6. ELIMINATION OF ANGLE (θ) ①
BY USING TRIGONOMETRIC IDENTIES

Class: VIII. MATHEMATICS
FOUNDATION ‡. SOLUTIONS

TEACHING TASK
JEE MAINS

01. $\sec\theta - \tan\theta = x \rightarrow (1)$

$\sec\theta + \tan\theta = \frac{1}{x} \rightarrow (2)$

$(1)+(2) \Rightarrow 2\sec\theta = x + \frac{1}{x} = \frac{1+x^2}{x}$

$\therefore \sec\theta = \frac{1+x^2}{2x}$ Ans. A

02 $\sec^2\theta - \tan^2\theta = 1 \Rightarrow x^2 - y^2 = 1$

$\Rightarrow x^2 = 1 + y^2$ (OR)

$\Rightarrow y^2 = x^2 - 1$ Ans. C

03 $\frac{x}{a} = \sec\theta, \ \frac{y}{b} = \tan\theta$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Ans. A

04 $x = a(1+\cos\theta) \quad | \quad y = a\sin\theta$

$\frac{x}{a} - 1 = \cos\theta \quad | \quad \frac{y}{a} = \sin\theta$

$\Rightarrow \left(\frac{x-a}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = 1$

$\Rightarrow (x-a)^2 + y^2 = a^2$ Ans. A

05 Let $\theta = 45^\circ$

$LHS = \tan\theta = \tan 45^\circ = 1$

$$RHS = \frac{1-\cos 2\theta}{\sin 2\theta} = \frac{1-\cos 2\times 45^\circ}{\sin 2\times 45^\circ} = \frac{1-\cos 90^\circ}{\sin 90^\circ}$$

$$= \frac{1-0}{1} = 1$$

Ans. A

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06 shortcut method

Given $x = \cot\theta + \tan\theta$ $\quad y = \sec\theta - \cos\theta$

Let $\theta = 45^\circ$

$\Rightarrow x = 1+1$

$\Rightarrow x = 2$

$y = \sqrt{2} - \frac{1}{\sqrt{2}}$

$y = \frac{1}{\sqrt{2}}$

Now, $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}}$

$= \left(4\cdot\frac{1}{\sqrt{2}}\right)^{\frac{2}{3}} - \left(2\cdot\frac{1}{2}\right)^{\frac{2}{3}}$

$= (2\sqrt{2})^{\frac{2}{3}} - (1)^{\frac{2}{3}} = 2-1 = 1$

NOTE: option: A, C are same

Ans. A, C

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07 shortcut method

$x = a(\text{cosec}\alpha + \cot\alpha)$

Let $\alpha = 45^\circ$

$x = a(\text{cosec}45^\circ + \cot 45^\circ)$

$\Rightarrow x = a(\sqrt{2}+1)$

$$y = \frac{b(1-\cos\alpha)}{\sin\alpha}$$

$$= \frac{b(1-\cos 45^\circ)}{\sin 45^\circ}$$

$$= \frac{b\left(1-\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$$

$y = b(\sqrt{2}-1)$

Now, $xy = ab(\sqrt{2}+1)(\sqrt{2}-1)$

$\Rightarrow xy = ab(2-1) \Rightarrow xy = ab$

Ans: C

08 $a\sec\theta + b\tan\theta + c = 0 \longrightarrow (1)$

$p\sec\theta + q\tan\theta + r = 0 \longrightarrow (2)$

$(1)\times q \Rightarrow aq\sec\theta + bq\tan\theta + cq = 0$

$(2)\times b \Rightarrow bp\sec\theta + bq\tan\theta + br = 0$

$\Rightarrow (aq - bp)\sec\theta + (cq - br) = 0$

$\Rightarrow \sec\theta = \dfrac{br - cq}{aq - bp}$

Similarly, $\tan\theta = \dfrac{pc - ar}{aq - bp}$

Now $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \left(\dfrac{br - cq}{aq - bp}\right)^2 - \left(\dfrac{pc - ar}{aq - bp}\right)^2 = 1$

$\Rightarrow (br - cq)^2 - (pc - ar)^2 = (aq - bp)^2$

Ans: C

09 Given $x = a\sec\theta.\cos\phi$ | Similarly $\dfrac{y}{b} = \sec\theta.\sin\phi$ | $\dfrac{z}{c} = \tan\theta$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = \sec^2\theta\cos^2\phi + \sec^2\theta\sin^2\phi - \tan^2\theta$

$= \sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta$

$= \sec^2\theta - \tan^2\theta$

$= 1$

Ans. A

10 $x = a\sec\theta$, $y = b\tan\theta$ (4)

$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = \sec^2\theta - \tan^2\theta = 1$ Ans. B

JEE ADVANCED LEVEL

01 $\frac{x}{a} = \tan\theta$; $\frac{y}{a} = \sec\theta$

$\left(\frac{y}{a}\right)^2 - \left(\frac{x}{a}\right)^2 = \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow y^2 - x^2 = a^2$

Also, $\frac{x}{a} = \tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{x}{a}\right)$

Also, $\frac{y}{a} = \sec\theta \Rightarrow \theta = \sec^{-1}\left(\frac{y}{a}\right)$ Ans. A, B, C

02 $\sec\theta + \tan\theta = x \rightarrow (1)$

$\sec\theta - \tan\theta = \frac{1}{x} \rightarrow (2)$

$(1) + (2) \Rightarrow 2\sec\theta = x + \frac{1}{x}$

$\Rightarrow \sec\theta = \frac{x^2+1}{2x} = \frac{1}{2}\left(x + \frac{1}{x}\right)$

$(1) - (2) \Rightarrow 2\tan\theta = x - \frac{1}{x}$

$\therefore \tan\theta = \frac{1}{2}\left(x - \frac{1}{x}\right)$ Ans: A, C

03 $(a\sec\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

put $\theta = 45^\circ$

$\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)(1+1)$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 2 = 1$ Ans: B

04. Statement I: $x = a\tan\theta,\ y = a\sec\theta$

$\Rightarrow \frac{x}{a} = \tan\theta,\ \frac{y}{a} = \sec\theta$

$\therefore \left(\frac{y}{a}\right)^2 - \left(\frac{x}{a}\right)^2 = 1 \Rightarrow y^2 - x^2 = a^2$

$\Rightarrow x^2 - y^2 = -a^2$ (True)

Statement II: Conceptual (True)

Ans. A

05 Statement I

$x = a\sec\theta + b\tan\theta \rightarrow (1)$

$y = a\tan\theta + b\sec\theta \rightarrow (2)$

$(1)^2 - (2)^2 \Rightarrow a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$

$x^2 - y^2 =$

$\therefore x^2 - y^2 = a^2 - y^2$ (True)

Ans:

Statement II: $\sec\theta + \tan\theta = a$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{a}$

$\Rightarrow \tan\theta - \sec\theta = \frac{-1}{a}$ (True)

Ans: A

06 Assertion: $x = a\cos\theta\ ;\ y = a\sin\theta$

$x^2 + y^2 = a^2$ (True)

Reason: Conceptual (True)

Ans: A

07 Assertion: Conceptual (False)

Reason: Conceptual (True)

Ans: D

08 $x = a\cos\theta\ ;\ y = a\sin\theta$

$x^2 + y^2 = a^2$

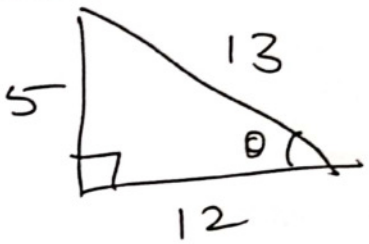
Ans: B

09. $\cos^2\theta + \sin^2\theta = 1$ Ans: B

10. $x^2 + y^2 = a^2$

This is equation of circle centre at (0,0) with radius a Ans.. C

11. $\tan\theta = \frac{5}{12}$



$\sin\theta = \frac{5}{13}$

$\Rightarrow \sin^2\theta = \frac{25}{169}$ Ans.. A

12 $\sec\theta = \frac{13}{12} \Rightarrow \sec^2\theta = \frac{169}{144}$ Ans.. B

13 $x = a\cos^2\theta$; $y = a\sin^2\theta$

$\frac{x}{a} + \frac{y}{a} = 1 \Rightarrow x + y = a = 10$ Ans: 10

14 NOTE: Please, discard this question as it has insufficient data.

15 a) $x = a\sin\theta,\ y = a\cos\theta$

$\Rightarrow x^2 + y^2 = a^2 \rightarrow \sin^2\theta + \cos^2\theta = 1$ (q)

b) $x = a\tan\theta,\ y = a\sec\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$ (p)

c) $\sin^2\theta + \cos^2\theta = 1$ (q)

d) $\sin^2\theta + \cos^2\theta = 1$ (q)

Ans: q, p, q, q

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16 a) short cut method

$\cot\theta + \tan\theta = m$

let $\theta = 45^\circ$

$1 + 1 = m$

$\Rightarrow \boxed{m = 2}$

$\sec\theta - \cos\theta = n$

$\sqrt{2} - \frac{1}{\sqrt{2}} = n$

$\Rightarrow \boxed{\frac{1}{\sqrt{2}} = n}$

Now, $(m^2 n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}}$

$= \left(4 \cdot \frac{1}{\sqrt{2}}\right)^{\frac{2}{3}} - \left(2 \cdot \frac{1}{2}\right)^{\frac{2}{3}}$

$= (2\sqrt{2})^{\frac{2}{3}} - (1)^{\frac{2}{3}}$

$= 1 - 1 = 0$ (q)

b) Given $m = \csc\theta - \sin\theta$ | $n = \sec\theta - \cos\theta$

Let $\theta = 45^\circ$ | Let $\theta = 45^\circ$

$m = \sqrt{2} - \frac{1}{\sqrt{2}}$ | $n = \sqrt{2} - \frac{1}{\sqrt{2}}$

$m = \frac{1}{\sqrt{2}}$ | $n = \frac{1}{\sqrt{2}}$

Now $(m^2 n)^{\frac{2}{3}} + (m n^2)^{\frac{2}{3}}$

$= \left(\frac{1}{2} \cdot \frac{1}{\sqrt{2}}\right)^{\frac{2}{3}} + \left(\frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right)^{\frac{2}{3}}$

$= \left(\frac{1}{2^{3/2}}\right)^{\frac{2}{3}} + \left(\frac{1}{2^{3/2}}\right)^{\frac{2}{3}} = \frac{1}{2} + \frac{1}{2} = 1$ (r)

c) short cut method

$x = a\cos^2\alpha + y\sin^2\alpha$

Let $\alpha = 0^\circ$

$\Rightarrow x = a \Rightarrow x - a = 0$

Now, $(x-a)(y-a) + (x-y)^2 \sin^2\alpha \cdot \cos^2\alpha$

$= (0)(y-a) + (x-y)^2 \cdot \sin^2 0 \cdot \cos^2 0^\circ = 0$ (q)

d) short cut method

Let $\alpha = \beta = \gamma = 0^\circ$

$\therefore x = r,\ y = 0,\ z = 0,\ \gamma = 0$

Now, $x^2 + y^2 + z^2 + \gamma^2 = r^2$ (s)

Ans: q, r, q, s

LEARNERS TASK (CURLS

01 $xy = \tan\theta \cdot \cot\theta = 1$ — Ans. B

02 $x^2 + y^2 = a^2$ — Ans. A

03 $y = \cos 2\theta = 2\cos^2\theta - 1 = 2x^2 - 1$ — Ans. C

04 $x^2 + y^2 = r^2$, $r^2 = r^2$

$\Rightarrow x^2 + y^2 = r^2$ — Ans. D

05 $x^2 + y^2 = 1$ — Ans. A

06 $x = \cot\theta + \operatorname{cosec}\theta \rightarrow (1)$, ~~$y = \cot\theta$~~

$y = \cot\theta - \operatorname{cosec}\theta \rightarrow (2)$

$(1) + (2) \Rightarrow x + y = 2\cot\theta$

$(1) - (2) \Rightarrow x - y = 2\operatorname{cosec}\theta$

Short cut method

$x = \cot\theta + \operatorname{cosec}\theta$

Let $\theta = 45^\circ$

$x = 1 + \frac{1}{\sqrt{2}}$

$y = \cot\theta - \operatorname{cosec}\theta$

$y = 1 - \frac{1}{\sqrt{2}}$

$x + y = 2$; $x - y = \sqrt{2}$ $\Rightarrow x^2 - y^2 = 2\sqrt{2}$

$(x^2 - y^2)^2 = 8$

:

optim: B $16xy = 16\left(1 - \frac{1}{2}\right) = 16\left(\frac{1}{2}\right) = 8$

Ans. B

07. $x = a\sin\theta$ | $y = b\tan\theta$

$\Rightarrow \frac{a}{x} = \csc\theta$ | $\frac{b}{y} = \cot\theta$

$\Rightarrow \frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

Ans: A

08 $x^2 - y^2 = 4$

Ans. B

09 $(\sin\theta+\cos\theta)^2 + (\sin\theta-\cos\theta)^2$

$= 2(\sin^2\theta + \cos^2\theta) = 2(1) = 2$

Ans: A

10 $xy = (at)\left(\frac{a}{t}\right) = a^2$

Ans. B

JEE MAINS LEVEL

01 $x = a\sin\theta,\ y = b\cos\theta$

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

Ans: B

02 $\sin A = \frac{1}{\sqrt{2}} \Rightarrow A = 45^\circ$

$\therefore x = \tan 45^\circ = 1$

Also

Ans: [illegible]

03 $x^2 + y^2 = r^2$

Ans: C

04 $\frac{x}{a} = \sin\theta\cos\theta$ | $y = a\sin^2\theta$

$\Rightarrow \frac{x^2}{a^2} = \sin^2\theta\cos^2\theta$ | $\Rightarrow \frac{y}{a} = \sin^2\theta$

$\therefore \cos^2\theta = 1 - \frac{y}{a}$

$\Rightarrow \frac{x^2}{a^2} = \frac{y}{a}\left(1 - \frac{y}{a}\right)$

Ans: D

05 $x = a\cos^2\theta; \quad y = a\sin^2\theta$

$x + y = a(\cos^2\theta + \sin^2\theta)$

$\Rightarrow x + y = a$

Ans. A

06 $x = r\cos\theta\cos\phi$

$y = r\cos\theta\sin\phi$

$z = r\sin\theta$

put $\theta = \phi = 0^\circ$

$\therefore x = r, \; y = 0, \; z = 0$

$\therefore x^2 + y^2 + z^2 = r^2$

Ans: C

07 $\frac{x}{a} = \sin^4\theta \quad \left| \quad \frac{y}{b} = \sin^4\theta \right.$

$\Rightarrow \sqrt{\frac{x}{a}} = \sin^2\theta \quad \left| \quad \sqrt{\frac{y}{b}} = \sin^2\theta \right.$

$\Rightarrow \sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$

Ans. C

08 $x\sec\theta + y\tan\theta = a \quad \left| \quad x\tan\theta + y\sec\theta = b \right.$

$x^2\sec^2\theta + y^2\tan^2\theta + 2\sec\theta\tan\theta xy = a^2 \quad \left| \quad x^2\tan^2\theta + y^2\sec^2\theta + 2xy\sec\theta\tan\theta = b^2 \right.$

$a^2 - b^2 = x^2(\sec^2\theta - \tan^2\theta) - y^2(\sec^2\theta - \tan^2\theta)$

$\Rightarrow a^2 - b^2 = x^2 - y^2$

Ans: B

09 Short cut method put $\phi = \theta = 0^\circ$

$a = 1, \; b = 0, \; c = 0 \Rightarrow a^2 + b^2 + c^2 = 1$

Ans: D

10 $x = a\cos^2\theta \cdot \sin\theta$, $y = a\sin^2\theta \cdot \cos\theta$ (12)

let $\theta = 45^\circ$

$x = \frac{a}{2\sqrt{2}}$; $y = \frac{a}{2\sqrt{2}}$

$x^2 + y^2 = \frac{a^2}{4}$ | $x^2y^2 = \frac{a^4}{64}$

$\Rightarrow (x^2+y^2)^3 = \frac{a^6}{64}$ |

$$\frac{(x^2+y^2)^3}{x^2y^2} = \frac{\left(\frac{a^6}{64}\right)}{\left(\frac{a^4}{64}\right)} = a^2$$ Ans. C

JEE ADVANCED LEVEL

01. $x = a\cos\theta + b\sin\theta \rightarrow (1)$

$y = a\sin\theta - b\cos\theta \rightarrow (2)$

$(1)^2 + (2)^2 \Rightarrow x^2 + y^2 = a^2 + b^2$

$(1)^2 - (2)^2 \Rightarrow x^2 - y^2 = (a\cos\theta + b\sin\theta)^2 - (a\sin\theta - b\cos\theta)^2$

$= a^2(\cos^2\theta - \sin^2\theta) + b^2(\cos^2\theta - \sin^2\theta) + 4ab\sin\theta\cos\theta$

$= a^2\cos 2\theta - b^2\cos 2\theta + 2ab\sin 2\theta.$

Ans: A

02 $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

Let $\theta = 45^\circ$

$=(1+1+\sqrt{2})(1+1-\sqrt{2})$

$=(2+\sqrt{2})(2-\sqrt{2})$

$=4-2=2$

Ans. C

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03 Statement I: $x+y=a$ (True)

Statement II: Conceptual (True)

Ans. A

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04. Statement I: $\operatorname{cosec}\theta-\sin\theta=l$

Let $\theta=45^\circ$

$\Rightarrow \sqrt{2}-\frac{1}{\sqrt{2}}=l$

$\Rightarrow \boxed{l=\frac{1}{\sqrt{2}}}$

$\sec\theta-\cos\theta=m$

Let $\theta=45^\circ$

$\Rightarrow \sqrt{2}-\frac{1}{\sqrt{2}}=m$

$\Rightarrow \boxed{\frac{1}{\sqrt{2}}=m}$

Now, $l^2m^2(l^2+m^2+3)$

$=\frac{1}{2}\cdot\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}+3\right)=\frac{1}{4}(4)=1$ (True)

Statement II: $LHS=\frac{\cos\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}$

$=\frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$

$=\frac{1+\sin\theta}{\cos\theta}=RHS$ ✓

Given $\frac{\sin\theta}{1+\sin\theta}=\frac{\cos\theta}{1+\cos\theta}$

$\Rightarrow \sin\theta(1+\cos\theta) = \cos\theta(1+\sin\theta)$

$\Rightarrow \sin\theta + \sin\theta\cos\theta = \cos\theta + \cos\theta\sin\theta$

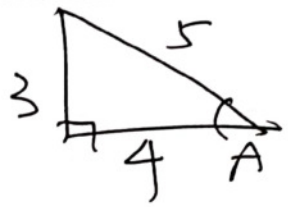
$\Rightarrow \sin\theta = \cos\theta$ which is not always true (False)

Ans: C

05 Assertion: $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$ (True)

Reason: Conceptual (True) Ans: A

06 Assertion $\sin A = \frac{3}{5}$



$\therefore \tan A = \frac{3}{4}$

(True)

Reason: Mathematically (True) Ans. A

07 $x = a\sec\theta$, $y = a\tan\theta$

$\Rightarrow x^2 - y^2 = a^2$ Ans. B

08 $\sec^2\theta - \tan^2\theta = 1$ Ans. A

09 $x^2 - y^2 = a^2$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$ (Hyperbola) Ans: B

10. $\sec\theta = \frac{5}{4}$ Ans. B

11. $\tan^2\theta + 1 = \left(\frac{3}{4}\right)^2 + 1 = \frac{25}{16}$ Ans. A

12. $x^2 + y^2 = a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25$ Ans. 25

13. NOTE: Please, discard this question as it has insufficient data.

14. a) $x^2 + y^2 = a^2 \ldots (s)$

b) $\frac{x}{a} = \tan\theta, \frac{y}{a} = \sec\theta$

$\Rightarrow \left(\frac{y}{a}\right)^2 - \left(\frac{x}{a}\right)^2 = 1 \Rightarrow y^2 - x^2 = a^2 \ldots (q)$

c) $\frac{x}{a} = 1 - \cos\theta \quad \Big| \quad \frac{y}{a} = \sin\theta$

$\Rightarrow \cos\theta = 1 - \frac{x}{a}$

$\Rightarrow \left(\frac{y}{a}\right)^2 + \left(\frac{a-x}{a}\right)^2 = 1 \Rightarrow (x-a)^2 + y^2 = a^2 \ldots (r)$

d) ~~$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = 1 \Rightarrow x^2 + y^2 = a^2 \ldots (s)$~~

$\frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow x + y = a \ldots (p)$

Ans:

Ans: s, q, r, p

11 (a) (t) $\frac{x-h}{r} = \sin\theta$; $\frac{y-k}{r} = \cos\theta$

$\Rightarrow \left(\frac{x-h}{r}\right)^2 + \left(\frac{y-k}{r}\right)^2 = 1$

$\Rightarrow (x-h)^2 + (y-k)^2 = r^2$

b) (r) $\frac{x+h}{r} = \cos\theta$; $\frac{y+k}{r} = \sin\theta$

$\Rightarrow \left(\frac{x+h}{r}\right)^2 + \left(\frac{y+k}{r}\right)^2 = 1$

$\Rightarrow (x+h)^2 + (y+k)^2 = r^2$

c) (q) $\frac{x-k}{r} = \cos\theta$, $\frac{y-h}{r} = \sin\theta$

$\Rightarrow (x-k)^2 + (y-h)^2 = r^2$

d) (p) $\frac{x+k}{r} = \sin\theta$, $\frac{y+h}{r} = \cos\theta$

$\Rightarrow (x+k)^2 + (y+h)^2 = r^2$ Ans: t, r, q, p

ADDITIONAL PRACTICE

01 $xy = a^2b^2(\sec\theta - \tan\theta)^2 \Rightarrow xy = a^2b^2$ Ans. A

02 $x = 1^2(2+\sqrt{3})^2 = 7+4\sqrt{3}$; $y = 7-4\sqrt{3}$

$x^2+y^2 = (7+4\sqrt{3})^2 + (7-4\sqrt{3})^2 = 2(49+48) = 194$ Ans. C

03 $x = \frac{3+2\sqrt{2}}{64}$; $y = \frac{3-2\sqrt{2}}{64}$ | $x^2 - y^2 = \frac{3\sqrt{2}}{2}$ Ans. C

$\Rightarrow$ THE END $\in$