6. TRIGONOMETRIC EQUATIONS – I

Class: IX . MATHEMATICS (1)

FOUNDATION+ : SOLUTIONS

JEEMAINS LEVEL (2025|26)

TEACHING TASK

01. Given $2\sin^2 x - 3\sin x + 1 = 0$.

option verification technique.

(A) $n\pi \pm \frac{\pi}{6}$, let $n = 0 \Rightarrow \theta = \frac{\pi}{6}$

$= 2\sin^2\frac{\pi}{6} - 3\sin\frac{\pi}{6} + 1$

$= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1$

$= \frac{1}{2} - \frac{3}{2} + 1 = 0$ ✓

let $\theta = \frac{\pi}{2}$

$LHS = 2\sin^2\frac{\pi}{2} - 3\sin\frac{\pi}{2} + 1$

$= 2(1)^2 - 3(1) + 1 = 2 - 3 + 1 = 0$ ✓

Option B: $n\pi + (-1)^n\frac{\pi}{2}$ and $n\pi + (-1)^n\frac{\pi}{6}$ ✓

Ans. B

02 $\sin\theta + \cos\theta = 1$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$\Rightarrow \sin 2\theta = 0$

$\Rightarrow \sin 2\theta = \sin n\pi$

$\Rightarrow 2\theta = n\pi, \ n \in \mathbb{Z}$

$\Rightarrow \theta = \frac{n\pi}{2}$

let $n = 0 \Rightarrow \theta = 0$ Now, $\sin 0^\circ + \cos 0^\circ = 1$ ✓

let $n = 1 \Rightarrow \theta = \frac{\pi}{2}$, Now, $\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1 + 0 = 1$ ✓

let $n = 4 \Rightarrow \theta = 2\pi$, Now, $\sin 2\pi + \cos 2\pi$
$= 0 + 1 = 1$ ✓

~~three~~ Two solutions

Ans: A

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03 $|\sin x| = \cos x$

option verification.

let $x = \frac{\pi}{4}$, $\left|\sin\frac{\pi}{4}\right| = \cos\frac{\pi}{4}$

$\Rightarrow \left|\frac{1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ ✓

let $x = \frac{7\pi}{4}$, $\left|\sin\frac{7\pi}{4}\right| = \cos\frac{7\pi}{4}$

$\therefore \left|\sin\left(2\pi - \frac{\pi}{4}\right)\right| = \cos\left(2\pi - \frac{\pi}{4}\right)$

$\Rightarrow \left|-\sin\frac{\pi}{4}\right| = \cos\frac{\pi}{4}$

$= \left|-\frac{1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

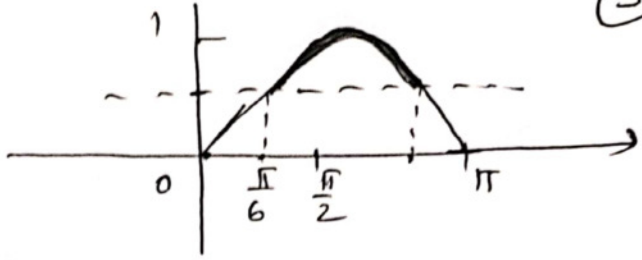
$= \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ ✓

$\therefore$ option: C $\left\{\frac{\pi}{4}, \frac{7\pi}{4}\right\}$

Ans. C

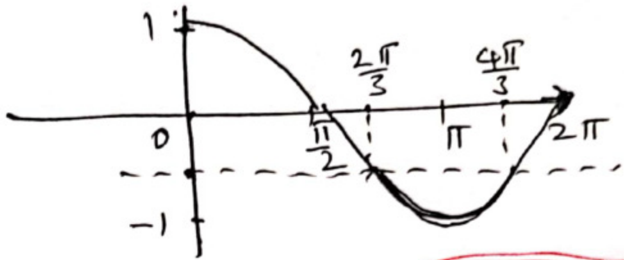
04. $\sin x > \frac{1}{2}$

$\Rightarrow x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$



$\cos x < -\frac{1}{2}$

$\Rightarrow x \in \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$



Common Interval

$x \in \left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$

Ans ___

05 Given $\tan x = \cot x$

$\Rightarrow \frac{\sin x}{\cos x} = \frac{\cos x}{\sin x}$

$\Rightarrow \sin^2 x = \cos^2 x$

$\Rightarrow \cos^2 x - \sin^2 x = 0$

$\Rightarrow \cos 2x = 0$

$\Rightarrow 2x = (2n+1)\frac{\pi}{2}$

$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{4}$

Ans: B

06 $2\sin x + \operatorname{cosec} x = 3$

$\Rightarrow 2\sin^2 x + 1 = 3\sin x$

$\Rightarrow 2\sin^2 x - 3\sin x + 1 = 0$

$\Rightarrow (\sin x - 1)(2\sin x - 1) = 0$

$\Rightarrow \sin x = 1$ or $\sin x = \frac{1}{2}$

$\Rightarrow \sin x = \sin\frac{\pi}{2}$ or $\sin x = \sin\frac{\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n \cdot \frac{\pi}{2}$ or $x = n\pi + (-1)^n \cdot \frac{\pi}{6}$ $n \in \mathbb{Z}$

Ans: D

07. <u>option Verification</u>

Let $x = \frac{\pi}{4}$

$LHS = \cos 2x = \cos 2 \cdot \frac{\pi}{4} = \cos\frac{\pi}{2} = 0$

$RHS = (\sqrt{2}+1)(\cos x - \frac{1}{\sqrt{2}})$

$= (\sqrt{2}+1)(\cos\frac{\pi}{4} - \frac{1}{\sqrt{2}}) = (\sqrt{2}+1)(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}) = 0$

Ans. D

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08 $\sin^{10} x - \cos^{10} x = 1$

opt A: $\sin^{10}(n\pi + \frac{\pi}{2}) - \cos^{10}(n\pi + \frac{\pi}{2})$

$= \sin^{10}(\pi + \frac{\pi}{2}) - \cos^{10}(\pi + \frac{\pi}{2})$ [$\because$ Let $n=1$]

$= \sin^{10}\frac{\pi}{2} - \cos^{10}\frac{\pi}{2}$

$= 1 - 0$

$= 1$ RHS

Ans. A

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09 $r\sin\theta = \sqrt{3}$

$\Rightarrow r = \frac{\sqrt{3}}{\sin\theta}$

$r + 4\sin\theta = 2(\sqrt{3}+1)$

$\Rightarrow \frac{\sqrt{3}}{\sin\theta} + 4\sin\theta = 2(\sqrt{3}+1)$

$\Rightarrow \sqrt{3} + 4\sin^2\theta = 2\sin\theta(\sqrt{3}+1)$

$\Rightarrow 4\sin^2\theta - 2\sin\theta + \sqrt{3} - 2\sqrt{3}\sin\theta = 0$

$\Rightarrow 2\sin\theta(2\sin\theta - 1) - \sqrt{3}(2\sin\theta - 1) = 0$

$\Rightarrow (2\sin\theta - 1)(2\sin\theta - \sqrt{3}) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$ or $\sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin\theta = \sin\frac{\pi}{6}$ or $\sin\theta = \sin\frac{\pi}{3}$

$\theta = n\pi + (-1)^n\frac{\pi}{6}$ or $n\pi + (-1)^n\frac{\pi}{6}$

$\therefore \theta \in \{\frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{2\pi}{3}\}$

Ans. B

10. $4\cos^2 x . \sin x - 2\sin^2 x - 3\sin x = 0$ (5)

$\Rightarrow \sin x\,(4\cos^2 x - 2\sin x - 3) = 0$

$\Rightarrow \sin x = 0$ or $4(1-\sin^2 x) - 2\sin x - 3 = 0$

$\boxed{x = n\pi}$ or $4\sin^2 x + 2\sin x - 1 = 0$

$\sin x = \dfrac{-1 \pm \sqrt{5}}{4}$

$\sin x = \dfrac{\sqrt{5}-1}{4}$ or $\sin x = \dfrac{-1-\sqrt{5}}{4}$

$\sin x = \sin\dfrac{\pi}{10}$ $\qquad$ $\sin x = \sin\left(-\dfrac{3\pi}{10}\right)$

$\boxed{x = n\pi + (-1)^n \dfrac{\pi}{10}}$ $\qquad$ $\boxed{x = n\pi + (-1)^n\left(-\dfrac{3\pi}{10}\right)}$

Ans: D

JEE ADVANCED LEVEL

01 $2\cos^2 x - 3\cos x + 1 = 0$

$2\cos^2 x - 2\cos x - \cos x + 1 = 0$

$\Rightarrow 2\cos^2 x - 2\cos x - 1(\cos x - 1) = 0$

$\Rightarrow 2\cos x(\cos x - 1) - 1(\cos x - 1) = 0$

$\Rightarrow (\cos x - 1)(2\cos x - 1) = 0$

$\Rightarrow \cos x = 1$ or $\cos x = \dfrac{1}{2}$

$\Rightarrow \cos x = \cos 0^\circ$ $\qquad$ $\cos x = \cos\dfrac{\pi}{3}$

$\Rightarrow x = 2n\pi \pm 0$ $\qquad$ $\boxed{x = 2n\pi \pm \dfrac{\pi}{3}}$

$\Rightarrow \boxed{x = 2n\pi}$

Let $n = 0$, $x = 0, \dfrac{\pi}{3}$ $\Rightarrow 2\cos^2 0 - 3\cos 0 + 1 = 0$ ✓

$2\cos^2\dfrac{\pi}{3} - 3\cos\dfrac{\pi}{3} + 1 = 0$ ✓

Let $n = 1$, $x = 2\pi$, $\left(2\pi \pm \dfrac{\pi}{3}\right) =$

$x = 2\pi, \left(2\pi + \frac{\pi}{3}\right), \left(2\pi - \frac{\pi}{3}\right)$

$2\cos^2 2\pi - 3\cos 2\pi + 1 = 2(1)^2 - 3(1) + 1 = 0$ ✓

$2\cos^2\left(2\pi - \frac{\pi}{3}\right) - 3\cos\left(2\pi - \frac{\pi}{3}\right) + 1$

$= 2\cos^2\frac{\pi}{3} - 3\cos\frac{\pi}{3} + 1 = 0$. ✓ (~~Repeated~~).

Here $x = 0, 2\pi$ treated as same

$\therefore$ Total No. of solutions in $[0, 2\pi]$ are 3

$\boxed{\text{Ans - A, B, C, D}}$

$\boxed{02}$ $\sin^3\theta + \sin\theta\cos\theta + \cos^3\theta = 1$

A) $\frac{n\pi}{2}$, let $n = 1 \Rightarrow \theta = \frac{\pi}{2}$

$\sin^3\frac{\pi}{2} + \sin\frac{\pi}{2}\cos\frac{\pi}{2} + \cos^3\frac{\pi}{2} = 1 + (1)(0) + 0 = 1$ ✓

B) $2n\pi + \frac{\pi}{2}$, let $n = 1 \Rightarrow \theta = 2\pi + \frac{\pi}{2}$

$\sin^3\left(2\pi + \frac{\pi}{2}\right) + \sin\left(2\pi + \frac{\pi}{2}\right)\cos\left(2\pi + \frac{\pi}{2}\right) + \cos^3\left(2\pi + \frac{\pi}{2}\right)$

$= \sin^3\frac{\pi}{2} + \sin\frac{\pi}{2}\cos\frac{\pi}{2} + \cos^3\frac{\pi}{2} = 1$ ✓

C) $2n\pi$, let $n = 1 \Rightarrow \theta = 2\pi$

$= \sin^3 2\pi + \sin 2\pi \cos 2\pi + \cos^3 2\pi$

$= (0)^3 + (0).(1) + (1)^3 = 1$ ✓

D) $n\pi$, let $n = 1 \Rightarrow \theta = \pi$

$\sin^3\pi + \sin\pi\cos\pi + \cos^3\pi$

$= (0)^3 + (0)(-1) + (-1)^3 = -1$ ✗

$\boxed{\text{Ans: A, B, C}}$

03 Statement I: $\sin\theta = \sin\frac{\pi}{6}$

$\Rightarrow \theta = n\pi + (-1)^n \cdot \frac{\pi}{6}, n \in Z$ (True)

Statement II: Conceptual (True)

Ans: A

04 Statement I: $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \cdot \tan 2x} = 1$

$\Rightarrow \tan(3x - 2x) = 1$

$\Rightarrow \tan x = 1$

$\Rightarrow \tan x = \tan\frac{\pi}{4}$

$\Rightarrow x = n\pi + \frac{\pi}{4}$ (False)

Ans: D

Statement II: Conceptual (True)

05 Assertion: $\tan 2\theta = \cot\theta$

$\Rightarrow \tan 2\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow 2\theta = n\pi + \frac{\pi}{2} - \theta$

$\Rightarrow 3\theta = n\pi + \frac{\pi}{2}$

$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{6}$ (True)

Reason: Mathematically (True) Ans. A

06 Assertion: Conceptual (True)

Reason: $\cos x = -1$

$\Rightarrow \cos x = \cos\pi$

$\Rightarrow x = 2n\pi \pm \pi, n \in Z$ (False)

Ans: C

07 Given $2\cos^2\theta - 3\cos\theta + 1 = 0$

$\Rightarrow 2\cos^2\theta - 2\cos\theta - \cos\theta + 1 = 0$

$\Rightarrow 2\cos\theta(\cos\theta - 1) - 1(\cos\theta - 1) = 0$

$\Rightarrow (\cos\theta - 1)(2\cos\theta - 1) = 0$

$\Rightarrow \cos\theta = 1$ or $\cos\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \cos 0$ | $\cos\theta = \cos\frac{\pi}{3}$

$\Rightarrow \theta = 2n\pi \pm 0$ | $\Rightarrow \boxed{\theta = 2n\pi \pm \frac{\pi}{3}}$

$\Rightarrow \boxed{\theta = 2n\pi}$

$\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}$ satisfies the above equation

∴ Total No. of solutions $\cancel{4}$ 3 in $[0, 2\pi]$

$\boxed{\text{Ans: B}}$

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08 $\cos\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \cos\frac{\pi}{3}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$ Ans: B

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09 $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$

$(2\sin x - \cos x)(1 + \cos x) = 1 - \cos^2 x$

$(2\sin x - \cos x)(1 + \cos x) = (1 + \cos x)(1 - \cos x)$

$[1 + \cos x][2\sin x - \cos x - 1 + \cos x] = 0$

$1 + \cos x = 0$ or $2\sin x - 1 = 0$ | $\cos\alpha = \frac{\sqrt{3}}{2}$

$\cos x = -1$ or $\sin x = \frac{1}{2}$ | $\boxed{\cos\alpha = \frac{\sqrt{3}}{2}}$

$x = \pi$ (X)

9

$3\cos^2 x - 10\cos x + 3 = 0$

$\Rightarrow 3\cos^2 x - 9\cos x - \cos x + 3 = 0$

$\Rightarrow 3\cos x(\cos x - 3) - 1(\cos x - 3) = 0$

$\Rightarrow (\cos x - 3)(3\cos x - 1) = 0$

$\Rightarrow \cos x = 3$ ✗ or $\cos x = \frac{1}{3}$

$$\boxed{\cos\beta = \frac{1}{3}}$$

Again $1 - \sin 2x = \cos x - \sin x$

$(\sin x - \cos x)^2 + (\sin x - \cos x) = 0$

$\Rightarrow (\sin x - \cos x)[\sin x - \cos x + 1] = 0$

✗ no need

$\Rightarrow \sin x - \cos x = 0$

$\Rightarrow \tan x = 1$

$\Rightarrow \cos x = \frac{1}{\sqrt{2}} \Rightarrow \boxed{\cos\gamma = \frac{1}{\sqrt{2}}}$

Now, $\cos\alpha + \cos\beta + \cos\gamma$

$= \frac{\sqrt{3}}{2} + \frac{1}{3} + \frac{1}{\sqrt{2}}$

$= \frac{3\sqrt{6} + 2\sqrt{2} + 6}{6\sqrt{2}}$

Ans: A

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100. We have $\cos\alpha = \frac{\sqrt{3}}{2} \Rightarrow \sin\alpha = \frac{1}{2}$

$\cos\beta = \frac{1}{3} \Rightarrow \sin\beta = \frac{2\sqrt{2}}{3}$

$\cos\gamma = \frac{1}{\sqrt{2}} \Rightarrow \sin\gamma = \frac{1}{\sqrt{2}}$

$\therefore \sin\alpha + \sin\beta + \sin\gamma$

$= \frac{1}{2} + \frac{2\sqrt{2}}{3} + \frac{1}{\sqrt{2}}$

$= \frac{14+3\sqrt{2}}{6\sqrt{2}}$

Ans: A

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11. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{1}{2}\cdot\frac{1}{3} - \frac{\sqrt{3}}{2}\cdot\frac{2\sqrt{2}}{3}$

$= \frac{1-2\sqrt{6}}{6}$

Ans: C

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12 Given $x \in [0, 4\pi]$

$\sin x = -1$

$\Rightarrow \sin x = \sin\left(\frac{-\pi}{2}\right)$

$\Rightarrow x = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$

let $n=0 \Rightarrow x = -\frac{\pi}{2}$ ⊗

let $n=1 \Rightarrow x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$ ✓

let $n=2 \Rightarrow x = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$ ✓

let $n=3 \Rightarrow x = 3\pi + \frac{\pi}{2} = \frac{7\pi}{2}$ ✓

$\therefore$ only, $\frac{3\pi}{2}$ and $\frac{7\pi}{2}$ are the solutions

Ans: 2

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13 $\cos 4x = \cos 2(2x) = 2\cos^2 2x - 1$

$= 2(2\cos^2 x - 1)^2 - 1$

$= 2[4\cos^4 x - 4\cos^2 x + 1] - 1$

$= 8\cos^4 x - 8\cos^2 x + 1$

$\because \cos 4x = 1 - 8\cos^2 x + 8\cos^4 x$

$a_0 + a_1 \cos^2 x + a_2 \cos^4 x$

$\therefore a_0 = 1, \quad a_1 = -8, \quad a_2 = 8$

Now, $5a_0 + a_1 + a_2 = 5(1) - 8 + 8 = 5$ Ans. 5

14 a) $\sin\theta = \frac{1}{2} \Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}$ (P)

b) $\theta = 2n\pi \pm \frac{\pi}{6}$ (q)

c) $\theta = n\pi + \frac{\pi}{4}$ (r)

d) $\theta = n\pi + \frac{\pi}{6}$ (s)

Ans. P, q, r, s

15 a) $\tan^2 x + \cot^2 x = 2$

$\Rightarrow (\tan x - \cot x)^2 = 0$

$\Rightarrow \tan x - \cot x = 0$

$\Rightarrow \tan^2 x = 1$

$\Rightarrow \tan^2 x = \tan^2 \frac{\pi}{4}$

$\Rightarrow x = n\pi \pm \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

No. of solution $= 4$

b) $\sin^2 x - \cos x = \frac{1}{2}$

$\Rightarrow 2\sin^2 x - 2\cos x = 1$

$\Rightarrow 2(1 - \cos^2 x) - 2\cos x = 1$

$\Rightarrow 2 - 2\cos^2 x - 2\cos x = 1$

$\Rightarrow 2\cos^2 x + 2\cos x - 1 = 0$

$\cos x = \frac{-2 \pm \sqrt{4 + 8}}{4}$

$\cos x = \frac{-2 \pm 2\sqrt{3}}{4}$

$\cos x = \frac{-1 \pm \sqrt{3}}{2} \rightarrow$ Two solutions

c) $4\sin^2 x - 6\cos^2 x = 10$

$4\sin^2 x - 6(1-\sin^2 x) = 10$

$\Rightarrow 10\sin^2 x - 16 = 0$

$\Rightarrow \sin^2 x = \frac{8}{5} \Rightarrow \sin x = \frac{2\sqrt{2}}{\sqrt{5}} \approx 1.26$

$\therefore$ no solution.

d) $\sin x = 1$

$\Rightarrow \sin x = \sin\frac{\pi}{2}$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{2}$

$x = \frac{\pi}{2}$

No. of solutions $= 1$

Ans: s, p, q, r

LEARNERS TASK

CUE'S

01. $\tan x =$ not defined, $\sin x = -1$

option verification

B) $x = (4n+3)\frac{\pi}{2}$

let $n=0$, $x = \frac{3\pi}{2}$

$\tan\frac{3\pi}{2} =$ not defined; $\sin\frac{3\pi}{2} = \sin\left(2\pi - \frac{\pi}{2}\right)$

$= -\sin\frac{\pi}{2} = -1$

Ans B

02 $\sec x = -\sqrt{2} \Rightarrow \cos x = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4}$

$\therefore x = 2n\pi \pm \frac{3\pi}{4}$

Ans. A

3. $\sin x + \cos x = \sqrt{2}$

option verification.

opt: A) $x = 2n\pi + \frac{\pi}{4}$

let $n=1 \Rightarrow x = 2\pi + \frac{\pi}{4}$

$\therefore LHS = \sin\left(2\pi + \frac{\pi}{4}\right) + \cos\left(2\pi + \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

Ans. A

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04 ~~opt~~ Given $\sin 3x = \cos 2x$

option verification

opt A: $x = \frac{n\pi}{5} + \frac{\pi}{10}$.

let $n=0 \Rightarrow x = \frac{\pi}{10}$

$LHS = \sin 3x = \sin\frac{3\pi}{10} = \sin\left(\frac{\pi}{2} - \frac{2\pi}{10}\right)$

$= \cos\frac{2\pi}{10} = \cos 2x = RHS$

Ans: A

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05 $a\sin x + b\cos x$ ~~= R sin(x+α)~~

We know the maximum value $= \sqrt{a^2+b^2}$

and minimum value $= -\sqrt{a^2+b^2}$

i.e. $-\sqrt{a^2+b^2} \leq c \leq \sqrt{a^2+b^2}$

$\Rightarrow c^2 \leq a^2+b^2$ (or)

$\Rightarrow a^2+b^2 \geq c^2$

Ans: A

06	$\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}$	(14) Ans: C
07	$\tan\theta = 0 \Rightarrow \theta = n\pi$	Ans: A
08	Conceptual	Ans: C
09	Conceptual	Ans: B
10	Conceptual	Ans: D

JEE MAINS LEVEL

01. Given $a\cos 2\theta + b\sin 2\theta = c$

$$\Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = c$$

$$\Rightarrow (a+c)\tan^2\theta - 2b\tan\theta + (c-a) = 0$$

$\because$ $\tan\alpha$, $\tan\beta$ are the roots

$$\therefore \tan\alpha \cdot \tan\beta = \frac{c-a}{c+a}$$

Ans: C

02. $\cos\theta = -\frac{\sqrt{3}}{2}$

$\cos\theta = \cos\frac{5\pi}{6}$

$$\Rightarrow \theta = 2n\pi \pm \frac{5\pi}{6}$$

Ans: A

03. $\tan x = 1 \longrightarrow [0, 3\pi]$

$\Rightarrow \tan x = \tan\frac{\pi}{4}$

$\Rightarrow x = n\pi + \frac{\pi}{4}$

Let $n=0 \Rightarrow x = \frac{\pi}{4}$ ✓

Let $n=1 \Rightarrow x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$ ✓

Let $n=2 \Rightarrow x = 2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$ ✓

Let $n=3 \Rightarrow x = 3\pi + \frac{\pi}{4}$ ✗

Total 3 solution

Ans: C

04 $\tan^2 x = \cot^2 x$

$\Rightarrow \tan^4 x = 1$

$\Rightarrow \tan^4 x - 1 = 0$

$\Rightarrow (\tan^2 x - 1)(\tan^2 x + 1) = 0$

$\Rightarrow \tan^2 x = 1$ or $\tan^2 x = -1$ (X)

$\Rightarrow \tan^2 x = \tan^2 \frac{\pi}{4}$

$\Rightarrow x = n\pi \pm \frac{\pi}{4}$

$\boxed{\text{Ans: B}}$

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05 $\cot x$ = Not defined $\sec x = -1$

<u>option verification</u>

A) $x = (2n+1)\pi$

let $n=0 \Rightarrow x = \pi$

$\cot\pi$ = Not defined, $\sec\pi = -1$

Ans: A

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06 $\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2}$

$\Rightarrow \frac{4\cos^3\theta - 3\cos\theta}{2(2\cos^2\theta - 1) - 1} = \frac{1}{2}$

$\Rightarrow \frac{\cos\theta(4\cos^2\theta - 3)}{4\cos^2\theta - 3} = \frac{1}{2}$

$\Rightarrow 2\cos\theta(4\cos^2\theta - 3) - (4\cos^2\theta - 3) = 0$

$\Rightarrow (4\cos^2\theta - 3)(2\cos\theta - 1) = 0$

$\Rightarrow \cos^2\theta = \frac{3}{4}, \cos\theta = \frac{1}{2}$

$\Rightarrow \cos^2\theta = \cos^2\frac{\pi}{3}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

$\cos\theta = \cos\frac{\pi}{3}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$

Ans: B

07. $\tan^2\theta - \tan\theta - \sqrt{3}\tan\theta + \sqrt{3} = 0$

$\Rightarrow (\tan\theta - 1)(\tan\theta - \sqrt{3}) = 0$

$\Rightarrow \tan\theta = 1$ or $\tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan\frac{\pi}{4}$

$\Rightarrow \theta = n\pi + \frac{\pi}{4}$

$\tan\theta = \tan\frac{\pi}{3}$

$\theta = n\pi + \frac{\pi}{3}$

Ans: C

08 $\sqrt{1-\cos x} = \sin x$

$\Rightarrow 1 - \cos x = \sin^2 x$

$\Rightarrow 1 - \cos x = 1 - \cos^2 x$

$\Rightarrow \cos^2 x - \cos x = 0$

$\Rightarrow \cos x(\cos x - 1) = 0$

$\Rightarrow \cos x = 0$ or $\cos x = 1$

$\Rightarrow x = (2n+1)\frac{\pi}{2}$ or

$\cos x = \cos 0$

$x = 2n\pi \pm 0$

$x = 2n\pi$

Ans: A

09 $\tan\theta - \cot\theta = 1$

$\Rightarrow \tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \tan^2\frac{\pi}{4}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$

Ans: D

10 $\sin 6\theta = \sin 4\theta - \sin 2\theta$

option verification

$\theta = 0$ satisfies this equation.

option: B $\theta = \frac{n\pi}{4}$.

Ans: B

when $n=0 \Rightarrow \theta = 0$.

JEE ADVANCED LEVEL

01 Given $\sin 2x = \cos 3x$

option verification:

Substitute $n=0$ in all the options, then

A) $x = \frac{\pi}{10}$ B) $x = \frac{-\pi}{2}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$.

Now option: A) $x = \frac{\pi}{10}$

$\sin 2x = \sin \frac{2\pi}{10} = \sin \frac{\pi}{5}$;

$\cos 3x = \cos \frac{3\pi}{10} = \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right) = \sin \frac{\pi}{5}$ ✓

option C: $x = \frac{\pi}{2}$

$\sin 2x = \sin 2\times\frac{\pi}{2} = \sin \pi = 0$

$\cos 3x = \cos 3\times\frac{\pi}{2} = \cos \frac{3\pi}{2} = 0$ ✓

$\therefore$ Ans: A, C

02 $a\tan\theta + b\sec\theta = c$

$\Rightarrow a\tan\theta - c = -b\sec\theta$

$\Rightarrow a^2\tan^2\theta - 2ac\tan\theta + c^2 = b^2\sec^2\theta$

$$\Rightarrow a^2\tan^2\theta - 2ac\tan\theta + c^2 = b^2(1+\tan^2\theta)$$

$$\Rightarrow (a^2-b^2)\tan^2\theta - 2ac\tan\theta + c^2 - b^2 = 0$$

$$\tan\alpha + \tan\beta = \frac{2ac}{a^2-b^2}$$

$$\tan\alpha \cdot \tan\beta = \frac{c^2-b^2}{a^2-b^2}$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta}$$

$$= \frac{\left(\frac{2ac}{a^2-b^2}\right)}{1-\left(\frac{c^2-b^2}{a^2-b^2}\right)} = \frac{2ac}{a^2-c^2}$$

Ans. A

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03 Statement I: $f(x) = a\sin\theta + b\cos\theta$

We know, max. value $= \sqrt{a^2+b^2}$

min. value $= -\sqrt{a^2+b^2}$

$$\therefore -\sqrt{a^2+b^2} \le f(x) \le \sqrt{a^2+b^2}$$

$$-\sqrt{a^2+b^2} \le c \le \sqrt{a^2+b^2}$$

$$c^2 \le a^2+b^2 \quad \text{(True)}$$

Statement II: Conceptual (True)

Ans. A

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04 Statement II: $3\sin x + 4\cos x = 7$

$a \quad b \quad c$

$c^2 = 7^2 = 49$

$a^2 + b^2 = 9 + 16 = 25$

$c^2 \le a^2+b^2$

$49 \le 25$ (~~False~~)

(True)

Statement II: Conceptual (True) (19)
Ans. A

05 Assertion: $\sin\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \sin\frac{\pi}{6}$

$\Rightarrow \theta = n\pi + (-1)^n \cdot \frac{\pi}{6}$ (True)

Reason: Conceptual (True) Ans. A

06 Assertion: Conceptual (True)
Reason: Conceptual (False) Ans: C

$\boxed{07}$ $\sin\theta = \frac{1}{2} \longrightarrow [0, 5\pi]$

$\Rightarrow \sin\theta = \sin\frac{\pi}{6}$

$\Rightarrow \theta = n\pi + (-1)^n \cdot \frac{\pi}{6}$

Let $n = 0 \Rightarrow \theta = \frac{\pi}{6}$ ✓

$n = 1 \Rightarrow \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ ✓

$n = 2 \Rightarrow \theta = 2\pi + \frac{\pi}{6} = \frac{13\pi}{6}$ ✓

$n = 3 \Rightarrow \theta = 3\pi - \frac{\pi}{6} = \frac{17\pi}{6}$ ✓

$n = 4 \Rightarrow \theta = 4\pi + \frac{\pi}{6} = \frac{25\pi}{6}$ ✓

$n = 5 \Rightarrow \theta = 5\pi - \frac{\pi}{6} = \frac{29\pi}{6}$ ✓

$\therefore$ Total No. of Solutions $= 6$ $\boxed{\text{Ans: D}}$

08 $\tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan\frac{\pi}{3}$

$\Rightarrow \theta = n\pi + \frac{\pi}{3}$

Ans: A

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09 $2\sin^2\theta = \sin\theta \longrightarrow (0, \pi)$

$\Rightarrow 2\sin^2\theta - \sin\theta = 0$

$\Rightarrow \sin\theta\,(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi$	$\sin\theta = \sin\frac{\pi}{6}$
$\Rightarrow \theta = \pi, 2\pi\ \boxed{\times}$	$\theta = n\pi + (-1)^n \frac{\pi}{6}$

$n = 0 \Rightarrow \theta = \frac{\pi}{6}$ ✓

$n = 1 \Rightarrow \theta = \pi - \frac{\pi}{6}$ ✓

$= \frac{5\pi}{6}$ ✓

Ans: C

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10 $\sin 2x = \sin x \longrightarrow [0, 2\pi]$

$2\sin x\cos x - \sin x = 0$

$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2}$

$x = n\pi$	$\cos x = \cos\frac{\pi}{3}$
$x = 0$ ✓	$x = 2n\pi \pm \frac{\pi}{3}$
$= \pi$ ✓	$x = \frac{\pi}{3}$ ✓
$= 2\pi$ ✓	$= 2\pi - \frac{\pi}{3}$ ✓

Total = 5 Solution

Ans: C

11. $\sin x = \frac{1}{2} \longrightarrow [0, 5\pi]$

* Repeated : Q.No: 7

12 $(\sqrt{3}+1)^{2x} + (\sqrt{3}-1)^{2x} = 2^{3x}$

$= (\sqrt{3}+1)^{2x} + (\sqrt{3}-1)^{2x} = (2\sqrt{2})^{2x}$

$\Rightarrow \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^{2x} + \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^{2x} = 1$

$\Rightarrow (\sin 75^\circ)^{2x} + (\cos 75^\circ)^{2x} = 1$

$\Rightarrow$ This is possible only when

$x = 1$

$\therefore$ No. of solutions $= 1$

Ans. 1

13 a) $\tan^2\theta = 1 \Rightarrow \tan^2\theta = \tan^2\frac{\pi}{4} \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$ (q)

b) $\cos^2\theta = \frac{1}{4} \Rightarrow \cos^2\theta = \left(\frac{1}{2}\right)^2$

$= \cos^2\theta = \cos^2\frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$ (r)

c) $\sin^2\theta = \frac{1}{4} \Rightarrow \sin^2\theta = \left(\frac{1}{2}\right)^2$

$\Rightarrow \sin^2\theta = \sin^2\frac{\pi}{6} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$ (p)

d) $\operatorname{cosec}^2\theta = 1 \Rightarrow \sin^2\theta = 1$

$\sin^2\theta = \sin^2\frac{\pi}{2} \Rightarrow \theta = n\pi \pm \frac{\pi}{2}$ (s)

Ans: q, r, p, s

14 (N) NOTE: please avoid this question, as it has no meaning (data insufficient)

ADDITIONAL PRACTICE QUESTIONS

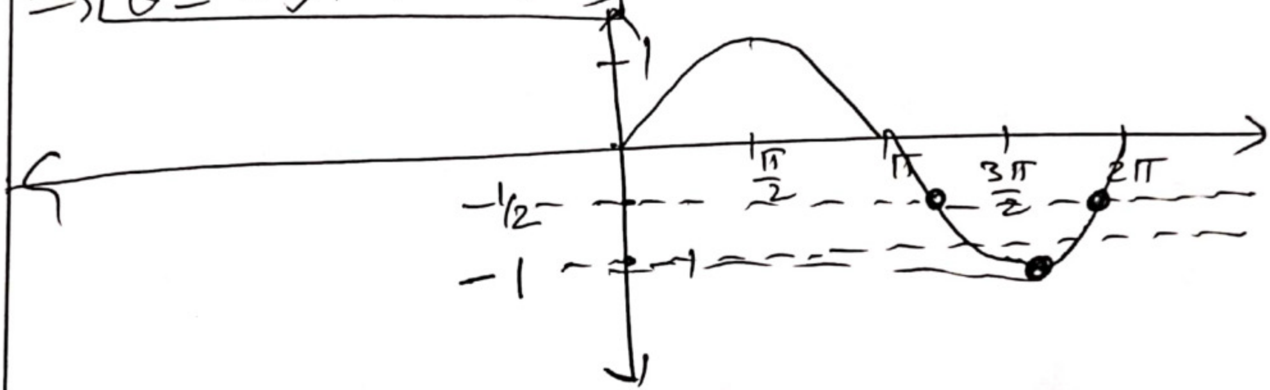
01. $2\sin^2\theta + 3\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta + 1)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = -1$ or $\sin\theta = -\frac{1}{2}$

$\Rightarrow \cancel{\sin\theta = \sin\left(-\frac{\pi}{2}\right)}$

$\Rightarrow \cancel{\theta = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)}$



No. of solutions = 3 Ans: C

02 $\sin(x + 28^\circ) = \cos(3x - 78^\circ)$

$\Rightarrow x + 28^\circ + 3x - 78^\circ = 90^\circ$

$\Rightarrow x = 35^\circ$

Also $x = 8^\circ \Rightarrow \sin 36^\circ = \cos 54^\circ$ (True)

Ans: B

03

03 $3\tan(\theta-15^\circ)=\tan(\theta+15^\circ)$

Let $\theta=\frac{\pi}{4}$

$LHS=3\tan(45^\circ-15^\circ)$
$=3\tan 30^\circ$
$=3\times\frac{1}{\sqrt{3}}$
$=\sqrt{3}$

$RHS=\tan(45^\circ+15^\circ)$
$=\tan 60^\circ$
$=\sqrt{3}$

Ans: B

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04 $1+\sin\alpha+\sin^2\alpha+\cdots\infty=4+2\sqrt{3}$

$\Rightarrow \frac{1}{1-\sin\alpha}=4+2\sqrt{3}$

$\Rightarrow \sin\alpha=\frac{3+2\sqrt{3}}{4+2\sqrt{3}}=\frac{\sqrt{3}(\sqrt{3}+2)}{2(2+\sqrt{3})}$

$=\frac{\sqrt{3}}{2}$

$\therefore \alpha=\frac{\pi}{3},\frac{2\pi}{3}$

NOTE: In G.P $S_\infty=\frac{a}{1-r}$

Ans: D

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05 $(1+\tan\alpha)(1+\tan 4\alpha)=2$

$1+\tan\alpha+\tan 4\alpha+\tan\alpha\cdot\tan 4\alpha=2$

$\Rightarrow \frac{\tan\alpha+\tan 4\alpha}{1-\tan\alpha\cdot\tan 4\alpha}=1$

$\Rightarrow \tan(\alpha+4\alpha)=\tan\frac{\pi}{4}$

$\Rightarrow 5\alpha=\frac{\pi}{4}$

$\Rightarrow \alpha=\frac{\pi}{20}$

Ans: A

$\Rightarrow$ THE END $\in$