COMPOUND ANGLES

Class: VIII, Mathematics

ADVANCED SOLUTIONS

TEACHING TASK

01. $A+B+C=\pi$

$\Rightarrow A+B=\pi-C$

$\Rightarrow \tan(A+B)=\tan(\pi-C)$

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\cdot\tan B}=-\tan C$

$\Rightarrow \tan A+\tan B+\tan C=\tan A\tan B\cdot\tan C$

$\Rightarrow \sum \tan A=\pi\tan C$

Ans. A

02 $\frac{\tan 225^\circ-\cot 81^\circ\cdot\cot 69^\circ}{\cot 261^\circ+\tan 21^\circ}$

$=\frac{1-\cot 81^\circ\cdot\cot 69^\circ}{\cot 81+\cot 69^\circ}$

$=-\left(\frac{\cot 81^\circ\cdot\cot 69^\circ-1}{\cot 81^\circ+\cot 69^\circ}\right)$

$=-\cot(81^\circ+69^\circ)$

$=-\cot 150^\circ$

$=-\cot(180^\circ-30^\circ)$

$=\cot 30^\circ$

$=\sqrt{3}$

$\tan 225^\circ=\tan(180^\circ+45^\circ)$

$=\tan 45^\circ$

$=1$

$\cot 261^\circ=\cot(180^\circ+81^\circ)$

$=\cot 81^\circ$

Ans: C

23 $\frac{(1+\tan 13^\circ)(1+\tan 32^\circ)}{(1+\tan 12^\circ)(1+\tan 33^\circ)}$

NOTE! (2)

$A+B = 45^\circ$

$\Rightarrow (1+\tan A)(1+\tan B) = 2$

$= \frac{2}{2}$

$= 1$

Ans: A

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04 $A+B = 300^\circ$

Let $A = B = 150^\circ$

$\cot 150^\circ = \cot(180^\circ - 30^\circ) = -\cot 30^\circ = -\sqrt{3}$

$\therefore (1+\sqrt{3}\cot A)(1+\sqrt{3}\cot B)$

$= (1+\sqrt{3}(-\sqrt{3}))(1+\sqrt{3}(-\sqrt{3}))$

$= (1-3)(1-3) = (-2)(-2)$

$= 4$

Ans: A

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05 $A+B+C = 360^\circ$

Let $A = B = C = 120^\circ$

Now $\frac{A}{4} = \frac{B}{4} = \frac{C}{4} = 30^\circ$

$\cot\frac{A}{4} + \cot\frac{B}{4} + \cot\frac{C}{4} = K \cdot \cot\frac{A}{4} \cdot \cot\frac{B}{4} \cdot \cot\frac{C}{4}$

$\Rightarrow \sqrt{3} + \sqrt{3} + \sqrt{3} = K \cdot \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3}$

$\Rightarrow 3\sqrt{3} = K \cdot 3\sqrt{3}$

$\Rightarrow K = 1$

Ans: D

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06 $\frac{\tan 160^\circ - \tan 110^\circ}{1+\tan 160^\circ \cdot \tan 110^\circ} = \tan(160^\circ - 110^\circ) = \tan 50^\circ = \cot 40^\circ$

$= \frac{1}{\tan 40^\circ} = \frac{1}{\tan 2\times 20^\circ} = \frac{1}{\left(\frac{2\tan 20^\circ}{1-\tan^2 20^\circ}\right)} = \frac{1-p^2}{2p}$

Ans. B

07 $A+B = 225^\circ \Rightarrow \cot(A+B) = \cot 225^\circ$ (3)

$\Rightarrow \frac{\cot A \cot B - 1}{\cot A + \cot B} = 1$

$\Rightarrow \cot A \cdot \cot B - 1 = \cot A + \cot B$.

Now $\frac{\cot A}{1+\cot A} \cdot \frac{\cot B}{1+\cot B} = \frac{\cot A \cdot \cot B}{1+\cot A + \cot B + \cot A \cdot \cot B}$

$= \frac{\cot A \cdot \cot B}{\cot A \cdot \cot B + \cot A + \cot B}$

$= \frac{\cot A \cdot \cot B}{2 \cdot \cot A \cdot \cot B} = \frac{1}{2}$

08 $A+B+C = 180^\circ$

$\Rightarrow A+B = 180^\circ - C$

$\Rightarrow \sin(A+B) = \sin(180^\circ - C)$

$\Rightarrow \sin A \cos B + \cos A \sin B = \sin C$

$\Rightarrow \frac{3}{5} \cdot \frac{12}{13} + \left(-\frac{4}{5}\right) \cdot \frac{5}{13} = \sin C$

$\Rightarrow \frac{36-20}{65} = \sin C$

$\Rightarrow \sin C = \frac{16}{65}$

Since A is obtuse

$\cos A = -\frac{4}{5}$

Ans: C

09 In $\triangle ABC$, let $A = B = C = 60^\circ$

$\sum \frac{\sin(60^\circ+60^\circ) \cdot \sin(60^\circ-60^\circ)}{\sin^2 60^\circ \cdot \sin^2 60^\circ} = 0$.

Ans: B

10. Let $A = 30°$, $B = 60°$ (4)

$$\frac{\cos A}{\cos B} = n \Rightarrow \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = n \Rightarrow \boxed{n = \sqrt{3}}$$

$$\frac{\sin A}{\sin B} = m \Rightarrow \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = m \Rightarrow \boxed{m = \frac{1}{\sqrt{3}}}$$

Given $(m^2 - n^2)\sin^2 B$

$$= \left(\frac{1}{3} - 3\right) \cdot \frac{3}{4} = \frac{-8}{3} \times \frac{3}{4} = -2$$

opt: A) $1 - n^2$

$= 1 - (\sqrt{3})^2$

$= -2$

Ans: A

11. $LR = \frac{\pi}{2} \Rightarrow P + Q = 90°$

$\Rightarrow \frac{P}{2} + \frac{Q}{2} = 45°$

$\tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1$

$$\Rightarrow \frac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right)}{1 - \tan\left(\frac{P}{2}\right) \cdot \tan\left(\frac{Q}{2}\right)} = 1$$

$$\Rightarrow \frac{\left(\frac{-b}{a}\right)}{1 - \frac{c}{a}} = 1$$

$$\Rightarrow \frac{b}{c - a} = 1$$

$\Rightarrow b = c - a$

$\Rightarrow a + b = c$

Ans: A

12 $\sin^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$

$= \sin\left(52\frac{1}{2}^\circ + 22\frac{1}{2}^\circ\right) \cdot \sin\left(52\frac{1}{2}^\circ - 22\frac{1}{2}^\circ\right)$

$= \sin(75^\circ) \cdot \sin 30^\circ$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}+1}{4\sqrt{2}}$ or $\frac{1}{2\sqrt{2}(\sqrt{3}-1)}$

Ans: A, D

13 $\sin(\theta+\alpha) = \cos(\theta+\alpha)$

$\Rightarrow \tan(\theta+\alpha) = 1$

$\Rightarrow \frac{\tan\theta + \tan\alpha}{1 - \tan\theta \cdot \tan\alpha} = 1$

$\Rightarrow \tan\theta + \tan\alpha = 1 - \tan\theta \cdot \tan\alpha$

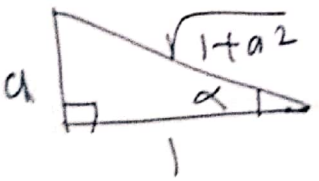
$\Rightarrow \tan\theta + \tan\alpha + \tan\theta \cdot \tan\alpha = 1$

$\Rightarrow \tan\alpha = \frac{1-\tan\theta}{1+\tan\theta}$

Ans. B

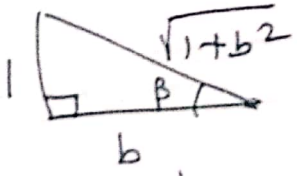
14 S.I

$\sin\alpha = \frac{a}{\sqrt{1+a^2}}$



$\tan\alpha = a$

$\cos\beta = \frac{b}{\sqrt{1+b^2}}$



$\tan\beta = \frac{1}{b}$

$\tan(\alpha+\beta) = \frac{a+\frac{1}{b}}{1-\frac{a}{b}} = \frac{ab+1}{b-a}$ (false)

Statement II:

$\tan 2B = \tan\left((A+B)-(A-B)\right) = \frac{m-n}{1+mn}$ (false)

Ans: B

15. **Statement I:**

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{12}{13}\left(\frac{-3}{5}\right) + \left(\frac{5}{13}\right)\left(-\frac{4}{5}\right)$$

$$= \frac{-56}{65} \quad \text{(False)}$$

Statement II:

$$\sin\phi = \frac{1}{\sqrt{10}}$$

$$\therefore \tan 2\phi = \frac{2\left(\frac{1}{3}\right)}{1-\frac{1}{9}} = \frac{3}{4}$$

$$\text{Now } \tan(\theta+2\phi) = \frac{\tan\theta + \tan 2\phi}{1 - \tan\theta\cdot\tan 2\phi}$$

$$= \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}} = \frac{25}{21} = 1 = \tan 45^\circ$$

Hence $\theta + 2\phi = 45^\circ$ (True) Ans. 1

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16 **Statement I:** $m\cos(\theta+\alpha) = n\cos(\theta-\alpha)$

$$\Rightarrow \frac{m}{n} = \frac{\cos(\theta-\alpha)}{\cos(\theta+\alpha)} \Rightarrow \frac{m+n}{m-n} = \frac{\cos(\theta-\alpha)+\cos(\theta+\alpha)}{\cos(\theta-\alpha)-\cos(\theta+\alpha)}$$

$$= \frac{2\cos\theta\cos\alpha}{2\sin\theta\sin\alpha} = \cot\theta\cot\alpha$$

(False)

$\Rightarrow$

Statement II: $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{a+b}{a-b}$

$$\Rightarrow \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = \frac{(a+b)+(a-b)}{(a+b)-(a-b)}$$

$\frac{\tan\alpha}{\tan\beta} = \frac{a}{b}$

(True) Ans. 1

17 $\cos^2\left(\frac{\pi}{4}+x\right)-\sin^2\left(\frac{\pi}{4}-x\right)$

$=\cos\left(\frac{\pi}{4}+x+\frac{\pi}{4}-x\right)\cdot\cos\left(\frac{\pi}{4}+x-\frac{\pi}{4}+x\right)$

$=\cos\frac{\pi}{2}\cdot\cos 2x$

$=0$

Ans: D

18 $\cos^2\left(\frac{\pi}{6}+\theta\right)-\cos^2\left(\frac{\pi}{6}-\theta\right)$

$=\sin\left(\frac{\pi}{6}+\theta+\frac{\pi}{6}-\theta\right)\cdot\sin\left(\frac{\pi}{6}-\theta-\frac{\pi}{6}-\theta\right)$

$=\sin\frac{\pi}{3}\cdot\sin(-2\theta)=-\frac{\sqrt{3}}{2}\cdot\sin 2\theta$

Ans: B

19 $\cos(A+B)\cdot\cos(A-B)+\sin(A+B)\cdot\sin(A-B)$

$=\cos^2A-\sin^2B+\sin^2A-\sin^2B$

$=1-2\sin^2B$

$=1-2(1-\cos^2B)$

$=2\cos^2B-1$

Ans: C

20 $\tan 2A=\tan[(A+B)+(A-B)]$

$=\frac{P+q}{1-Pq}$

Ans: C

21. $\cot 2B=\cot[(A+B)-(A-B)]$

$=\frac{Pq+1}{P-q}$

Ans: C

22. $\tan 2A \cdot \cot 2B + \tan 2B \cdot \cot 2A$

$= \frac{p+q}{1-pq} \cdot \frac{pq+1}{p-q} + \frac{p-q}{pq+1} \cdot \frac{1-pq}{p+q}$

let $p=1, q=0$

$= \frac{1}{1} \cdot \frac{1}{1} + \frac{-1}{1} \cdot \frac{1}{1}$

$= 2$

Ans: D

23. $\tan(A+B) = \frac{\frac{n}{n+1} + \frac{1}{2n+1}}{1 - \frac{n}{n+1} \cdot \frac{1}{2n+1}}$

put $n=1$

$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = 1$ hence $A+B = 45^\circ$

Ans: 45°

24

24. a) $A+B+C=180^\circ$

Let $A=B=C=60^\circ$

$$\sum \frac{\cot A+\cot B}{\tan A+\tan B}$$

$$= \sum \frac{\left(\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}+\sqrt{3}} = \sum \frac{1}{3} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1.$$

b) In $\triangle ABC$, Let $A=B=C=60^\circ$

$$\sum \frac{\cos(B-C)}{\sin B \cdot \sin C} = \sum \frac{1}{\left(\frac{\sqrt{3}}{2}\right)\cdot\left(\frac{\sqrt{3}}{2}\right)} = \sum \frac{4}{3}$$

$$= 3\times\frac{4}{3} = 4$$

c) In $\triangle ABC$, Let $A=B=C=60^\circ$

$$\sum \frac{\sin(A-B)}{\cos A \cdot \cos B} = \sum \frac{\sin(60^\circ-60^\circ)}{\cos 60^\circ \cdot \cos 60^\circ} = 0$$

d) In $A+B+C=90^\circ$, Let $A=B=C=30^\circ$

$$\frac{\cot 30^\circ+\cot 30^\circ+\cot 30^\circ}{\cot 30^\circ \cdot \cot 30^\circ \cdot \cot 30^\circ} = \frac{3\times\sqrt{3}}{(\sqrt{3})^3} = \frac{3\sqrt{3}}{3\sqrt{3}} = 1$$

Ans: t, q, r, t

25 (10)

a) $$\frac{\sin\theta}{\sin\left(\frac{\pi}{8}+\frac{\theta}{2}+\frac{\pi}{8}-\frac{\theta}{2}\right)\cdot\sin\left(\frac{\pi}{8}+\frac{\theta}{2}-\frac{\pi}{8}+\frac{\theta}{2}\right)}$$

$$= \frac{\sin\theta}{\sin\frac{\pi}{4}\cdot\sin\theta} = \sqrt{2}$$

b) $$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)=a$$

let $\theta = 0^\circ$

$\Rightarrow a = 2$

Now $\tan^2\left(\frac{\pi}{4}+\theta\right)+\tan^2\left(\frac{\pi}{4}-\theta\right)$

let $\theta = 0$

$= 2 = a^2 - 2 = (2)^2 - 2 = 2$

c) $\cot A + 2\tan(A-B)$

$$= \cot A + 2\left(\frac{\tan A - \tan B}{1+\tan A\cdot\tan B}\right)$$

$$= \frac{\cot A(1+\tan A\cdot\tan B)+2(\tan A-\tan B)}{1+\tan A\cdot\tan B}$$

$$= \frac{\cot A+\tan B+2\tan A-2\tan B}{1+\tan A\cdot\tan B}$$

$$= \frac{(2\tan A+\cot A)-\tan B}{1+\tan A\cdot\tan B} = \frac{\tan B-\tan B}{1+\tan A\tan B} = 0$$

d) $A+B+C = 90^\circ$

$\Rightarrow \tan A\tan B+\tan B\tan C+\tan C\cdot\tan A = 1$

Ans: t, σ, ϑ, ς

LEARNER'S TASK

01. $\sin 40^\circ \cdot \cos 50^\circ + \cos 40^\circ \cdot \sin 50^\circ = \sin(40^\circ + 50^\circ) = \sin 90^\circ = 1$ Ans: A

02. $\cos 70^\circ \cdot \cos 20^\circ - \sin 70^\circ \cdot \sin 20^\circ = \cos(70^\circ + 20^\circ) = \cos 90^\circ = 0$ Ans: B

03 $\dfrac{\tan 20^\circ + \tan 25^\circ}{1 - \tan 20^\circ \cdot \tan 25^\circ} = \tan(20^\circ + 25^\circ) = \tan 45^\circ = 1$ Ans: A

04 $\tan(A+B) = \dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \dfrac{5}{5} = 1 = \tan\dfrac{\pi}{4}$ Ans: C

05 $\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ Ans: B

06 $\tan\left(\dfrac{\pi}{4} + 45^\circ\right) > \tan 90^\circ =$ Not defined Ans: D

07 $\cos(A+B) \cdot \cos(A-B) = \cos^2 A - \sin^2 B$
$= \cos^2 B - \sin^2 A$ Ans: A

08 $\cos^2 45^\circ - \sin^2 15^\circ$
$= \cos(45^\circ + 15^\circ) \cdot \cos(45^\circ - 15^\circ)$
$= \cos 60^\circ \cdot \cos 30^\circ = \dfrac{1}{2} \times \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4}$ Ans: C

09 $\cos(120^\circ + A) + \cos(120^\circ - A)$
$= 2\cos 120^\circ \cdot \cos A = 2\cos(180^\circ - 60^\circ) \cdot \cos A = -2 \times \dfrac{1}{2} \cos A$
$= -\cos A$ Ans: B

10 $\sin(B+C) \cdot \sin(C-B)$
$= -\sin(B+C) \cdot \sin(B-C)$
$= -(\sin^2 B - \sin^2 C) = \sin^2 C - \sin^2 B$ Ans: B

JEE MAINS LEVEL

01 $\sin 75^\circ \cdot \cos 75^\circ$

$= \frac{1}{2}(2\sin 75^\circ \cdot \cos 75^\circ) = \frac{1}{2}\sin 150^\circ = \frac{1}{2}\sin(180^\circ - 30^\circ)$

$= \frac{1}{2}\sin 30^\circ = \frac{1}{4}$ Ans: C

02 $\tan^2 75^\circ + \cot^2 75^\circ$

$= (2+\sqrt{3})^2 + (2-\sqrt{3})^2 = 2\left((2)^2 + (\sqrt{3})^2\right) = 2 \times 7 = 14$

Ans: A

03. $\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}\cdot\frac{5}{12}} = \frac{63}{16}$ Ans: D

03. $\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}\cdot\frac{5}{12}} = \frac{63}{16}$ Ans: D

04 $A + B = 45^\circ$

$\Rightarrow \tan(A+B) = 1$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1$

$\tan A + \tan B + \tan A \cdot \tan B = 1$

$\Rightarrow (1+\tan A)(1+\tan B) = 2$

Ans: B

05 $20^\circ = 55^\circ - 35^\circ$

$\Rightarrow \tan 20^\circ = \frac{\tan 55^\circ - \tan 35^\circ}{1 + \tan 55^\circ \cdot \tan 35^\circ}$

$\Rightarrow \tan 20^\circ = \frac{\tan 55^\circ - \tan 35^\circ}{1 + 1}$

$\Rightarrow 2\tan 20^\circ + \tan 35^\circ = \tan 55^\circ$

$\therefore \alpha = 55^\circ$

Since $\tan 35^\circ = \cot 55^\circ$

Ans: C

JEE MAINS LEVEL

01 $\sin 75^\circ \cdot \cos 75^\circ$

$= \frac{1}{2}(2\sin 75^\circ \cdot \cos 75^\circ) = \frac{1}{2}\sin 150^\circ = \frac{1}{2}\sin(180^\circ - 30^\circ)$

$= \frac{1}{2}\sin 30^\circ = \frac{1}{4}$ Ans: C

02 $\tan^2 75^\circ + \cot^2 75^\circ$

$= (2+\sqrt{3})^2 + (2-\sqrt{3})^2 = 2\left((2)^2 + (\sqrt{3})^2\right) = 2 \times 7 = 14$

Ans: A

03. $\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} = \frac{63}{16}$ Ans: D

03. $\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} = \frac{63}{16}$ Ans: D

04 $A + B = 45^\circ$

$\Rightarrow \tan(A+B) = 1$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1$

$\tan A + \tan B + \tan A \cdot \tan B = 1$

$\Rightarrow (1+\tan A)(1+\tan B) = 2$

Ans: B

05 $20^\circ = 55^\circ - 35^\circ$

$\Rightarrow \tan 20^\circ = \frac{\tan 55^\circ - \tan 35^\circ}{1 + \tan 55^\circ \cdot \tan 35^\circ}$

$\Rightarrow \tan 20^\circ = \frac{\tan 55^\circ - \tan 35^\circ}{1+1}$

$\Rightarrow 2\tan 20^\circ + \tan 35^\circ = \tan 55^\circ$

$\therefore x = 55^\circ$

Since $\tan 35^\circ = \cot 55^\circ$

Ans: C

06. $\cos^2 48^\circ - \sin^2 12^\circ$

$= \cos(48^\circ+12^\circ)\cdot\cos(48^\circ-12^\circ)$

$= \cos 60^\circ \cdot \cos 36^\circ$

$= \frac{1}{2} \times \frac{1+\sqrt{5}}{4} = \frac{1+\sqrt{5}}{8}$

Ans: A

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07. $x = \frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}$

$x = \frac{\sin A\cos B - \cos A \sin B}{\cos A \cdot \cos B}$

$x = \frac{\sin(A-B)}{\cos A \cdot \cos B}$

$y = \frac{\cos B}{\sin B} - \frac{\cos A}{\sin A}$

$y = \frac{\sin A\cos B - \cos A \sin B}{\sin B \cdot \sin A}$

$y = \frac{\sin(A-B)}{\sin A \sin B}$

$\frac{1}{x} + \frac{1}{y} = \frac{\cos A\cos B + \sin A \sin B}{\sin(A-B)} = \frac{\cos(A-B)}{\sin(A-B)}$

$= \cot(A-B)$

Ans. A

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08. $\tan(A-B) = \frac{7}{24}$

$\Rightarrow \frac{\tan A - \tan B}{1+\tan A \cdot \tan B} = \frac{7}{24}$

$\Rightarrow \frac{\frac{4}{3} - \tan B}{1+\frac{4}{3}\cdot\tan B} = \frac{7}{24}$

$\Rightarrow \tan B = \frac{3}{4}$

$\tan(A+B) = \frac{\frac{4}{3}+\frac{3}{4}}{1-\frac{4}{3}\cdot\frac{3}{4}}$

$= \frac{(\ )}{0}$

$=$ not defined

$A+B = \frac{\pi}{2}$

Ans: D

09 $\tan(A+B+C) = \dfrac{\tan(1+2+3) - (1\cdot 2\cdot 3)}{1-(1\cdot 2+2\cdot 3+3\cdot 1)} = 0$

$\therefore \quad A+B+C = \pi$

Ans : B

10 $A+B+C = 90^\circ \iff \tan A\cdot\tan B + \tan B\cdot\tan C + \tan C\cdot\tan A = 1$

$A+B+C = 180^\circ$

$\Rightarrow \dfrac{A}{2}+\dfrac{B}{2}+\dfrac{C}{2} = 90^\circ$

$\therefore \tan\dfrac{A}{2}\cdot\tan\dfrac{B}{2} + \tan\dfrac{B}{2}\cdot\tan\dfrac{C}{2} + \tan\dfrac{C}{2}\cdot\tan\dfrac{A}{2} = 1$

$\Rightarrow \dfrac{5}{6}\cdot\dfrac{20}{37} + \dfrac{20}{37}\cdot\tan\dfrac{C}{2} + \tan\dfrac{C}{2}\cdot\dfrac{5}{6} = 1$

$\Rightarrow \tan\dfrac{C}{2} = \dfrac{122}{305}$

Ans : A

11. $A+B = \dfrac{\pi}{4} \Rightarrow (1+\tan A)(1+\tan B) = 2$

$(1+\cot A)(1+\cot B) = 2\cot A\cot B$

Ans. A, C

12 $\dfrac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \dfrac{1+\tan 9^\circ}{1-\tan 9^\circ}$

$= \dfrac{\tan 45^\circ + \tan 9^\circ}{1-\tan 45^\circ\cdot\tan 9^\circ} = \tan 54^\circ$

$= \cot 36^\circ$

Ans. A, C

13 Statement I: $\tan A = x\tan B \Rightarrow \dfrac{\tan A}{\tan B} = x$

$\dfrac{\sin(A-B)}{\sin(A+B)} = \dfrac{\sin A\cos B - \cos A\sin B}{\sin A\cos B + \cos A\sin B} = \dfrac{\frac{\tan A}{\tan B} - 1}{\frac{\tan A}{\tan B} + 1}$

Statement II: Conceptual (True) $= \dfrac{x-1}{x+1}$ (True)

Ans : A.

14 $A+B = 45^\circ$

$\cot(A+B) = 1$

$\Rightarrow \dfrac{\cot A \cot B - 1}{\cot A + \cot B} = 1$

$\Rightarrow \cot A \cot B - 1 = \cot A + \cot B$

$\Rightarrow \cot A \cdot \cot B - \cot A - \cot B = 1$

$\Rightarrow (\cot A - 1)(\cot B - 1) = 2$

Ans: C

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15 $A = 22\frac{1}{2}^\circ$

$\Rightarrow 2A = 45^\circ$

$\Rightarrow \tan 2A = \tan 45^\circ$

$\Rightarrow \dfrac{2\tan A}{1-\tan^2 A} = 1$

$\Rightarrow \dfrac{2x}{1-x^2} = 1$

$\Rightarrow 2x = 1 - x^2$

$\Rightarrow x^2 + 2x - 1 = 0$

$x = \dfrac{-2 + \sqrt{4+4}}{2} = \dfrac{-2+2\sqrt{2}}{2}$

$= \sqrt{2} - 1$

Ans: C

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16 $A = 67\frac{1}{2}^\circ$

$\Rightarrow 2A = 135^\circ$

$\Rightarrow \tan 2A = \tan(135^\circ)$

$\Rightarrow \dfrac{2\tan A}{1-\tan^2 A} = -1$

$\Rightarrow \dfrac{2x}{1-x^2} = -1$

$\Rightarrow 2x = -1 + x^2$

$\Rightarrow x^2 - 2x - 1 = 0$

$\Rightarrow x = \dfrac{2 + \sqrt{4+4}}{2} = \dfrac{2+2\sqrt{2}}{2}$

$= 1 + \sqrt{2}$

Ans: B

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17 $80^\circ - 10^\circ = 70^\circ$

$\Rightarrow \dfrac{\tan 80^\circ - \tan 10^\circ}{1 + \tan 80^\circ \cdot \tan 10^\circ} = \tan 70^\circ$

$\Rightarrow \tan 80^\circ - \tan 10^\circ = 2\tan 70^\circ$

$\dfrac{\tan 80^\circ - \tan 10^\circ}{\tan 70^\circ} = 2$

Ans: 2

18 In $\triangle ABC$, $A = B = C = 60^\circ$

$$\frac{A}{2} = \frac{A}{2} = \frac{C}{2} = 30^\circ$$

$\tan 30^\circ \tan 30^\circ + \tan 30^\circ \tan 30^\circ + \tan 30^\circ \tan 30^\circ$

$= 3 \tan^2 30^\circ$

$= 3\left(\frac{1}{\sqrt{3}}\right)^2 = 1$ Ans : 1

19 A) $\cos\theta + \cos(120^\circ + \theta) + \cos(120^\circ - \theta)$

Let $\theta = 0^\circ$

$1 + \cos 120^\circ + \cos 120^\circ = 1 - \cos 60^\circ - \cos 60^\circ$

$= 1 - \frac{1}{2} - \frac{1}{2} = 0$

B) $\cos\theta + \cos(240^\circ + \theta) + \cos(240^\circ - \theta)$

Let $\theta = 0^\circ$

$= 1 + \cos 240^\circ + \cos 240^\circ = 1 - \frac{1}{2} - \frac{1}{2} = 0$

C) $\cos\theta - \cos(60^\circ + \theta) - \cos(60^\circ - \theta)$

Let $\theta = 0^\circ$

$= 1 - \cos 60^\circ - \cos 60^\circ = 1 - \frac{1}{2} - \frac{1}{2} = 0$

D) $\sin\theta + \sin(120^\circ + \theta) + \sin(240^\circ + \theta)$

Let $\theta = 0^\circ$

$= 0 + \sin 120^\circ - \sin 60^\circ$

$= \sin 60^\circ - \sin 60^\circ$

$= 0$

Ans : A, C, D, B

20 a) $20^\circ + 40^\circ = 60^\circ$

$$\Rightarrow \frac{\tan 20^\circ + \tan 40^\circ}{1 - \tan 20^\circ \cdot \tan 40^\circ} = \sqrt{3}$$

$$\Rightarrow \tan 20^\circ + \tan 40^\circ + \sqrt{3}\tan 20^\circ \cdot \tan 40^\circ = \sqrt{3}$$

b) $75^\circ - 30^\circ = 45^\circ$

$$\Rightarrow \frac{\tan 75^\circ - \tan 30^\circ}{1 + \tan 75^\circ \cdot \tan 30} = 1$$

$$\Rightarrow \tan 75^\circ - \tan 30^\circ - \tan 75^\circ \cdot \tan 30^\circ = 1$$

c) $$\frac{\cos 13^\circ - \sin 13^\circ}{\cos 13^\circ + \sin 13^\circ} + \frac{1}{\cot 148^\circ}$$

$$= \frac{1 - \tan 13^\circ}{1 + \tan 13^\circ} + \tan 148^\circ$$

$$= \frac{\tan 45^\circ - \tan 13^\circ}{1 + \tan 45^\circ \cdot \tan 13^\circ} + \tan(180^\circ - 32^\circ)$$

$$= \tan(45^\circ - 13^\circ) - \tan 32^\circ$$

$$= \tan 32^\circ - \tan 32^\circ = 0$$

d) $$\frac{\tan\alpha + \tan\beta}{\tan(\alpha+\beta)} + \frac{\tan\alpha - \tan\beta}{\tan(\alpha-\beta)}$$

$$= \frac{\tan\alpha + \tan\beta}{\left(\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}\right)} + \frac{\tan\alpha - \tan\beta}{\left(\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}\right)}$$

$$= 1 - \tan\alpha \cdot \tan\beta + 1 + \tan\alpha \cdot \tan\beta = 2$$

Ans: 5, β, δ, t

ADDITIONAL PRACTICE QUESTIONS

01. $\operatorname{Cosec} 15^\circ - \sec 15^\circ$

$= \dfrac{2\sqrt{2}}{\sqrt{3}-1} - \dfrac{2\sqrt{2}}{\sqrt{3}+1}$

$= \sqrt{2}(\sqrt{3}+1) - \sqrt{2}(\sqrt{3}-1)$

$= \sqrt{2}(\sqrt{3}+1-\sqrt{3}+1) = 2\sqrt{2}$

Ans: A

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02 $4x + 5x = 9x$

$\Rightarrow \dfrac{\tan 4x + \tan 5x}{1 - \tan 4x \cdot \tan 5x} = \tan 9x$

$\Rightarrow \tan 4x + \tan 5x - \tan 9x = -\tan 4x \cdot \tan 5x \cdot \tan 9x$

$\therefore K = -1$

Ans: B

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03 $\tan\left(\dfrac{\pi}{4}+\theta\right) \cdot \tan\left(\dfrac{3\pi}{4}+\theta\right)$

Let $\theta = 0^\circ$

$= \tan\dfrac{\pi}{4} \cdot \tan\dfrac{3\pi}{4}$

$= 1 \times (-1) = -1$

Ans: B

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04 $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$

$\Rightarrow \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = 0 \rightarrow (1)$

$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta = 0 \rightarrow (2)$

$(1)+(2) \Rightarrow 1+1+2(\cos(\alpha-\beta)) = 0 \rightarrow \cos(\alpha-\beta) = -1$

Ans: C

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05 $\dfrac{\cos A\cos B + \sin A\sin B}{\cos A\cos B - \sin A\sin B} + \dfrac{\cos C\cos D - \sin C\sin D}{\cos C\cos D + \sin C\sin D} = 0$

$= \dfrac{1+\tan A\tan B}{1-\tan A\tan B} + \dfrac{1-\tan C\tan D}{1+\tan C\cdot\tan D} = 0$

$\Rightarrow \tan A\cdot\tan B\cdot\tan C\cdot\tan D = 1$

Ans: B