Chapter: Elimination Of Angle (θ) by Using Trigonometric Identities

TEACHING TASK

01. Given $\sin\theta + \cos\theta = p$ and $\tan\theta + \cot\theta = q$

Let $\theta = 45^\circ$

$\sin 45^\circ + \cos 45^\circ = p$ and $\tan 45^\circ + \cot 45^\circ = q$

$\therefore p = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$q = 1 + 1 = 2$

Now $q(p^2 - 1) = (2)\left((\sqrt{2})^2 - 1\right) = 2(2-1) = 2$

Ans: B

02. Given $\theta = 45^\circ$

$a = x\cos\theta + y\sin\theta$

$a = x\cos 45^\circ + y\sin 45^\circ$

$a = x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right)$

$a = \frac{x+y}{\sqrt{2}}$

$b = x\sin\theta - y\cos\theta$

$b = x\sin 45^\circ - y\cos 45^\circ$

$b = x\left(\frac{1}{\sqrt{2}}\right) - y\left(\frac{1}{\sqrt{2}}\right)$

$b = \frac{x-y}{\sqrt{2}}$

Now, $a - b = \frac{x+y}{\sqrt{2}} + \frac{x-y}{\sqrt{2}} = \frac{2x}{\sqrt{2}} = \sqrt{2}\,x$

Ans: B

03. Given $\sec\theta = a + \frac{1}{4a}$

$\Rightarrow 2\sec\theta = 2a + \frac{1}{2a}$

$\Rightarrow (\sec\theta + \tan\theta) + (\sec\theta - \tan\theta) = 2a + \frac{1}{2a}$

$\therefore \sec\theta + \tan\theta = 2a$

Ans: B

04. Given $m = a\cos^3\theta + 3a\cos\theta.\sin^2\theta$

Let $\theta = 0^\circ$

$\therefore \quad m = a\cos^3 0^\circ + 3a\cos 0^\circ.\sin^2 0^\circ$

$\Rightarrow \quad m = a$

Given $n = a\sin^3\theta + 3a\sin\theta.\cos^2\theta$

Let $\theta = 0^\circ$

$\therefore \quad n = a\sin^3 0^\circ + 3a\sin 0^\circ.\cos^2 0^\circ$

$\Rightarrow \quad n = 0$

Now $(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}}$

$= (a+0)^{\frac{2}{3}} + (a-0)^{2/3}$

$= 2a^{\frac{2}{3}}$

Ans. D.

05 Given $x = a\cos^3\theta.\sin^2\theta$

$\Rightarrow x^2 = a^2\cos^6\theta.\sin^4\theta$

$y = a\sin^3\theta.\cos^2\theta$

$\Rightarrow y^2 = a^2\sin^6\theta.\cos^4\theta$

Now $x^2+y^2 = a^2\cos^6\theta.\sin^4\theta + a^2\sin^6\theta.\cos^4\theta$

$= a^2\cos^4\theta\sin^4\theta(\cos^2\theta+\sin^2\theta)$

$= a^2\cos^4\theta.\sin^4\theta$

$xy = a\cos^3\theta.\sin^2\theta.a\sin^3\theta.\cos^2\theta$

$= a^2\cos^5\theta.\sin^5\theta$

Now x^2y

$$(x^2+y^2)^{\frac{1}{4}} = a^{\frac{1}{2}}.\cos\theta.\sin\theta$$

Also $(xy)^{\frac{1}{5}} = a^{\frac{2}{5}}.\cos\theta.\sin\theta$

$$\therefore \frac{(x^2+y)^{\frac{1}{4}}}{(xy)^{\frac{1}{5}}} = \frac{a^{\frac{1}{2}}.\cos\theta.\sin\theta}{a^{\frac{2}{5}}.\cos\theta.\sin\theta} = a^{\frac{1}{10}}$$, which is Independent of θ

$\therefore p = \frac{1}{4}$, $q = \frac{1}{5}$

$\Rightarrow 4p = 5q$

Ans: A

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06 Given $x = \cot\theta + \tan\theta$ $\quad y = \sec\theta - \cos\theta$

Let $\theta = 45^\circ$

$\therefore x = \cot 45^\circ + \tan 45^\circ$

$x = 1+1$

$x = 2$

$y = \sec 45^\circ - \cos 45^\circ$

$y = \sqrt{2} - \frac{1}{\sqrt{2}}$

$y = \frac{1}{\sqrt{2}}$

Now $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}}$

$= \left(4.\frac{1}{\sqrt{2}}\right)^{\frac{2}{3}} - \left(2.\frac{1}{2}\right)^{\frac{2}{3}}$

$= (2\sqrt{2})^{\frac{2}{3}} - 1$

$= \left(2^{\frac{3}{2}}\right)^{\frac{2}{3}} - 1$

$= 2 - 1$

$= 1$

Ans: C

07. Given $x = a(\operatorname{cosec}\alpha + \cot\alpha)$ | $y = \dfrac{b(1-\cos\alpha)}{\sin\alpha}$

Let $\alpha = 45^\circ$

$\therefore x = a(\operatorname{cosec}45^\circ + \cot 45^\circ)$ | $y = \dfrac{b(1-\cos 45^\circ)}{\sin 45^\circ}$

$\Rightarrow x = a(\sqrt{2}+1)$ | $y = \dfrac{b(1-\frac{1}{\sqrt{2}})}{\frac{1}{\sqrt{2}}}$

$y = b(\sqrt{2}-1)$

Now $xy = a(\sqrt{2}+1)\cdot b(\sqrt{2}-1)$

$\Rightarrow xy = ab(2-1)$

$\Rightarrow xy = ab$

Ans. D

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08. Given $a\sec\theta + b\tan\theta + c = 0 \rightarrow (1)$

$p\sec\theta + q\tan\theta + r = 0 \rightarrow (2)$

$(1)\times q \Rightarrow aq\sec\theta + bq\tan\theta + cq = 0 \rightarrow (3)$

$(2)\times b \Rightarrow bp\sec\theta + bq\tan\theta + br = 0$

$(aq-bp)\sec\theta + (cq-br) = 0$

$\Rightarrow \sec\theta = \dfrac{br-cq}{aq-bp}$

Similarly $\tan\theta = \dfrac{pc-ar}{aq-bp}$

$\operatorname{ec}\alpha + \cot\alpha)$

$= 45^\circ$

$^\circ + \cot 45^\circ)$

1)

$$y = \frac{b(1-\cos\alpha)}{\sin\alpha}$$

$$y = \frac{b(1-\cos 45^\circ)}{\sin 45^\circ}$$

$$y = \frac{b\left(1-\frac{1}{\sqrt{2}}\right)}{\frac{1}{\sqrt{2}}}$$

$$y = b(\sqrt{2}-1)$$

1). $b(\sqrt{2}-1)$

$-1)$

Ans. D

$mx + c = 0 \rightarrow (1)$

Now $\sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow \left(\frac{br-qc}{aq-bp}\right)^2 - \left(\frac{pc-ar}{aq-bp}\right)^2 = 1$$

$$\Rightarrow (br-qc)^2 - (pc-ar)^2 = (aq-bp)^2$$

Ans : C

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09 Given $x = a\sec\theta.\cos\phi$ | $y = b\sec\theta.\sin\phi$ | $z = c\tan\theta$

$\Rightarrow \frac{x}{a} = \sec\theta.\cos\phi$ | $\frac{y}{b} = \sec\theta.\sin\phi$ | $\frac{z}{c} = \tan\theta$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = \sec^2\theta.\cos^2\phi + \sec^2\theta.\sin^2\phi - \tan^2\theta$$

$$= \sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta$$

$$= \sec^2\theta - \tan^2\theta$$

$$= 1$$

Ans : A

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10. Given $x = a\sec\theta$; $y = b\tan\theta$

$\Rightarrow \frac{x}{a} = \sec\theta$; $\frac{y}{b} = \tan\theta$

$\Rightarrow \frac{x^2}{a^2} = \sec^2\theta$, $\frac{y^2}{b^2} = \tan^2\theta$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Ans : B

11. Given $\sin A + \cos A = p$, $\sin^3 A + \cos^3 A = q$

Let $A = 0^\circ$

$\therefore \sin 0^\circ + \cos 0^\circ = p$, $\sin^3 0^\circ + \cos^3 0^\circ = q$

$\therefore p = 1$, $q = 1$

Now $p^3 - 3p + 2q = (1)^2 - 3(1) + 2(1)$

$= 1 - 3 + 2$

$= 0$

Ans: C

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12. Given $\sec\theta + \tan\theta = x \longrightarrow (1)$

$\Rightarrow \sec\theta + \tan\theta = \frac{1}{x} \longrightarrow (2)$

$(1)+(2) \Rightarrow 2\sec\theta = x + \frac{1}{x}$

$\Rightarrow \sec\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$

$(1)-(2) \Rightarrow 2\tan\theta = x - \frac{1}{x}$

$\Rightarrow \tan\theta = \frac{1}{2}\left(x - \frac{1}{x}\right)$

Ans: A, C

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13. Given $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

Let $\theta = 45^\circ$

$(\operatorname{cosec} 45^\circ - \sin 45^\circ)(\sec 45^\circ - \cos 45^\circ)(\tan 45^\circ + \cot 45^\circ)$

$= \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)(1+1)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(2) = 1$

Ans: B

14. $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

Let $\theta = 45^\circ$

$= (1+\tan 45^\circ+\sec 45^\circ)(1+\cot 45^\circ-\operatorname{cosec} 45^\circ)$

$= (1+1+\sqrt{2})(1+1-\sqrt{2})$

$= (2+\sqrt{2})(2-\sqrt{2})$

$= (4-2) = 2.$

Ans: C

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15 Statement – I

Given $\sin\theta+\cos\theta = x$

Let $\theta = 0^\circ$

$\therefore \sin 0^\circ + \cos 0^\circ = x$

$\therefore x = 1$

$\sin^6\theta + \cos^6\theta = \sin^6 0^\circ + \cos^6 0^\circ$

$= 1$

$\frac{4-3(x^2-1)}{4} = \frac{4-3(1^2-1)}{4}$

$= 1$

$\therefore$ Statement – I is correct.

Statement – II $\sin^6\theta + \cos^6\theta = 1-3\sin^2\theta\cdot\cos^2\theta$

Let $\theta = 0^\circ$

$\sin^6 0^\circ + \cos^6 0^\circ = 1$

$1-3\sin^2 0^\circ\cdot\cos^2 0^\circ = 1$

$\therefore$ Statement – II is Correct

Ans: A

16. Statement-I

Given

$x = a\sec\theta + b\tan\theta$

$\Rightarrow x^2 = a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta$

Again $y = a\tan\theta + b\sec\theta$

$\Rightarrow y^2 = a^2\tan^2\theta + b^2\sec^2\theta + 2ab\tan\theta\sec\theta$

Now $x^2 - y^2 = a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$

$\Rightarrow x^2 - y^2 = a^2 - b^2$

Statement-I is Correct.

Statement-II

Given $\sec\theta + \tan\theta = a$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{a}$

$\Rightarrow \tan\theta - \sec\theta = -\frac{1}{a}$

Statement-II is Correct.

Ans: A

17. Given $l = \operatorname{cosec}\theta - \sin\theta$ | $m = \sec\theta - \cos\theta$

Let $\theta = 45^\circ$

$l = \operatorname{cosec} 45^\circ - \sin 45^\circ$ | $m = \sec 45^\circ - \cos 45^\circ$

$\Rightarrow l = \sqrt{2} - \frac{1}{\sqrt{2}}$ | $m = \sqrt{2} - \frac{1}{\sqrt{2}}$

$\Rightarrow l = \frac{1}{\sqrt{2}}$ | $m = \frac{1}{\sqrt{2}}$

Now $l^2m^2(l^2+m^2+3)$

$= \left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2\left(\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+3\right)$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}+\frac{1}{2}+3\right)$

$= \frac{1}{4}(4) = 1$

∴ Statement I is true

Statement II:

$$\frac{\cos\theta}{1-\sin\theta} = \frac{\cos\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

Again $\frac{\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta} = \frac{\sin\theta(1-\sin\theta)}{1-\sin^2\theta}$

$= \sin\theta(1-\sin\theta)$

$$\frac{\sin\theta}{1+\sin\theta} = \frac{\cos\theta}{1+\cos\theta}$$

Let $\theta = 0^\circ$ $\quad \frac{\sin 0^\circ}{1+\sin 0^\circ} = \frac{\cos 0^\circ}{1+\cos 0^\circ}$

$\frac{0}{1+0} = \frac{1}{1+1}$

$0 = \frac{1}{2}$ (False)

∴ statement II : False

Ans: C

17. Given $\cos A + \cos^2 A = 1$

$\Rightarrow \cos A = 1 - \cos^2 A$

$\Rightarrow \cos A = \sin^2 A$

18. $\sin^2 A + \sin^4 A$

$\Rightarrow \cos A + \cos^2 A = 1$ Ans: B

19. Now, $\sin^4 A - 2\sin^2 A - \sin^8 A$

$= \cos^2 A - 2\cos A - \cos^4 A$

$= \cos^2 A - \cos^4 A - 2\cos A$

$= \cos^2 A (1 - \cos^2 A) - 2\cos A$

$= \cos^2 A (\cos A) - 2\cos A$

$= \cos A (\cos^2 A - 2)$

$= \cos A (1 - \cos A - 2)$

$= \cos A (-1 - \cos A) = -\cos A - \cos^2 A = -1$

Ans: C

20. Now, $\cos^6 A + 4\cos^3 A$

$= \cos^3 A (\cos^3 A + 4)$

$= \cos^2 A \cdot \cos A (\cos^2 A \cdot \cos A + 4)$

$= (1 - \cos A)\cos A ((1 - \cos A)\cos A + 4)$

$$= (\cos A - \cos^2 A)(\cos A - \cos^2 A + 4)$$

$$= (\cos A - 1 + \cos A)(\cos A - 1 + \cos A + 4)$$

$$= (2\cos A - 1)(2\cos A + 3)$$

$$= 4\cos^2 A + 4\cos A - 3$$

$$= 4(1 - \cos A) + 4\cos A - 3$$

$$= 4 - 4\cos A + 4\cos A - 3 = 1$$

Ans : B

21. Given $2\sec\theta = x + \frac{1}{x}$, $2\tan\theta = x - \frac{1}{x}$

$\Rightarrow 4\sec^2\theta = \left(x + \frac{1}{x}\right)^2$, $4\tan^2\theta = \left(x - \frac{1}{x}\right)^2$

$\Rightarrow 4(\sec^2\theta - \tan^2\theta) = \left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2$

$\therefore \left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2 = 4.$

Ans: 4

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22. a) Given $m = \cot\theta + \tan\theta$,

$\theta = 45^\circ$

$\therefore m = \cot 45^\circ + \tan 45^\circ$,

$\Rightarrow m = 1 + 1$

$\Rightarrow m = 2$

$n = \sec\theta - \cos\theta$

$n = \sec 45^\circ - \cos 45^\circ$

$n = \sqrt{2} - \frac{1}{\sqrt{2}}$

$n = \frac{1}{\sqrt{2}}$

Now $(m^2 n)^{\frac{2}{3}} - (m n^2)^{\frac{2}{3}}$

$= \left(4 \cdot \frac{1}{\sqrt{2}}\right)^{\frac{2}{3}} - \left(2 \cdot \frac{1}{2}\right)^{\frac{2}{3}}$

$= (2\sqrt{2})^{\frac{2}{3}} - (1)^{\frac{2}{3}}$

$= \left(2^{\frac{3}{2}}\right)^{\frac{2}{3}} - 1$

$= 2 - 1$

$= 1$

b) Given

Given $m = \cos\theta\sec\theta - \sin\theta$

Let $\theta = 45^\circ$

$\Rightarrow m = \sqrt{2} - \frac{1}{\sqrt{2}}$

$\Rightarrow m = \frac{1}{\sqrt{2}}$

$n = \sec\theta - \cos\theta$

Let $\theta = 45^\circ$

$n = \sqrt{2} - \frac{1}{\sqrt{2}}$

$n = \frac{1}{\sqrt{2}}$

Now $(m^2 n)^{\frac{2}{3}} + (m n^2)^{\frac{2}{3}}$

$= \left(\frac{1}{2} \cdot \frac{1}{\sqrt{2}}\right)^{\frac{2}{3}} + \left(\frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right)^{\frac{2}{3}}$

$= \left(\frac{1}{2^{3/2}}\right)^{\frac{2}{3}} + \left(\frac{1}{2^{3/2}}\right)^{\frac{2}{3}}$

$= \frac{1}{2} + \frac{1}{2} = 1$

c) Given $a = x\cos^2\alpha + y\sin^2\alpha$

Now $(x-a)(y-a) + (x-y)^2 \sin^2\alpha \cdot \cos^2\alpha$

$= (x - x\cos^2\alpha - y\sin^2\alpha)(y - x\cos^2\alpha - y\sin^2\alpha)$

$+ (x-y)^2 \cdot \sin^2\alpha \cdot \cos^2\alpha$

$= (x(1-\cos^2\alpha) - y\sin^2\alpha)(y(1-\sin^2\alpha) - x\cos^2\alpha)$

$+ (x-y)^2 \cdot \sin^2\alpha \cdot \cos^2\alpha$

$= (x\sin^2\alpha - y\sin^2\alpha)(y\cos^2\alpha - x\cos^2\alpha)$

$+ (x-y)^2 \cdot \sin^2\alpha \cdot \cos^2\alpha$

$= (x-y)\sin^2\alpha\,(y-x)\cos^2\alpha + (x-y)^2 \cdot \sin^2\alpha \cdot \cos^2\alpha$

$= -(x-y)^2 \sin^2\alpha \cdot \cos^2\alpha + (x-y)^2 \cdot \sin^2\alpha \cdot \cos^2\alpha$

$= 0$.

d) Given $x = r\cos\alpha\cos\beta\cos\gamma$, $y = r\cos\alpha\cos\beta\sin\gamma$

$\mu = r\sin\beta$ and $z = r\sin\alpha\cdot\cos\beta$

~~Now $x^2+y^2+z^2+\mu^2$~~

~~$= r^2\cos^2\alpha\cos^2\beta$~~

Let $\alpha = \beta = \gamma = 45^\circ$

$\therefore x = r\cdot\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$x = \frac{r}{2\sqrt{2}}$ Similarly $y = \frac{r}{2\sqrt{2}}$, $\mu = \frac{r}{\sqrt{2}}$, $z = \frac{r}{2}$

Now

$x^2+y^2+z^2+\mu^2 = \frac{r^2}{8} + \frac{r^2}{8} + \frac{r^2}{2} + \frac{r^2}{4}$

$= \frac{r^2+r^2+4r^2+2r^2}{8}$

$= \frac{8r^2}{8}$

$= r^2$

Ans: a → r, b → r, c → q, d → s

LEARNERS TASK

01. $x = m\sec\theta$, $y = n\tan\theta$

$\frac{x}{m} = \sec\theta$, $\frac{y}{n} = \tan\theta$

$\Rightarrow \frac{x^2}{m^2} - \frac{y^2}{n^2} = \sec^2\theta - \tan^2\theta$

$\Rightarrow \frac{x^2}{m^2} - \frac{y^2}{n^2} = 1$

Ans: B

02. Given $x = \operatorname{cosec}\theta + \cot\theta$, $y = \operatorname{cosec}\theta - \cot\theta$

$\therefore xy = (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)$

$\Rightarrow xy = \operatorname{cosec}^2\theta - \cot^2\theta$

$\Rightarrow xy = 1$

Ans: B

03. Given $a = x\operatorname{cosec}\theta$ and $b = y\cot\theta$

$\Rightarrow \frac{a}{x} = \operatorname{cosec}\theta$, $\frac{b}{y} = \cot\theta$

$\Rightarrow \frac{a^2}{x^2} - \frac{b^2}{y^2} = \operatorname{cosec}^2\theta - \cot^2\theta$

$\Rightarrow \frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

Ans: B

04. Given $x = a(\sec\theta + \tan\theta)^2$

$y = b(\sec\theta - \tan\theta)^2$

$\Rightarrow xy = a^2(\sec\theta + \tan\theta)^2 b(\sec\theta - \tan\theta)^2$

$\Rightarrow xy = a^2 b(\sec^2\theta - \tan^2\theta)^2$

$\Rightarrow xy = a^2 b \Rightarrow x^2y^2 = a^2b^2$ opt: 1

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05. Given $x = a(\operatorname{cosec}\theta + \cot\theta)$

$y = b(\operatorname{cosec}\theta - \cot\theta)$

$\Rightarrow xy = ab(\operatorname{cosec}^2\theta - \cot^2\theta)$

$\Rightarrow xy = ab$ Ans: C

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06 $x = \cot\theta + \operatorname{cosec}\theta$, $y = \cot\theta - \operatorname{cosec}\theta$

Let $\theta = 45^\circ$

$x = \cot 45^\circ + \operatorname{cosec} 45^\circ$, $y = \cot 45^\circ - \operatorname{cosec} 45^\circ$

$\Rightarrow x = 1 + \frac{1}{\square}$, $y = 1 - \frac{1}{\square}$

06. Given $x = \cot\theta + \operatorname{cosec}\theta$; $y = \cot\theta - \operatorname{cosec}\theta$

$$x = \frac{\cos\theta}{\sin\theta} + \operatorname{cosec}\theta$$

$x + y = 2\cot\theta$, $\quad x - y = 2\operatorname{cosec}\theta$

$$(x+y)(x-y) = 4\cot\theta \cdot \operatorname{cosec}\theta$$

$$(x^2 - y^2)^2 = 16\cot^2\theta \cdot \operatorname{cosec}^2\theta$$

$$= 16\frac{\cos^2\theta}{\sin^2\theta}(1 - \sin^2\theta)$$

$$= 16\left(\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta\right)$$

$$= 16(\cot^2\theta - \cos^2\theta)$$

$$= 16(\cot\theta + \operatorname{cosec}\theta)(\cot\theta - \operatorname{cosec}\theta)$$

$$= 16xy$$

Ans: B

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07. $x = a\sin\theta$ and $y = b\tan\theta$

$$\Rightarrow \frac{a}{x} = \operatorname{cosec}\theta, \quad \frac{b}{y} = \cot\theta$$

$$\Rightarrow \frac{a^2}{x^2} - \frac{b^2}{y^2} = \operatorname{cosec}^2\theta - \cot^2\theta$$

$$= 1$$

Ans: A

02. $x = p(\operatorname{cosec}\theta + \cot\theta)$, $y = q(\cot\theta - \operatorname{cosec}\theta)$

$\Rightarrow xy = -pq(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)$

$\Rightarrow xy = -pq(\operatorname{cosec}^2\theta - \cot^2\theta)$

$\Rightarrow xy = -pq$

$\Rightarrow \frac{x}{p} = \frac{-q}{y}$

Ans: B

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03. $p = \cos\theta + \sin\theta$, $q = \sin\theta - \cos\theta$

$p^2 + q^2 = (\cos\theta + \sin\theta)^2 + (\sin\theta - \cos\theta)^2$

$= 2(\cos^2\theta + \sin^2\theta)$

$= 2$

$\therefore p^2 - 2 = -q^2$

Ans: B

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04. $x = p\cos\theta + q\sin\theta$, $y = p\sin\theta - q\cos\theta$

$x^2 + y^2 = p^2\cos^2\theta + q^2\sin^2\theta + 2pq\cos\theta\sin\theta +$
$p^2\sin^2\theta + q^2\cos^2\theta - 2pq\sin\theta\cos\theta$

$\therefore x^2 + y^2 = p^2 + q^2$

$\Rightarrow x^2 - p^2 = q^2 - y^2$

Ans. C

08 Given $x = 2\operatorname{cosec}\theta$, $y = 2\cot\theta$

$\Rightarrow \frac{x}{2} = \operatorname{cosec}\theta$, $\frac{y}{2} = \cot\theta$

$\Rightarrow \frac{x^2}{4} - \frac{y^2}{4} = \operatorname{cosec}^2\theta - \cot^2\theta = 1$ Ans: B

$\Rightarrow x^2 - y^2 = 4$

09. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$= 2(\sin^2\theta + \cos^2\theta)$

$= 2(1)$ Ans. A

$= 2$

10 Given $x = at$, $y = \frac{a}{t}$

$\Rightarrow xy = (at)\left(\frac{a}{t}\right)$

$\Rightarrow xy = a^2$ Ans: B

JEE MAIN LEVEL QUESTIONS

01. $x = a\tan\theta$, $y = b\sec\theta$

$\Rightarrow \frac{x}{a} = \tan\theta$, $\frac{y}{b} = \sec\theta$

$\Rightarrow \frac{y^2}{b^2} - \frac{x^2}{a^2} = \sec^2\theta - \tan^2\theta$

$\Rightarrow \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ | $\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Ans. A

05 $x^2 = a^2\cos^2\theta$; $y^2 = b^2\sin^2\theta$

$\Rightarrow \frac{x^2}{a^2} = \cos^2\theta$, $\frac{y^2}{b^2} = \sin^2\theta$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Ans : B

06 $x^2+y^2+z^2 = r^2\cos^2\theta.\cos^2\phi + r^2\cos^2\theta.\sin^2\phi + r^2\sin^2\theta$

$= r^2\cos^2\theta(\cos^2\phi + \sin^2\phi) + r^2\sin^2\theta$

$= r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2(\cos^2\theta + \sin^2\theta)$

$= r^2$

opt: C

07 $x = a\sin^4\theta$, $y = b\cos^4\theta$

$\frac{x}{a} = \sin^4\theta$, $\frac{y}{b} = \cos^4\theta$

$\Rightarrow \sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = \sin^2\theta + \cos^2\theta = 1$

Ans : C

08 $a = x\sec\theta + y\tan\theta$; $b = x\tan\theta + y\sec\theta$

$a^2 = x^2\sec^2\theta + y^2\tan^2\theta + 2xy\sec\theta.\tan\theta$

$b^2 = x^2\tan^2\theta + y^2\sec^2\theta + 2xy\tan\theta.\sec\theta$

$a^2 - b^2 = x^2(\sec^2\theta - \tan^2\theta) - y^2(\sec^2\theta - \tan^2\theta)$

$\therefore a^2 - b^2 = x^2 - y^2$

Ans : C

09. Let $\phi = \psi = \delta = 45^\circ$

$\therefore a = \cos 45^\circ . \cos 45^\circ + \sin 45^\circ . \sin 45^\circ . \cos 45^\circ$

$\Rightarrow a = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$

$\Rightarrow a = \frac{1}{2} + \frac{1}{2\sqrt{2}}$

Similarly $b = 0$, $c = \frac{1}{2}$

Now $a^2 + b^2 + c^2 = \left(\frac{1}{2} + \frac{1}{2\sqrt{2}}\right)^2 + 0^2 + \left(\frac{1}{2}\right)^2$

$= \frac{1}{4} + \frac{1}{8} + \frac{1}{2\sqrt{2}} + \frac{1}{4}$

Similarly $b = \frac{1}{2} - \frac{1}{2\sqrt{2}}$ and $c = \frac{1}{2}$

Now $a^2 + b^2 + c^2 = \left(\frac{1}{2} + \frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2} - \frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$

$= 2\left(\frac{1}{4} + \frac{1}{8}\right) + \frac{1}{4}$

$= 2\left(\frac{3}{8}\right) + \frac{1}{4}$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

Ans: D

10. Given $x = a\cos^2\theta \cdot \sin\theta$, $y = a\sin^2\theta \cdot \cos\theta$

Let $\theta = 45^\circ$

$\therefore x = a\cos^2 45^\circ \cdot \sin 45^\circ$, $y = a\sin^2 45^\circ \cdot \cos 45^\circ$

$\Rightarrow x = a\left(\frac{1}{\sqrt{2}}\right)^2 \cdot \left(\frac{1}{\sqrt{2}}\right)$, $y = a\left(\frac{1}{\sqrt{2}}\right)^2 \cdot \left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow x = \frac{a}{2\sqrt{2}}$, $y = \frac{a}{2\sqrt{2}}$

Now $x^2 + y^2 = \frac{a^2}{4} \Rightarrow (x^2+y^2)^3 = \frac{a^6}{64}$

$x^2y^2 = \frac{a^4}{64}$

$\therefore \frac{(x^2+y^2)^3}{x^2y^2} = \frac{a^6}{a^4} = a^2$ Ans. C

ADVANCED LEVEL QUESTIONS

11. Given $x = \sqrt{a}\,\text{cosec}\,\theta$, $y = \sqrt{a}\cot\theta$

$\Rightarrow \frac{x}{\sqrt{a}} = \text{cosec}\,\theta$, $\frac{y}{\sqrt{a}} = \cot\theta$

$\Rightarrow \frac{x^2}{a} = \text{cosec}^2\theta$, $\frac{y^2}{a} = \cot^2\theta$

$\Rightarrow \frac{x^2}{a} - \frac{y^2}{a} = 1$

$\Rightarrow x^2 - y^2 = a$

$\Rightarrow (x+y)(x-y) - a = 0$ Ans: B, C

12. Statement I:

Given $l = p\sec\theta.\cos\phi$

$\Rightarrow \frac{l}{p} = \sec\theta.\cos\phi$

Similarly $\frac{m}{q} = \sec\theta.\sin\phi$, $\frac{n}{r} = \tan\theta$

Now, $\frac{l^2}{p^2} + \frac{m^2}{q^2} + \frac{n^2}{r^2} = \sec^2\theta.\cos^2\phi + \sec^2\theta.\sin^2\phi + \tan^2\theta$

$= \sec^2\theta(\cos^2\phi + \sin^2\phi) + \tan^2\theta$

$= \sec^2\theta + \tan^2\theta \neq 1.$

Statement - I is false.

Statement - II

(i) $\frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta} = \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} = \frac{1+\cos\theta}{\sin\theta}.$

(ii) We know $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \tan\theta + \sec\theta = \frac{1}{\sec\theta - \tan\theta}$

∴ Statement - II is false

Ans: D

Given $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$, $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$ (2?)

13. $\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right)^2 = (1)^2$

$\Rightarrow \frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{2xy}{ab}\cos\theta\sin\theta = 1 \rightarrow (1)$

Again $\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right)^2 = (1)^2$

$\Rightarrow \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - \frac{2xy}{ab}\sin\theta\cos\theta = 1 \rightarrow (2)$

$(1)+(2) \Rightarrow \frac{x^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

$\Rightarrow a^2y^2 + b^2x^2 = 2a^2b^2$ Ans: D

14. We have $a^2y^2 + b^2x^2 = 2a^2b^2$

Given $a = 2, \quad b = 2$

$\therefore 4y^2 + 4x^2 = 2 \times 4 \times 4$

$\Rightarrow x^2 + y^2 = 8$ Ans: C

15. $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ Given $\theta = 45^\circ$, $a = b = 2$

$\Rightarrow \frac{x}{2}\cos 45^\circ + \frac{y}{2}\sin 45^\circ = 1$

$$\Rightarrow \frac{x}{2\sqrt{2}} + \frac{y}{2\sqrt{2}} = 1$$

$$\Rightarrow x + y = 2\sqrt{2}$$

Ans: A

~~Sir Lady x - y = 2√2~~

16. Given $U = l \tan\theta$ | $V = m \sin\theta$

$\Rightarrow \cot\theta = \frac{l}{U}$ | $\Rightarrow \operatorname{cosec}\theta = \frac{m}{V}$

$$\Rightarrow \frac{m^2}{V^2} - \frac{l^2}{U^2} = \operatorname{cosec}^2\theta - \cot^2\theta$$

$$= 1$$

Ans: 1

17.

19. a) $x = at^2, \quad y = 2at$

$\Rightarrow y^2 = 4a^2t^2$

$= 4a(at^2)$

$= 4ay$

$\therefore y^2 = 4ay$

b) $x = at, \quad y = \frac{a}{t}$

$\Rightarrow xy = (at)\left(\frac{a}{t}\right)$

$\Rightarrow xy = a^2$

c) $x = a\sec\theta, \quad y = a\tan\theta$

$\Rightarrow \frac{x}{a} = \sec\theta, \quad \frac{y}{a} = \tan\theta$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$

$\Rightarrow x^2 - y^2 = a^2$

d) $x = a\sin\theta, \quad y = b\cos\theta$

$\Rightarrow \frac{x}{a} = \sin\theta, \quad \frac{y}{b} = \cos\theta$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore$ Ans: $a \to r, \; b \to q, \; c \to p, \; d \to s$

20) a) $(x-h)^2+(y-k)^2=r^2$

let $x=h+r\cos\theta$, $y=k+r\sin\theta$

$\Rightarrow (x-h)=r\cos\theta$, $y-k=r\sin\theta$

$\Rightarrow (x-h)^2+(y-k)^2=r^2\cos^2\theta+r^2\sin^2\theta$

$=r^2(\cos^2\theta+\sin^2\theta)$

$=r^2$

b) $(x+h)^2+(y+k)^2=r^2$

let $x=-h+r\cos\theta$, $y=-k+r\sin\theta$

$\Rightarrow x+h=r\cos\theta$, $y+k=r\sin\theta$

$\Rightarrow (x+h)^2+(y+k)^2=r^2$

c) $(x-k)^2+(y-h)^2=r^2$

let $x=k+r\cos\theta$, $y=h+r\sin\theta$

$\Rightarrow x-k=r\cos\theta$, $y-h=r\sin\theta$

$\Rightarrow (x-k)^2+(y-h)^2=r^2$

d) $(x+k)^2+(y+h)^2=r^2$

let $x=-k+r\sin\theta$, $y=-h+r\cos\theta$

$\Rightarrow (x+k)=r\sin\theta$, $y+h=r\cos\theta$

$\Rightarrow (x+k)^2+(y+h)^2=r^2$

Ans: a → t, b → s

c → q, d → p

ADDITIONAL PRACTICE QUESTIONS FOR STUDE

01 Given $x = a^2(\sec\theta + \tan\theta)^2$, $y = b^2(\sec\theta - \tan\theta)^2$

$\Rightarrow xy = a^2b^2(\sec^2\theta - \tan^2\theta)^2$

$\Rightarrow xy = a^2b^2$ Ans : A

02. Given $a = b = 1$, $\theta = 60^\circ$

$\therefore x = 1^2(\sec 60^\circ + \tan 60^\circ)^2$

$x = (2+\sqrt{3})^2 = 7+4\sqrt{3}$

Similarly $y = (2-\sqrt{3})^2 = 7-4\sqrt{3}$

Now $x^2+y^2 = (7+4\sqrt{3})^2 + (7-4\sqrt{3})^2$

$= 2(7^2 + (4\sqrt{3})^2)$

$= 194$ Ans : C

03 ~~$x^2-y^2 = (7+4\sqrt{3})^2 - (7-4\sqrt{3}$~~

Given $a = b = \frac{1}{2}$, $\theta = 45^\circ$

$\therefore \frac{1}{2}^2(\sqrt{2}+1)^2 = 3+2\sqrt{2}$

03) $x = a^2(\sec\theta + \tan\theta)^2$

$x = \left(\frac{1}{2}\right)^2 (\sec 45^\circ + \tan 45^\circ)^2$

$= \frac{1}{4}(\sqrt{2}+1)^2$

$= \frac{3+2\sqrt{2}}{4}$

$y = b^2(\sec\theta - \tan\theta)^2$

$y = \frac{3-2\sqrt{2}}{4}$

Now $x^2 - y^2 = \left(\frac{3+2\sqrt{2}}{4}\right)^2 - \left(\frac{3-2\sqrt{2}}{4}\right)^2$

$= \frac{4.3.2\sqrt{2}}{16} = \frac{3\sqrt{2}}{2}$ opt: C.