

ws-12- work qth foundation +
Task

Given
Force

①

Given Force $F = 7 \text{ N}$ displacement $s = 8$
 F and s are in same direction so $\theta = 0^\circ$

$$\begin{aligned} W &= F s \cos \theta \\ &= 7 \times 8 \times \cos 0^\circ \\ &\Rightarrow 56 \times 1 = 56 \text{ J} \rightarrow D \end{aligned}$$

②

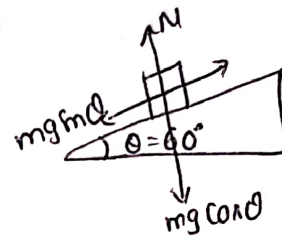
Given mass $= 2 \text{ kg}$

displacement $= 1 \text{ m}$

work done $= F s$

$$= mg \sin \theta s$$

$$\Rightarrow 2 \times 10 \times \sin 60^\circ \Rightarrow 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ J} \rightarrow B$$



③

mass $= 200 \text{ kg}$; $h = 5 \text{ m}$

work done in raising a bucket full of water

$$W = mgh = 200 \times 9.8 \times 5$$

$$= 9800 \text{ J} \rightarrow B$$

④

Given

$$F = 3t \hat{i} + 5 \hat{j} \quad \text{and} \quad \vec{s} = 4t \hat{i}$$

$$W = \int F \cdot ds = \int (3t \hat{i} + 5 \hat{j}) \cdot (4 \hat{i} dt)$$

$$= \int (3t)(4) dt$$

$$= \int 12t dt$$

$$\Rightarrow 12 \left[\frac{t^2}{2} \right] \Rightarrow 6t^2 \quad \text{at } t = 2 \text{ sec}$$

$$\Rightarrow W = 6(2)^2 = 24 \text{ J} \rightarrow A$$

$$d\vec{s} = 4 \hat{i} dt$$

$$[\hat{i} \cdot \hat{i} = 1 ; \hat{i} \cdot \hat{j} = 0]$$

$$\frac{d}{dx} \int x^n = \frac{x^{n+1}}{n+1}$$

③

Given $\vec{F} = 3x^2 \hat{i} + 2y \hat{j}$ N; $\vec{r}_1 = 2\hat{i} + 3\hat{j}$ m, $\vec{r}_2 = 4\hat{i} + 6\hat{j}$ m.
Force is a variable force. $\vec{r} = x\hat{i} + y\hat{j}$

$$W = \int \vec{F} \cdot d\vec{r} = \int (3x^2 \hat{i} + 2y \hat{j}) (dx \hat{i} + dy \hat{j})$$

$$= \int_2^4 (3x^2) dx + \int_3^6 2y dy \quad \int x^n dx = \frac{x^{n+1}}{n+1}$$

$$= 3 \left[\frac{x^{2+1}}{2+1} \right]_2^4 + 2 \left[\frac{y^{1+1}}{1+1} \right]_3^6$$

$$= 3 \left(\frac{x^3}{3} \right)_2^4 + 2 \left(\frac{y^2}{2} \right)_3^6 = (4^3 - 2^3) + (6^2 - 3^2)$$

$$= 64 - 8 + 36 - 9 = 83 \text{ J}$$

⑥

Given length $l = 2\text{ m}$, mass $m = 0.5 \text{ kg}$, $\theta = 60^\circ$

work done in raising other end of the string

$$= m g \frac{l}{2} (1 - \cos \theta)$$

$$= 0.5 \times 10 \times \frac{2}{2} (1 - \cos 60^\circ)$$

$$= 5 \left(1 - \frac{1}{2} \right) = \frac{5}{2} = 2.5 \text{ J} \rightarrow B$$

⑦

Given Force $F = -2x$ N.

$$\text{work done by variable force} = \int F \cdot dx = \int (-2x) dx$$

$$= -2 \int x dx = -2 \frac{x^2}{2}$$

$$= -x^2$$

displacement of the block is from $2 \rightarrow -4 \text{ m}$

$$W = -[(-4)^2 - 2^2] = -[16 - 4] = -12 \text{ J} \rightarrow C$$



(8)

Region OA has positive work because slope of a vs x is increasing [velocity is increasing]

Region AB has work zero because slope $\left[\frac{v}{a}\right] = \text{constant}$
 $a = 0$.

Region BC has negative work because slope constantly decrease. [acceleration = -ve]

(9)

Given

$$F = 10 + 0.5x \text{ N}$$

work done $W = \int F dx = \int_0^2 (10 + 0.5x) dx$

$$= \int_0^2 10 dx + \int_0^2 (0.5)x dx$$

$$\approx 10[x]_0^2 + (0.5)\left[\frac{x^2}{2}\right]_0^2 \quad \left(\int x^n dx = \frac{x^{n+1}}{n+1}\right)$$

$$\approx 10[2-0] + (0.5)\left(\frac{2^2}{2}\right)$$

$$\approx 20 + 0.5 \times 2 = 21 \text{ J} \rightarrow A$$

(10)

Given velocity $v = ax^{3/2}$

$$: a = 5 \text{ m/s}^2, m = 0.5 \text{ kg}$$

velocity at $x=0 \Rightarrow v_1 = 5(0)^{3/2} = 0$

also $W = 25 \text{ k}$

velocity at $x=2 \text{ m} \Rightarrow v_2 = 5(2)^{3/2} =$

According to work energy theorem

$$W = \Delta KE$$

$$= \frac{1}{2} m (v_2^2 - v_1^2)$$

$$= \frac{1}{2} \times 0.5 [(5(2)^{3/2})^2 - 0^2]$$

$$= \frac{1}{2} \times 0.5 (25 \times 2^3)$$

$$W = 50 \text{ J}$$

$$\Rightarrow 25 \text{ k} = 50 \text{ J}$$



(14)

From graph. $x=0 \rightarrow x=3$

work done = Area of F-s graph

$$= \frac{1}{2} b_1 h_1 + b_2 h_2$$

$$= \frac{1}{2} \times 2 \times 2 + 1 \times 2 = 4 \text{ J}$$

(15)

From graph for $4 \rightarrow 6 \text{ m}$

work done = Area under F-s graph

$$= \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 2 \times (-1) = -1 \text{ J}$$

(16)

From graph for $x=3 \rightarrow 4$ displacement $\neq 0$

$$F \cos \theta = 0$$

$$\text{Since } W = F S \cos \theta = 0 \times S \cos \theta = 0$$

(17)

Given

$$F = 2\hat{i} + 3\hat{j} \text{ N}; \text{ line eqn } 3y + kx = 5$$

$$\Rightarrow y + \frac{k}{3}x = \frac{5}{3} \text{ compare with}$$

$$y = mx + c$$

$$\therefore \text{ slope } m = \frac{k}{3}; c = \frac{5}{3}$$

$$\Rightarrow \frac{dy}{dx} = \frac{k}{3}$$

$$\Rightarrow \text{ displacement} = x\hat{i} + y\hat{j} = 3\hat{i} + k\hat{j}$$

Given

$$W = 0$$

$$\Rightarrow (F \cdot S) = 0$$

$$\Rightarrow (2\hat{i} + 3\hat{j}) \cdot (3\hat{i} + k\hat{j}) = 0$$

$$\Rightarrow 2 \times 3 + 3k = 0 \Rightarrow 3k = -6 \Rightarrow k = -2$$



(18)

Given $m = 1 \text{ kg}$; length $l = 1.6 \text{ m}$

$$\text{work done in lifting the chain} = - \frac{mgl}{2}$$

$$= 1 \times 10 \times \frac{1.6}{2} = 8 \text{ J}$$

(19)

Given $m = 5 \text{ kg}$, $v = 3t$; time $t = 2 \text{ sec}$

$$\text{acceleration } a = \frac{dv}{dt} = \frac{d(3t)}{dt} = 3 \text{ m/s}^2$$

$$\text{From Newton's 2nd law } F = ma = 5(3) = 15 \text{ N}$$

velocity at 2 sec

$$v = 3 \cdot 2 = 6 \text{ m/s}$$

$$\text{displacement after 2 sec From } v^2 = u^2 + 2as$$

where $u = 0$ (initial velocity) because body starts from rest

$$v^2 = 2as \Rightarrow s = \frac{v^2}{2a} = \frac{6^2}{2 \times 3} = 6 \text{ m}$$

work done by the force

$$W = F \cdot s \\ = 15 \times 6 = 90 \text{ J}$$

LTASK
CUBA

(4)

There is a electrostatic force of repulsion between the two charges, it means force acts in opposite direction of the movement.

The agent is applying a force to move the charge towards the fixed +ve charge. This force and displacement are in same direction so $\theta = 0^\circ \Rightarrow \cos \theta = \cos 0 = 1$

$$W = FS \cos \theta = FS \cos 0^\circ = FS = \text{positive.}$$

(4)

(6), (7)

we know that $W = FS \cos \theta$

when θ is in between $90 \rightarrow 180$: $\cos \theta = -ve$

so work done = -ve

when force is acting in a direction opposite to motion of the body. then $\theta = 180^\circ$

(8), (9)

we know that $W = FS \cos \theta$

when θ is in between $0 \rightarrow 90^\circ$ $\cos \theta = +ve$

so work done = +ve.

if $\theta = 0^\circ$ $W = FS \cos 0 = FS \rightarrow \text{maximum}$

(10)

when a body is in circular motion then displacement and centripetal force are perpendicular to each other

$\theta = 90^\circ$ $W = FS \cos \theta = FS \cos 90^\circ$

$$W = FS(0) = 0$$

See main level

(11)

Given $F = 50 \text{ N}$: $\theta = 30^\circ$: distance = 1m

$$\therefore \text{work done } W = FS \cos \theta = 50 \times 1 \times \cos 30$$

$$= 50 \times \frac{\sqrt{3}}{2}$$

$$\approx 25\sqrt{3} \text{ J}$$



②

Given mass $m = 5 \text{ kg}$; distance $= 4 \text{ m}$; acceleration $= 3 \text{ m/s}^2$

$$\text{Force } F = ma = 5 \times 3 = 15 \text{ N}$$

$$\text{work done} = F \times \text{displacement} \Rightarrow 15 \times 4 = 60 \text{ J} \rightarrow \text{D}$$

③

Given

$$\text{work} = 64,000 \text{ J} ; \text{Force} = 8,000 \text{ N}$$

$\therefore$ Here the displacement and force are in same direction

$$W = F \times S$$

$$\Rightarrow 64,000 = 8000 \times S$$

$$\Rightarrow S = \frac{64}{8} = 8 \text{ m} \rightarrow \text{C}$$

④

mass $m = 3 \text{ kg}$; initial velocity $u = 0$

displacement $x = 6 \text{ m}$; acceleration $a = 4 \text{ m/s}^2$

$$\text{work done} = F \times x = m a x$$

$$= 3 \times 4 \times 6 \quad \text{Area of Graph}$$

$$\Rightarrow 3 \left[4 \times 4 + \frac{1}{2} \times 4 \times 2 \right]$$

$$\Rightarrow 3[16 + 4] = 3 \times 20 = 60 \text{ J} \rightarrow \text{D}$$

⑤

Given

$T = 200 \text{ N}$; distance $= 20 \text{ m}$, $\theta = 60^\circ$

$$\text{work done} = F s \cos \theta = (200)(20) \cos 60^\circ$$

$$= 4000 \times \frac{1}{2} = 2000 \text{ J} \rightarrow \text{B}$$



Given

mass $m = 2 \text{ kg}$; displacement $x = \frac{t^3}{3}$

$$v = \frac{dx}{dt} = \frac{d}{dt} \left[\frac{t^3}{3} \right] = \frac{t^2}{1} = t^2$$

At $t=0 \rightarrow$ initial velocity $u = (0)^2 = 0$

At $t=2 \text{ sec} \rightarrow$ Final velocity $v = (2)^2 = 4 \text{ m/s}$

$$\therefore \text{work done} = \frac{1}{2} m v^2 = \frac{1}{2} \times 2 \times 4^2 = 16 \text{ J} \rightarrow B$$

(7)

Given mass $= m = 5 \text{ kg}$; $\theta = 30^\circ$; length $l = 10 \text{ m}$

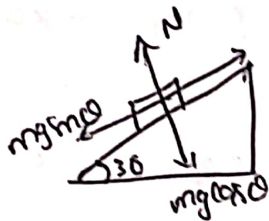
work done by gravity $= mg l \cos \theta$

$$= 5 \times 9.8 \times 10 \times \cos 60$$

[$\theta = 60^\circ$, it is the angle between force and displacement]

$$= 490 \times \frac{1}{2}$$

$$= 245 \text{ J} \rightarrow A$$



(8)

From given data displacement $= (3-1)\hat{i} + (3-3)\hat{j} + (6-2)\hat{k}$
 $= 2\hat{i} + 0\hat{j} + 4\hat{k} = 2\hat{i} + 4\hat{k}$

Net force acting $F = \vec{F}_1 + \vec{F}_2 = \hat{i} + 2\hat{j} + 2\hat{k} + 3\hat{j} + 4\hat{k}$
 $= 5\hat{j} + \hat{i} + 6\hat{k}$

$\therefore$ work done $= \vec{F} \cdot \vec{s} = (\hat{i} + 5\hat{j} + 6\hat{k}) \cdot (2\hat{i} + 4\hat{k})$

$$\Rightarrow 2 + 6(4) = 2 + 24$$

$$= 26 \text{ J}$$

⑧

⑨

while moving from $(0,0) \rightarrow (a,0)$

along + x-axis, $\vec{F} = -kx\hat{j}$

$$y=0$$

i.e. Force is -ve, y-direction, while displacement is in +ve x-direction

Because $F \perp$ to displacement

Then particle moves from $(a,0)$ to (a,a) along a line parallel to y-axis ($x=a$) during this $\vec{F} = -k(y\hat{i} + a\hat{j})$

The first component of force $-ky\hat{i}$ will not contribute any work because this component is along -ve x-direction ($-\hat{i}$) while displacement is in +ve y-direction $(a,0)$ to (a,a) .

The second component of force $-ka\hat{j}$ will perform -ve work.

$$\therefore w_2 = (-ka\hat{j}) \cdot (a\hat{j}) = (-ka)(a) = -ka^2$$

$$\therefore \text{net work} = w = w_1 + w_2 = 0 + (-ka^2) = \underline{-ka^2}$$

IInd method

$$\text{Given } \vec{F} = -k(x\hat{j} + y\hat{i})$$

$$\text{we know that } w = \int \vec{F} \cdot d\vec{s} = \int_{(0,0)}^{(a,a)} -k(y\hat{i} + x\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$w = -k \left[\int_{(0,0)}^{(a,a)} y dx + \int_{(0,0)}^{(a,a)} x dy \right]$$

$$= -k \int_{(0,0)}^{(a,a)} d(xy) = -k [xy]_{(0,0)}^{(a,a)} = -k(a^2)$$

$$= -ka^2$$



Given

$$M_{\text{bucket}} = 20 \text{ kg}; \quad \text{depth } (d) = 20 \text{ m}$$

$$\text{or } x = 20 \text{ m.}$$

weight of the bucket at the top $m = 10 \text{ kg}$.

$$\text{The rate of weight loss per meter} = \frac{\text{Total weight loss}}{\text{depth}}$$

$$= \frac{10}{20} = 0.5 \text{ kg/m.}$$

mass of the bucket at depth x can be expressed as

$$m(x) = 20 - 0.5x.$$

$$\therefore \text{Force } F(x) = (20 - 0.5x)g = (20 - 0.5x)10$$
$$= 200 - 5x \text{ N.}$$

work done

$$W = \int_0^{20} F \, dx = \int_0^{20} (200 - 5x) \, dx$$

$$= \int_0^{20} 200 \, dx - \int_0^{20} 5x \, dx$$

$$= 200[x]_0^{20} - 5\left[\frac{x^2}{2}\right]_0^{20} = 200(20-0) - \frac{5}{2}(20^2-0)$$

$$= 4000 - \frac{5}{2} \times 400$$

$$= 4000 - 1000 = 3000 \text{ J}$$

(ii)

we know

$$W = FS \cos \theta$$

$$\text{If } F \perp S \Rightarrow \theta = 90^\circ \Rightarrow W = FS \cos 90^\circ$$
$$= FS(0) = 0$$

$$\text{If } S = 0 \Rightarrow W = F(0) \cos \theta = 0.$$

(14), (15), (16)

Given

$$M_{\text{block}} = 2.5 \text{ kg} ; \text{ displacement } S_x = 2.02 \text{ m}$$

$$\text{Force} = 16 \text{ N}$$

$$\theta = 45^\circ$$

work done by applied force

$$W = F S \cos \theta$$

$$= 16 \times 2.02 \times \cos 45^\circ$$

$$= 16 \times 2.02 \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow 16 \times 1.01 \times \sqrt{2}$$

$$\Rightarrow 24.0886 \Rightarrow 24.09 \text{ J}$$

work done by gravity $W_g = F_g S \cos \theta$

Here gravity and displacement are perpendicular

$$\theta = 90^\circ \Rightarrow \cos 90^\circ = 0 \Rightarrow W_g = F_g S (0) = 0 \text{ J}$$

Here Normal force and displacement are perpendicular

$$\text{so } W = 0$$

(17)

Given

$$S = t^2 + 2t \Rightarrow \text{velocity } v = \frac{ds}{dt} = \frac{d}{dt} [t^2 + 2t]$$

$$m = 2 \text{ kg}$$

$$v = 2t + 2$$

$$\text{Acceleration } a = \frac{dv}{dt} = \frac{d}{dt} [2t + 2] = 2 \frac{dt}{dt} + \frac{d}{dt} (2) = 2 + 0 = 2 \text{ m/s}^2$$

$\therefore$ work done by the force

$$W = \int F ds$$

$$\left(\because v = \frac{ds}{dt} \right)$$

$$ds = v dt$$

$$= \int m a v dt$$

$$= \int_2^4 2 \times 2 \times (2t + 2) dt$$

$$= \int_2^4 (8t + 8) dt$$

$$\Rightarrow \int_2^4 8t dt + \int_2^4 8 dt$$

$$\Rightarrow 8 \left[\frac{t^2}{2} \right]_2^4 + 8 [t]_2^4$$

$$\Rightarrow 8 \left[\frac{4^2 - 2^2}{2} \right] + 8 [4 - 2] = 64 \text{ J}$$