow MV + 0 = m(0) + (M-m) u_2$

$\Rightarrow MV = (M-m) V_2 \Rightarrow u_2 = \frac{MV}{(M-m)}$

(7)

Given $m_1 = m$; $m_2 = 2m$; $m_3 = 3m$; $m_4 = 4m$; $m_5 = 5m$

$u_1 = V$; $u_2 = 2V$; $u_3 = 3V$; $u_4 = 4V$; $u_5 = 5V$

After collision all bodies move with same velocity

$V_1 = V_2 = V_3 = V_4 = V_5 = V_{com}$

According to law of conservation of linear momentum

$$m_1u_1+m_2u_2+m_3u_3+m_4u_4+m_5u_5 = m_1V_1+m_2V_2+m_3V_3+m_4V_4+m_5V_5$$

$$\Rightarrow mv+2m(2V)+3m(3V)+4m(4V)+5m(5V) = (m+2m+3m+4m+5m)\,V_{com}$$

$$\Rightarrow mv+4mv+9mv+16mv+25mv = 15m\,V_{com}$$

$$\Rightarrow 55\cancel{m}v = 15\cancel{m}\,V_{com} \Rightarrow V_{com} = \frac{55}{15}V = \frac{11}{3}v\ m/s$$

(8)

$M_{man} = 64\ kg$; $m_{cart} = 32\ kg$

$u_{man} = 5.4\ kmph$; $u_{cart} = 1.8\ kmph$

$V_{man} = V_{cart} = V_{com}$.

After approaching both man and cart are moving with same velocity

According to law of conservation of linear momentum

$$\Rightarrow M_{man}u_{man} + m_{cart}u_{cart} = M_{man}V_{man} + m_{car}V_{cart}$$

$$\Rightarrow 64\times 5.4 + 32(1.8) = 64V_{com}+32V_{com}$$

$$\Rightarrow \cancel{32}\times \overset{0.6}{\cancel{1.8}}[2\times 3+1] = \cancel{96}\,V_{com}$$

$$\Rightarrow 0.6\times[6+1] = V_{com} \Rightarrow V_{com} = 0.6\times 5 = 3\ kmph$$

(9)

$M_{ball} = 2\ kg$; $m_{cannon} = 198\ kg$.

$u_{ball} = 50\ m/s$ $V_{recoil} = ?$

According to law of conservation of linear momentum

$$V_{recoil} = -\frac{m_{ball}\times V_{ball}}{m_{cannon}} = -\frac{2\times 50}{198} \simeq 0.5\ m/s$$

(10)

$m_{Block} = 11\, m_{bullet}$

$u_{block} = 0$; $u_{bullet} = 120$ m/s

After bullet is fired, it get's embedded into the block and both are moving with same velocity $V_{block} = V_{bullet} = V_{com}$

According to law of conservation of linear momentum

$\Rightarrow m_{block}\, u_{block} + m_{bullet}\, u_{bullet} = m_{block} V_{com} + m_{bullet} V_{com}$

$\Rightarrow m_{block} \times 0 + m_{bullet} \times 120 = (11\, m_{bullet} + m_{bullet}) V_{com}$

$\Rightarrow \overset{10}{\cancel{120}} \times \cancel{m_{bullet}} = \cancel{12}\, \cancel{m_{bullet}} \times V_{com}$

$\Rightarrow V_{com} = 10$ m/s

(16) (i)

$m_{bullet} = 50$ gm → ~~$u_{gun} = 30$ m/s~~ $u_{bullet} = 30$ m/s

$V_{recoil} = 1$ m/s $M_{gun} = ?$

According to law of conservation of linear momentum

$$V_{recoil} = -\frac{m_{bullet}\, u_{bullet}}{M_{gun}}$$

$$\Rightarrow \overset{M_{gun}}{\cancel{V_{recoil}}} = -\frac{50 \times 30}{V_{recoil}} = \frac{1500}{1} = 1500 \text{ gm}$$

$\Rightarrow M_{gun} = 1500 \times 10^{-3}$

$= 1.05$ kg.

(16) (ii)

$m_{gun} = 10\ kg$; $m_b = 10\ gm = 10^{-2}\ kg$; $u_b = 400\ m/s$

According to law of conservation of linear momentum

$$\Rightarrow m_{gun} V_{recoil} = -m_b u_b$$

$$\Rightarrow V_{recoil} = -\frac{m_b u_b}{m_{gun}} = -\frac{10^{-2} \times 40\cancel{0}}{1\cancel{0}} = 0.4\ kg\ m/s$$

(17)

$m_{W} = 10^5\ kg$; $u_{W} = 0.5\ m/s$

$m_{coal} = 2 \times 10^5\ kg$ let v be velocity of wagon after coal is dropped in it.

According to law of conservation of linear momentum

$\Rightarrow$ momentum of wagon with coal = momentum of wagon before coal dropped

$$\Rightarrow (M_w + M_c) \times V = m_w \times u_w$$

$$\Rightarrow (10^5 + 2 \times 10^5) V = 10^5 \times 0.5$$

$$\Rightarrow \cancel{10^5}(1+2) V = 0.5 \times \cancel{10^5} \Rightarrow 3V = 0.5 \Rightarrow V = \frac{0.5}{3} = 0.167\ m/s$$

(18)

$m_{block} = 13\ m_{shot}$; $u_{shot} = 140\ m/s$ $u_{block} = 0$

After collision $V_{block} = V_{shot} = V_{com}$

According to law of conservation of linear momentum

$$\Rightarrow m_{block} \times u_{block} + m_{shot} u_{shot} = M_{block} V_{block} + M_{shot} V_{shot}$$

$$\Rightarrow m_{block} \times 0 + m_{shot} \times 140 = 13\ m_{shot} V_{com} + m_{shot} V_{com}$$

$$\Rightarrow \frac{\cancel{m_{shot}} \times \cancel{140}}{10} = \cancel{14}\ \cancel{m_{shot}} V_{com}$$

$$\Rightarrow V_{com} = 10\ m/s = V_{shot} = V_{block}$$ After collision