moving down
acts in upward
M

WS-9 - Work

Think

①

$\vec{r_1} = 3\hat{i} + 2\hat{j} - 5\hat{k}$: $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$

$\therefore$ $W = F \cdot r \Rightarrow (2\hat{i} - \hat{j} - \hat{k}) \cdot (3\hat{i} + 2\hat{j} - 5\hat{k})$

$W = (2 \cdot 3) + (-1 \cdot 2) + (-1)(-5)$

$= 6 - 2 + 5 = 9\,J$

②

$r = 7m$: depth $h = 20m$: $d_w = 10m$

$m = d \times v = 10^3 \times \pi r^2 h = 10^3 \times \frac{22}{7} \times 7^2 \times 10 = 154 \times 10^4\,kg$

$W.D = \cancel{}\ \cancel{\tfrac{1}{2}}\ Mg\left[h + \frac{d_w}{2}\right]$

$= 154 \times 10^4 \times 10\,[20 + 5]$

$= 385 \times 10^6\,J = 385\,MJ$

③

$n = 20$

$m = 100\,gm = \frac{1}{10}\,kg$

$U_p = \cancel{n}\ mg\frac{h}{2}$

$h = 10\,cm = 10^{-2}\,m$

$= \frac{1}{10} \times 10 \times \frac{10}{2} \times 10^{-2} = 1\,J$

After Piling . $U_f = -M_{system}\, g\, h_{com\ of\ system}$

$= -20 \times \frac{1}{10} \times 10 \times .5 \times 20 \times \frac{1}{100}$

$= -20\,J$

$W = -U_f + U_p = -20 + 1 = 19\,J$

(4)

$F = \hat{i} + 2\hat{j} + 3\hat{k}$ $\quad S = 4\hat{k}$?

$W = F \cdot S = (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot 4\hat{k}$

$\Rightarrow 3 \times 4 = 12J$

(5)

$v = kx \Rightarrow \frac{dv}{dx} = k.$

acceleration $\quad a = v\frac{dv}{dx}.$

$\Rightarrow a = v \times k \Rightarrow (kx)k$

$\Rightarrow a = k^2 x$

$F = ma = mk^2 x$

work $= \int_{x_1}^{x_2} F dx = \int_0^d mk^2 x \, dx$

$= mk^2 \left[\frac{x^2}{2}\right]_0^d = \frac{mk^2 d^2}{2}$

(6)

$l = 5m$; $\theta = 30°$ $\quad m = 10$ kg.

$W = mg \sin\theta \, l$

$= 10 \times 9.8 \times \sin 30 \times 5$

$= 98 \times \frac{1}{2} \times 5$

$= 49 \times 5$

$= 245J$

(2)

$m = 10\,kg$; $v = 2\,m/s$.

$\uparrow a$ F $h = 10m$ mg

$\cancel{F = mg}$

Net force $= F - mg$

$\Rightarrow ma = F - mg$

$F = ma + mg$

$F = m(g + a) \Rightarrow$

$F = 10\,(9.8 + 0.2) = 100\,N$

From $v^2 - u^2 = 2as$

$\Rightarrow v^2 = 2as$

$\Rightarrow a = \frac{v^2}{2s} = \frac{2^2}{2 \times 10}$

$\Rightarrow a = 0.2\,m/s^2$

$W = F \times h = 100 \times 10 = 1000\,J$

(3)

$w_{bucket + Rope} = 20 + 0.2x$

$m_b = 20$

$h = 20m$

$\lambda = \frac{m}{l} = 0.2$

For x meter

$m_{Rope} = 0.2x$

work done in raising the rope + bucket by depth $dx = dw$

$\Rightarrow dw = mg\,dx$

Integrating on both sides from $0 \rightarrow 20m$

$$\Rightarrow w = \int_0^{20} (20 + 0.2x)\,g\,dx.$$

$$= \int_0^{20} 20g\,dx + 0.2\int_0^{20} xg\,dx$$

$$= 20 \times 10\,[x]_0^{20} + 0.2 \times 10\left[\frac{x^2}{2}\right]_0^{20}$$

$$\Rightarrow 200[20 - 0] + \cancel{2}\left(\frac{400 - 0}{\cancel{2}}\right) = 4000 + 400$$

$$= 4400\,J$$

(9)

Mass of block = M

$h = d$; $a = \frac{g}{3}$.

$F = m(a - g)$

$= m(\frac{g}{3} - g)$

$= -\frac{2mg}{3}$

$\omega = F \times d$.

$= -\frac{2}{3} mg d$.

(10)

$F = 5\hat{i} - 3\hat{j} + 2\hat{k}$ N : $\vec{r_1} = 2\hat{i} + 7\hat{j} + 4\hat{k}$ m

$r_2 = 5\hat{i} + 2\hat{j} + 8\hat{k}$ m

$\vec{s} = \vec{r_2} - \vec{r_1} = 5\hat{i} + 2\hat{j} + 8\hat{k} - 2\hat{i} - 7\hat{j} - 4\hat{k}$

$\vec{s} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

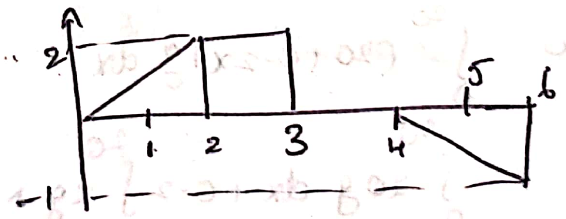
$\omega = \vec{F} \cdot \vec{s} = (5\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (3\hat{i} - 5\hat{j} + 4\hat{k})$

$\Rightarrow 5 \cdot 3 + (-3)(-5) + 2 \times 4$

$\Rightarrow 15 + 15 + 8 = 38$ J

(14), (15), (16)

From



(14)

From fig $x = 0 \to 3$ m $\omega = F \times d$ = Area of graph

$= \frac{1}{2} \times b_1 \times h_1 + b_2 h_2$

$= \frac{1}{2} \times \cancel{2} \times 2 + 1 \times 2 = 2 + 2 = 4$ J

(16)

From $x = 3 \to 4$ m $\Rightarrow$ displacement $= 0$

$\omega = F \times d = F \times 0 = 0$ J

(15) continuation

From $x = 4m$ to $6m$.

$W =$ Area of graph $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 2 \times (-1)$

$= -1J$

(17)

$F = 2\hat{i} + 3\hat{j}$ N. $\rightarrow$ (1)

Given $3y + kx = 5$ $\Rightarrow y = -\frac{k}{3}x + \frac{5}{3}$.

comparing with $y = mx + c$

so $m_1 = -\frac{k}{3}$ From (1) $\tan\theta =$ slope $m_2 = \frac{3}{2}$

$\therefore$ F $\perp^{er}$ to displacement work done is zero.

ie. $m_1 m_2 = -1$ $\Rightarrow \frac{3}{2} \times (-\frac{k}{3}) = -1$

$\Rightarrow k = 2$.

(18)

$m_{cycle} = 1$ kg

Length $= 1.6$ m

$g = 10 m/s^2$.

whenever the object like (ladder, stick, chain etc) are in contact with some other object or walls at the end point & its mass is divided into 2 equal p

work done $= \frac{mgh}{2}$

$\therefore W = \frac{1 \times 10 \times 1.6}{2}$

$W = 8J$

(19)

$v = 3t$

$\Rightarrow \frac{dx}{dt} = 3t$

$\Rightarrow dx = 3t\,dt$

$\therefore x = \int 3t\,dt$

$x = 3\frac{t^2}{2}$

acceleration $a = \frac{dv}{dt} \Rightarrow \frac{d}{dt}(3t)$

$a = 3\frac{dt}{dt} = 3\ m/s^2$

Given $m = 5$ kg

$W = FS$

$= maS$

$\Rightarrow W = 5 \times 3 \times \frac{3t^2}{2}$

At $t = 2$ sec

$\Rightarrow W = 15 \times 3 \times \frac{\cancel{2^2}}{\cancel{2}} = 15 \times 3 \times 2 = 90J$

<u>Task</u> <u>QUE's</u>

(6)

$W = FS\cos\theta$

if $\theta \to 90^\circ \to 180^\circ$

$\cos\theta = -ve.$

$\therefore W = -ve$

(8)

$W = FS\cos\theta$

if $\theta \Rightarrow 0 \to 90^\circ$

$\therefore W = +ve.$

$\cos\theta = +ve.$

(9)

$W = FS\cos\theta$

when $\theta = 0^\circ$

$\therefore W = $ maximum

$= FS$

$\cos\theta = 1 \to$ maximum value of $\cos\theta$.

(10) When body moves in circular path centripetal force and displacement are perpendicular $\theta = 90^\circ$

$W = FS\cos 90^\circ = \underline{0}.$

Scanned with OKEN Scanner

(1)

$M = 4\,kg$; $L = 2m$ length of hanging part $= 0.6m$

mass of hanging part $m = \frac{M}{L} \times l = \frac{4}{2} \times 0.6$

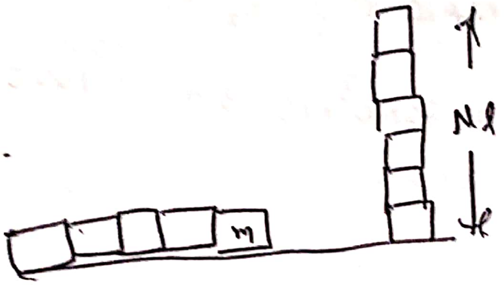
$m = 1.2\,kg.$

work done to pull hanging part $W = mg\frac{h}{2}$

$= 1.2 \times 10 \times \frac{0.6}{2}$

$= 3.6\,J$

(2)



P.E of each block $= mg\frac{l}{2}$

For n blocks.

$P.E_1 = n\,mg\frac{l}{2}$

Total length of arrangement $= \frac{nl}{2}$

$\therefore$ The final P.E of the system $PE_2 = nmg\left(\frac{nl}{2}\right)$

$\therefore$ work done $= P.E_2 - P.E_1$

$= n\,mg\,\frac{nl}{2} - mng\frac{l}{2}$

$= mg\frac{l}{2}\,n(n-1)$

(3)

Total mass $= m = 50 + 20 = 70$

height $h = 20 \times 0.2 = 4m$

work done $= mgh = 70 \times 10 \times 4$

$= 2800\,J$

(4)

mass = density × vol

$= 10^3 \times 6 \times 4 \times 2 = 72 \times 10^3$ kg

when face with large area is in contact with the ground its height is 2m

Centre of mass is at a height of $1m = h_1$

when face with small area is in contact with the ground it's height is 6m. • centre of mass height $h_2 = 3m$.

$w = \cancel{72 \times 10}\ mg(h_2 - h_1)$

$= 72 \times 10^3 \times 10 (3-1) = 1440$ kJ

(5)

Given $m = 5$ kg ; $u = 4$ m/s ; $v = 8$ m/s. $F = 10$ N.

$F = 10$ N

$a = \frac{F}{m} = \frac{10}{5} = 2$ m/s^2

From $v^2 - u^2 = 2as$

$\Rightarrow 64 - 16 = 2 \times 2 \times S$

$\Rightarrow 48 = 4S$

$\Rightarrow S = 12$ m

$W = F \cdot S$

$= 10 \times 12$

$= 120$ J

(6)

$m = 5$ kg ; $\theta = 30°$ $L = 2$ m.

work done $= mg \frac{L}{2} \sin\theta$

$= 5 \times 10 \times \frac{2}{2} \times \sin 30 = 50 \times \frac{1}{2} = \underline{25}$ J

Scanned with OKEN Scanner

$F_1 = mg = 300N$; $l = 5m$, C.G = 2m from A.

$W = F_1 d_1 + F_2 d_2$ $\quad F_2 = 80N$

$= 300 \times 2 + 80 \times 5$ $\quad d_1 = 2m$

$= 1000J$

8

$F = 4i + 6j$ N

slope of force $m_1 = \tan\theta = \frac{6}{4} = \frac{3}{2}$.

Given line $3y + kx = 5 \Rightarrow y = \frac{5 - kx}{3} = \frac{5}{3} - \frac{k}{3}x$

compare with $y = mx + c$

$\therefore$ slope $m = m_2 = \frac{-k}{3}$

Here F & displacement are Perpendicular to each oth

$\therefore m_1 m_2 = -1$

$\Rightarrow \frac{3}{2}\left(-\frac{k}{3}\right) = -1 \Rightarrow k = 2$

9

For motion of the Particle from (0,0) (a,0)

$\vec{F} = -k(0\hat{i} + a\hat{j}) = -ka\hat{j}$

Displacement $\vec{r} = (a\hat{i} + 0\hat{j}) = a\hat{i}$

$W = \vec{F} \cdot \vec{r} = -ka\hat{j} \cdot a\hat{i} = 0$.

For motion from (a,0) to (a,a)

$\vec{F} = -k(a\hat{i} + a\hat{j})$

9th continuation.

Displacement

$$\vec{r} = (a\hat{i} + a\hat{j}) - (a\hat{i} + 0\hat{j})$$
$$= a\hat{j}.$$

$\therefore$ work done $\quad W = \vec{F} \cdot \vec{r}$

$$= -k(a\hat{i} + a\hat{j}) \cdot a\hat{j}$$
$$= -ka^2$$

(10)

Total weight = 20 kg

As bucket is leaking for every meter 0.5 kg mass reduced

$$\therefore \quad F = (20 - 0.5x)\, g$$

For displacement dx

$$W = \int_0^{20} F\, dx. = \int_0^{20} (20 - 0.5x) g\, dx$$

$$\Rightarrow \left[20x - 0.5\frac{x^2}{2} \right]_0^{20} g$$

$$\Rightarrow \left(20(20-0) - \frac{0.5}{2}(20^2 - 0)\right] \times 10$$

$$\Rightarrow \left(400 - \frac{0.5}{\cancel{2}} \times \cancel{400}^{200}\right] \times 10$$

$$\Rightarrow (400 - 100) \times 10 = 3000\ kJ$$

$$= 3\ kJ$$

14, 15, 16

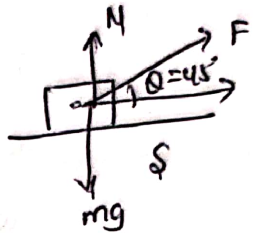
$m = 2.5\ kg$. $S = 2.2\ m$

14 $W = FS\cos\theta$ $F = 16N$

$= \cancel{mg}\ FS\cos 45$

$\Rightarrow 16 \times 2.2 \times \frac{1}{\sqrt{2}}$

$= 24.9\ J$



15 $W_g = mg\cos\theta \times S$ $\theta = 90°$

$\Rightarrow 2.5 \times 10 \times \cos 90 \times 2.2$

$= 0.$

16 $W_N = NS\cos\theta$ $N \perp S$ $\theta = 90°$

$= NS\cos 90$

$= 0.$

17

$S = t^2 + 2t.\ ; m = 2.$

$V = \frac{ds}{dt} = \frac{d}{dt}(t^2 + 2t)$

$= 2t + 2$

$V_1 = 2(2) + 2 \rightarrow t_1 = 2\ sec$

$= 6\ m/s$

For $t = 4$ sec

$V_2 = 2(4) + 2$

$= 10\ m/s$

W-E theorem

$W = \frac{1}{2} m (V_2^2 - V_1^2)$

From 2 → 4 sec

$W = \frac{1}{2} m \left(\cancel{(2t+2)^2 - (2t+2)}\right)$

$W = \frac{1}{2} \times 2 [10^2 - 6^2]$

$= [16] \times 4$

$= 64\ J$

(18)

$F = 10N$

$S = 10m$

$W = FS\cos\theta$

$= 10 \times 10 \times \cos 120°$

$= 100 \times (-\frac{1}{2})$

$= -50J$

(19)

$F = 100N$; $m = 10kg$; $S = 5m$

$t = 2 sec.$

weight $= mg = 10 \times 10 = 100N$

velocity $= \frac{s}{2} = \frac{5}{2} = 2.5 m/s$

$W = FS \cos\theta$ $(\theta = 0)$

$= 100 \times 5 \times \cos 0$

$= 500J$ +ve.