2. POLYNOMIALS – I

Class: IX, MATHEMATICS

INTEGRATED + SOLUTIONS

2025/26

JEE MAINS LEVEL

01. $x^2+2xy+y^2-z^2$

$= (x+y)^2-z^2$

$= (x+y+z)(x+y-z)$

Ans. B

02 $p(x) = x^3-px^2+6x-p$

$p(p) = p^3-p\cdot p^2+6\cdot p-p = 5p$

Ans. C

03 $f(x) = px^2+5x+r$

$f(2) = 4p+10+r=0 \rightarrow ①$

$f\left(\frac{1}{2}\right) = \frac{p}{4}+\frac{5}{2}+r=0$

$p+10+4r=0 \rightarrow ②$

$① - ② \Rightarrow p=r \Rightarrow p^2-r^2=0$

Ans. A

04 We have $a+b+c=0 \Rightarrow a^3+b^3+c^3=3abc$

$a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}}=0 \Rightarrow \left(a^{\frac{1}{3}}\right)^3+\left(b^{\frac{1}{3}}\right)^3+\left(c^{\frac{1}{3}}\right)^3 = 3a^{\frac{1}{3}}\cdot b^{\frac{1}{3}}\cdot c^{\frac{1}{3}}$

$\Rightarrow a+b+c = 3a^{\frac{1}{3}}\cdot b^{\frac{1}{3}}\cdot c^{\frac{1}{3}}$

$\Rightarrow (a+b+c)^3 = 27abc$

Ans. C

05 $a^{12}x^4 - a^4 x^{12}$

$= (a^6 x^2)^2 - (a^2 x^6)^2$

$= (a^6 x^2 + a^2 x^6)(a^6 x^2 - a^2 x^6)$

$= (a^6 x^2 + a^2 x^6)(a^3 x + a x^3)(a^3 x - a x^3)$

$= a^2 x^2 (a^4 + x^4)(ax)(a^2 + x^2)\, ax\, (a^2 - x^2)$

$= a^4 x^4 (a^4 + x^4)(a^2 + x^2)(a+x)(a-x)$ Ans.. A

06. option Verification

$lxb = (7y-3)(5y+4)$

$= 35y^2 + 28y - 15y - 12$

$= 35y^2 + 13y - 12$ Ans.. A

07 $(x-a)^3 + (a-b)^3 + (x-c)^3$

Now $x-a+x-b+x-c = 3x-(a+b+c)$

$= 3x - 3x = 0$

$\therefore (x-a)^3 + (x-b)^3 + (x-c)^3 = 3(x-a)(x-b)(x-c)$

Ans.. A

08 $\dfrac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)}$

$= (a+b)(b+c)(c+a)$ Ans: B

09. $f(x) = x^{140} + 2 \cdot x^{151} + 15$

$f(-1) = 1 - 2 + 15 = 0 \Rightarrow k = 1$ Ans. A

10 $= ay + yz + zx + y^2$ | $= x^2 + ay + yz + zx$

$= xy + zx + yz + y^2$ | $= x(x+y) + z(y+x)$

$= x(y+z) + y(z+y)$ | $= (x+y)(x+z)$

$= (x+y)(y+z)$

$\therefore$ L.C.M $= (x+y)(y+z)(z+x)$ Ans. B

JEE ADVANCED LEVEL

11. $p(x) = px^2 + 5x + r$

Since $p = r$, $p(x) = 0$ has reciprocal roots.

$\therefore$ The roots are $(2, \frac{1}{2})$ or $(-2, -\frac{1}{2})$

$\therefore$ factors are $(x+2)(x+\frac{1}{2})$ (or)

$(x-2)(x-\frac{1}{2})$

Ans: (A, C) (or) (B, D)

12 $p(x) = a_0 x^n + a_1 \cdot x^{n-1} + a_2 \cdot x^{n-2} + \ldots + a_n$

$p(1) = a_0 + a_1 + a_2 + \ldots + a_n = 0$

$p(-1) = a_0 - a_1 + a_2 + \ldots + (-1)^n a_n = 0.$

$\Rightarrow a_0 + a_2 + a_4 + \ldots = a_1 + a_3 + a_5 + \ldots$ Ans: A, D

13. $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ (4)

$f(1) = a+b+c+d+e = 0$

$f(-1) = a-b+c-d+e = 0$

$\Rightarrow a+c+e = b+d$

Ans: A, C

14 Statement I:

$a_7x^7 + a_6x^6 + a_5x^5 + \cdots a_1x + a_0 = (3x-1)^7$

put $x = 1$

$a_7 + a_6 + a_5 + \cdots + a_1 + a_0 = (2)^7 = 128$ (True)

Statement II: Conceptual (True)

Ans. A

15 $= \left(x - \frac{1}{2}\right)(x - \sqrt{2})$

$= x^2 - \left(\frac{1}{2} + \sqrt{2}\right)x + \frac{1}{2}\cdot\sqrt{2}$

$= x^2 - \left(\frac{1}{2} + \sqrt{2}\right)x + \frac{1}{\sqrt{2}}$

Ans: C

16 $\left(x - \frac{m}{n}\right)\left(x + \frac{m}{n}\right) = 0$

$\Rightarrow x^2 - \frac{m^2}{n^2} = 0$

$\Rightarrow n^2x^2 - m^2 = 0$

Ans: C

17. $\sqrt{2} \times \sqrt{3} = \sqrt{2\times3} = \sqrt{6}$

Ans. A

18 $x-y+z=0$

$\Rightarrow x+(-y)+z=0$

$\Rightarrow x^3+(-y)^3+z^3=3(x)(-y)(z)$

$\Rightarrow x^3-y^3+z^3=-3xyz$

$\Rightarrow x^3+z^3=y^3-3xyz$ Ans. D

19 $(x+y+z)^3=27xyz$ Ans: B

20 $l-m-n=0$

$l+(-m)+(-n)=0$

$l^3+(-m)^3+(-n)^3=3(l)(-m)(-n)$

$\Rightarrow l^3-m^3-n^3=-3lmn$

$\Rightarrow l^3=m^3+n^3+3lmn$ Ans. B

21 $\sum x^2(y^2-z^2)=x^2(y^2-z^2)+y^2(z^2-x^2)+z^2(x^2-y^2)$

$=0$ Ans. 0

22 a) $f(x)=x^3-px^2+6x-p$

$f(p)=p^3-p\cdot p^2+6p-p=5p$ (r)

b) $f(x)=x^3-px^2+5x-p$

$f(-p)=-p^3-p^3-5p-p=-2p(p^2+3)$ (t)

c) $f(x)=x^3-px^2+p^2x-1$

$f(1)=1-p+p^2-1=0=p^2-p$ (q)

d) $f(x)=p^3x^3-3px^2+3px+2$

$f(-1)=-p^3-3p-3p+2=-p^3-6p+2$ (p)

Ans. r, t, q, p

LEARNERS TASK (CUQ'S) (6)

01. Conceptual — Ans: B, C

02 Conceptual — Ans. B

03 $x^{-2}\left(x^5 + \frac{3}{x^{-4}} + x^{11}\right)$

$= x^3 + x^7 + x^9 \rightarrow$ degree $= 9$ — Ans: D

05 $A = 1, B = -1 \Rightarrow \frac{A}{B} = \frac{1}{-1} = -1$ — Ans. B

04) $x^3 + 2x^4y^2 - x^5y^3z + 13$

$\rightarrow$ degree $= 5+3+1 = 9$ — Ans. D

06 Conceptual — Ans. A

07 $3 \times 4 = 12$ — Ans. D

08 0 — Ans. C

09 $\frac{-1}{2}$ — Ans: C

10 $a \neq 0$ or $a > 0$ or $a < 0$ — Ans: B, C, D

JEE MAINS LEVEL

01. $p(x) = 3x^2 - 5x = x(3x-5)$

Zeroes are $x = 0$ or $\frac{5}{3}$ — Ans. B

optical

Ans.. A

(9)

$1) = ax^3 + bx^2 + cx + d$

$1) = a + b + c + d$

Ans: C

$+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(9)^2 = a^2 + b^2 + c^2 + 2(26)$

$a^2 + b^2 + c^2 = 81 - 52 = 29$

Ans: E

$+ \quad 5x - \frac{1}{x} = a$

$\Rightarrow a^2 + 4a + 4$

$= (a+2)^2$

$= \left(5x - \frac{1}{x} + 2\right)^2$

$= \left(\frac{5x^2 - 1 + 2x}{x}\right)^2$

Ans: A

$+2b - (a^2 - b^2)$

$+ 2b - a^2 + b^2$

$(1 + 2b + b^2) - a^2$

$= (1+b)^2 - a^2$

$= (1+b+a)(1+b-a)$

Ans: A

$x^3 - b^3 + 1 + 3ab$

$= a^3 + (-b)^3 + (1)^3 - 3(a)(-b)(1)$

$= (a - b + 1)(a^2 + b^2 + 1 + ab + bc - 1)$

one of the factors $a - b + 1$

Ans: B

02 Conceptual

Ans.. A

(1)

03 $f(x) = ax^3 + bx^2 + cx + d$

$f(1) = a + b + c + d$

Ans: C

04 $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(9)^2 = a^2 + b^2 + c^2 + 2(26)$

$\Rightarrow a^2 + b^2 + c^2 = 81 - 52 = 29$

Ans: B

05 Let $5x - \frac{1}{x} = a$

$\Rightarrow a^2 + 4a + 4$

$= (a+2)^2$

$= \left(5x - \frac{1}{x} + 2\right)^2$

$= \left(\frac{5x^2 - 1 + 2x}{x}\right)^2$

Ans: A

06 $1 + 2b - (a^2 - b^2)$

$= 1 + 2b - a^2 + b^2$

$= (1 + 2b + b^2) - a^2$

$= (1+b)^2 - a^2$

$= (1 + b + a)(1 + b - a)$

Ans: A

07 $a^3 - b^3 + 1 + 3ab$

$= a^3 + (-b)^3 + (1)^3 - 3(a)(-b)(1)$

$= (a - b + 1)(a^2 + b^2 + 1 + ab + bc - 1)$

$\therefore$ One of the factors $a - b + 1$

Ans: B

08 Option: C $30^3+20^3-50^3$

Since $30+20-50=0$

Ans: C

09 $f(x)=ax^3+3x^2-13$; $g(x)=2x^3-5x+a$

$f(-2)=g(-2)$

$-8a+12-13=-16+10+a$ $\quad -1+6$

$\Rightarrow 9a=5 \Rightarrow a=\frac{5}{9}$

Ans. D

10 $(x+y)(1-z)-(y+z)(1-x)$

$=(x-zx+y-yz)-(y-xy+z-zx)$

$=x-zx+y-yz-y+xy-z+zx$

$=x-z-yz+xy=(x-z)(1+y)$.

Ans. D

~~$=(x-z)-y(z-1)$~~

JEE ADVANCED LEVEL

11. $p(a)=0$, $p(b)=0$ Ans. A, C, D

12 $f(x)=2x^3-5x^2+ax+b$

$f(2)=16-20+2a+b=0$

$2a+b=4 \rightarrow (1)$

$f(0)=a\cdot 0+b=0 \Rightarrow b=0$

$(1) \Rightarrow a=2$

Ans: A, B

13 Statement I:

$f(x) = ax^3 - bx^2 + cx - d$

$f(1) = a - b + c - d = 0 \Rightarrow a + c = b + d$ (True)

Statement II: Conceptual (True) Ans: A

14 Statement I: $f(x) = 4x^2 - 1 = 0$

$\Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}$ (True)

Statement II: Conceptual (True) Ans: A

15 $f(x) = x^2 - 5x + 6$

$\Delta = (-5)^2 - 4.1.6 = 25 - 24 = 1 > 0$ Ans: C

16 $f(x) = x^2 + 4x + 8$

$\Delta = 4^2 - 4.1.8 = 16 - 32 = -16 < 0$ Ans: C

17 $b^2 - 4ac = 0 \Rightarrow$ roots equal $\Rightarrow \alpha = \beta$ Ans: C

18 $(-2)^2 - a(-2) + 12 = 0$

$\Rightarrow 4 + 2a + 12 = 0 \Rightarrow a = -8$ Ans: -8

19 $f(x) = ax^2 + bx + c$

$f(0) = 0 \Rightarrow c = 0$ Ans: 0

20 $a = 1, \; b = 0 \Rightarrow a + b = 1$ Ans: 1

21 A) $x^2y^2+2xyz^2-x^2z^2-y^2z^2$

$=(y^2-z^2)x^2+2yz^2x-y^2z^2$ (x)

B) $x^3+y^3-z^3+3xyz$

$\Rightarrow x^3+3xyz+(y^3-z^3)$ (P) (Change the option P)

C) $(y-z)x^3+(z-x)y^3+(x-y)z^3$

$=(y-z)x^3+zy^3-xy^3+xz^3-yz^3$ (S)

D) $x^2(y-z)+y^2(x-z)+z^2(x-y)+xyz$

$=x^2(y-z)+(y^2+z^2+yz)x-y^2z-z^2y$ (Q)

Ans: x, P, S, Q

ADDITIONAL QUESTIONS FOR PRACTICE

01. $x^{12}-y^{12}$

$=(x^6)^2-(y^6)^2$

$=(x^6+y^6)(x^6-y^6)$

$=[(x^2)^3+(y^2)^3]\left[(x^3)^2-(y^3)^2\right]$

$=(x^2+y^2)(x^4-x^2y^2+y^4)(x^3+y^3)(x^3-y^3)$

$=(x^2+y^2)(x^4-x^2y^2+y^4)(x+y)(x^2-xy+y^2)(x-y)$

(x^2+xy+y^2)

Ans. C

Total 6 factors

02 $p^3(q-r)^3 + q^3(r-p)^3 + r^3(p-q)^3$

(1) $= [p(q-r)]^3 + [q(r-p)]^3 + [r(p-q)]^3 \rightarrow (1)$

here $a+b$

$p(q-r) + q(r-p) + r(p-q) = 0$

By known formula

(1) $= 3p(q-r)q(r-p).r(p-q)$

$= 3pqr(p-q)(q-r)(r-p)$

∴ one of the factor $= pqr$

Ans. D

03. $f(x) = x^4 - 2x^3 + 3x^2 - ax - b$

$f(1) = 6$

$\Rightarrow 1 - 2 + 3 - a - b = 6$

$\Rightarrow a + b = -4$

Ans. C

04 put $x = 1$

Ans: C

05 $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$

$(a+b+c)(a-b+c)$

$= (a+c+b)(a+c-b)$

$= (a+c)^2 - b^2$

$= (a+c)^2 - ac$

$= a^2 + c^2 + ac$

Ans. B

06 $f(x) = x^4 + x^3$

Remainder $= f(-9) = (-9)^4 + (-9)^3 \rightarrow$ Constant

$\therefore$ degree $= 0$

Ans: B

07 $f(x) = x^3 + 3x^2 + 3x + 1$

$\therefore f(x) = (x+1)^3$

Remainder $\Rightarrow f(\pi) = (\pi+1)^3$

$= \left(\frac{22}{7} + 1\right)^3 = \left(\frac{29}{7}\right)^3$

Ans C

08 $3x^2 + 7x + 6 \overline{)\, 3x^3 + x^2 - 2x + 9 \,(x - 2}$

$3x^3 + 7x^2 - 6x$

$- 6x^2 + 4x + 9$

$- 6x^2 - 14x + 12$

$18x - 3$

Ans: A

09 $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f(1) = a + b + c + d + e = 0$

$f(-1) = a - b + c - d + e = 0 \Rightarrow a + c + e = b + d$

Ans: D

10 $c^2 x + d^2 = 0 \Rightarrow x = \frac{-d^2}{c^2} = -\left[\frac{d}{c}\right]^2$ Ans: B

$\Rightarrow$ THE END $\Leftarrow$