5. TRIGONOMETRIC IDENTITIES ①

Class: VIII, MATHEMATICS

FOUNDATION+. SOLUTIONS

TEACHING TASK (2025/26)

JEEMAINS LEVEL

01. We know, $\tan^2\theta = \sec^2\theta - 1$

$= \frac{169}{144} - 1 = \frac{25}{144}$

$\therefore \tan\theta = \frac{5}{12}$

Ans - D

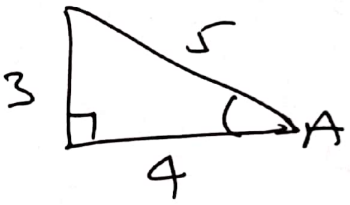
02. $\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{m^2}{n^2}}$

Ans. A

03. $\tan A = \frac{3}{4}$

$-\sin A = \frac{3}{5}$

$\Rightarrow \sin^2 A = \frac{9}{25}$



Ans. A

04. $y^2 - x^2 = \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow y^2 = x^2 + 1$

Ans. A

05. $\sec^2 A = 1 + \tan^2 A = 1 + \frac{p^2}{q^2} = \frac{q^2+p^2}{q^2}$

$\therefore \sec A = \frac{\sqrt{p^2+q^2}}{q}$

Ans. A

06. $\left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right)$ (2)

$=(1+\cot^2 A)(1+\tan^2 A)$

$=\operatorname{cosec}^2 A \cdot \sec^2 A$

Ans: A

07 $\sin^2 A \cos^2 B - \cos^2 A \cdot \sin^2 B$

$=(1-\cos^2 A)\cos^2 B - \cos^2 A\,(1-\cos^2 B)$

$=\cos^2 B - \cos^2 A\cos^2 B - \cos^2 A + \cos^2 A \cdot \cos^2 B$

$=\cos^2 B - \cos^2 A$

Ans. D

08 $\frac{\tan\theta}{(1+\tan^2\theta)^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$

Let $\theta = 45^\circ$

$=\frac{1}{(1+1^2)^2} + \frac{1}{(1+1^2)^2} = \frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

opt: c $\rightarrow \sin\theta\cdot\cos\theta = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \frac{1}{2}$

Ans: C

09 $\sec\theta + \tan\theta = p \rightarrow (1)$

$\sec\theta - \tan\theta = \frac{1}{p} \rightarrow (2)$

(1) + (2) $\Rightarrow 2\sec\theta = p + \frac{1}{p} = \frac{p^2+1}{p}$

$\Rightarrow \sec\theta = \frac{p^2+1}{2p}$

(1) − (2) $\Rightarrow 2\tan\theta = p - \frac{1}{p} = \frac{p^2-1}{p}$

$\Rightarrow \tan\theta = \frac{p^2-1}{2p}$

$\frac{2\tan\theta}{2\sec\theta} = \frac{p^2-1}{p^2+1}$

$\sin\theta = \frac{p^2-1}{p^2+1}$

Ans: B

10. $\frac{\operatorname{cosec}^2\alpha}{1+\cot^2\alpha} + \frac{\tan^2\alpha}{1+\tan^2\alpha} + \cos^2\alpha$

Let $\alpha = 45^\circ$

$= \Rightarrow \frac{(\sqrt{2})^2}{1+(1)^2} + \frac{(1)^2}{1+(1)^2} + \left(\frac{1}{\sqrt{2}}\right)^2 = 2$

Ans. D

11 $\sin^6\theta + \cos^6\theta + \sin^2\theta \cdot \cos^2\theta = a\sin^4\theta + b\cos^4\theta$

Let $\theta = 0^\circ$ | Let $\theta = 90^\circ$

$1 = b$ | $1 = a$

Now, $a + 2b = 1 + 2 \cdot 1 = 1 + 2 = 3$

Ans. C

JEE ADVANCED LEVEL

01. $\tan\theta = \frac{7}{24}$

A) $\sec\theta = \frac{25}{24}$ ✓

B) $\sin\theta = \frac{7}{25}$ ✓

C) $\cos\theta = \frac{24}{25}$ ✓, D) $\operatorname{cosec}\theta = \frac{25}{7}$

Ans.. A, B, C, D

02 $\sin(\alpha+\beta) = 1 \Rightarrow \alpha+\beta = 90^\circ \rightarrow (1)$

$\sin(\alpha-\beta) = \frac{1}{2} \Rightarrow \alpha-\beta = 60^\circ \rightarrow (2)$

$\therefore$ Solving (1) & (2) $\Rightarrow \alpha = 60^\circ, \beta = 30^\circ$ Ans: A, C

03 $a\sin^2\theta + b\cos^2\theta = c$

$\Rightarrow \frac{a\sin^2\theta}{\cos^2\theta} + \frac{b\cos^2\theta}{\cos^2\theta} = \frac{c}{\cos^2\theta}$

$\Rightarrow a\tan^2\theta + b = c\sec^2\theta$

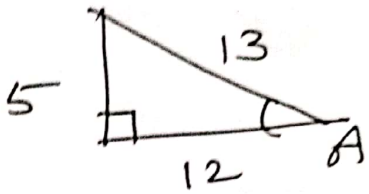
$\Rightarrow a\tan^2\theta + b = c(1+\tan^2\theta)$

$\Rightarrow \tan^2\theta = \frac{c-b}{a-c}$ Ans: B

---

04. Statement I: $\sin A = \frac{5}{13}$

$\Rightarrow \cos A = \frac{12}{13}$



Statement II: Conceptual (True) Ans: A

---

05. Statement I: $\sin^2 A + \sin^2\left(\frac{\pi}{2} - A\right)$

$= \sin^2 A + \cos^2 A = 1$ (True)

Statement II:

LHS $= \sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}$

$= \sin^2 10^\circ + \sin^2 20^\circ + \sin^2 70^\circ + \sin^2 80^\circ$

$= (\sin^2 10^\circ + \sin^2 80^\circ) + (\sin^2 20^\circ + \sin^2 70^\circ)$

$= 1 + 1 = 2$ (False) Ans: C

---

06 Statement I: Conceptual (True)

Statement II: LHS $= \frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A}$

$= \operatorname{cosec} A + \cot A - \operatorname{cosec} A$

$= \cot A$

RHS $= \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$

$= \operatorname{cosec} A - (\operatorname{cosec} A - \cot A) = \cot A$ (True)

Ans: A

07. Assertion: $\cos^4 x - \sin^4 x = \cos 2x$

put $x = 0^\circ$ $\cos^4 0^\circ - \sin^4 0^\circ = \cos 2.0^\circ$

$\Rightarrow 1 - 0 = 1 \Rightarrow 1 = 1$ (True)

(OR) $LHS = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$= \cos 2x = RHS$

Reason: Conceptual (True)

Ans: A

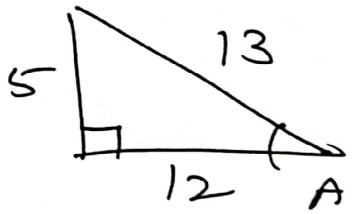
08 Assertion: $\sin^2\theta + \cos^2\theta = 1 \quad \forall \theta \in R$ (False)

Reason: Conceptual (False)

Ans: B

09 $\tan A = \frac{5}{12}$

$\sec A = \frac{13}{12}$



Ans: A

10. $\sin A = \frac{5}{13}$

Ans: A

11 $\sin\theta + \cos\theta = \frac{b}{a}$; $\sin\theta \cdot \cos\theta = \frac{c}{a}$

$(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)$

$= (\sin\theta + \cos\theta)\sqrt{(\sin\theta + \cos\theta)^2 - 4\sin\theta\cos\theta}$

$= \left(\frac{b}{a}\right)\sqrt{\left(\frac{b}{a}\right)^2 - \frac{4c}{a}}$

$= \left(\frac{b}{a}\right)\sqrt{\frac{b^2 - 4ac}{a^2}} = \frac{b\sqrt{b^2 - 4ac}}{a^2}$

Ans: D

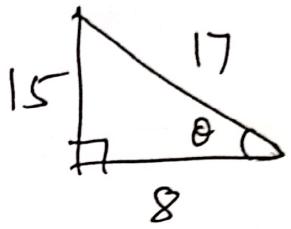
12. $\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$ (6)

$= 1 - 2\left(\frac{c}{a}\right)^2 = \frac{a^2 - 2c^2}{a^2}$

Ans: A

13. $\cos\theta = \frac{8}{17}$

$\therefore \sin\theta = \frac{15}{17}$



$\Rightarrow 289\sin^2\theta = 225$

Ans: 225

14 $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Ans: 1

15 a) Conceptual (q)

b) Conceptual (p)

c) Conceptual (r)

d) Conceptual (s)

Ans: q, p, r, s

16 a) $\sin(n\pi + \theta) = (-1)^n \cdot \sin\theta$ (t)

b) $\cos(n\pi + \theta) = (-1)^n \cdot \cos\theta$ (r)

c) $\tan\left((2n+1)\frac{\pi}{2} - \theta\right) = \cot\theta$ (p)

d) $\cos\left[(2n+1)\frac{\pi}{2} + \theta\right] = (-1)^{n+1} \cdot \sin\theta$ (q)

Ans: t, r, p, q

LEARNERS TASK

CONCEPTUAL UNDERSTANDING QUESTION (CUQ)

01 $\sec^2 x - \tan^2 x = 1$

Ans: C

02 $\cot A = 0 \Rightarrow$

$\operatorname{cosec}^2 A - \cot^2 A = 1$

$\operatorname{cosec}^2 A - (0)^2 = 1 \Rightarrow \operatorname{cosec} A = 1$ (Treat A as acute)

Ans. B

03 Conceptual

Ans: B

04. Conceptual

Ans. A

05 Conceptual

Ans: C

06 $\dfrac{\operatorname{cosec}\theta + 1}{\cot\theta} \times \dfrac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta - 1} = \dfrac{\operatorname{cosec}^2\theta - 1}{\cot\theta(\operatorname{cosec}\theta - 1)}$

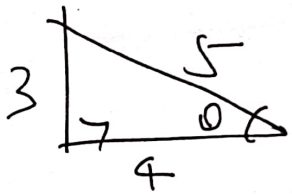
$= \dfrac{\cot^2\theta}{\cot\theta(\operatorname{cosec}\theta - 1)} = \dfrac{\cot\theta}{\operatorname{cosec}\theta - 1}$

Ans. B

07 $4\sin\theta - 3\cos\theta = 0$

$\Rightarrow 4\sin\theta = 3\cos\theta$

$\Rightarrow \tan\theta = \dfrac{3}{4}$



$\sin^2\theta - \cos^2\theta = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \dfrac{9-16}{25} = \dfrac{-7}{25}$

Ans. D

08 $\tan^2 45° + \dfrac{1}{\tan^2 45°} = 1 + \dfrac{1}{1} = 2$

Ans. B

09 $\cos\theta = \sec\theta$

$\Rightarrow \cos^2\theta = 1$

$\sin\theta = \operatorname{cosec}\theta$

$\sin^2\theta = 1$

$\therefore \sin^2\theta - \cos^2\theta = 0$

Ans. C

10 $(1 - \sin^2\theta)\sec^2\theta = \cos^2\theta \cdot \sec^2\theta = 1$

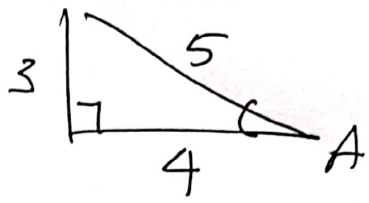
$\cos\theta \cdot \sec\theta = 1$ (Here θ is considered as acute angle)

Ans: D

JEE MAINS LEVEL

01 $\operatorname{cosec} A = \frac{5}{3}$

$\therefore \cot A = \frac{4}{3}$



Ans: A

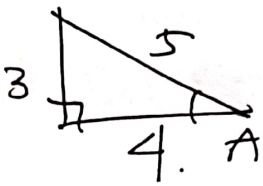
02 $\cos x = \frac{12}{13} \Rightarrow \sec x = \frac{13}{12}$

$\therefore \sec^2 x = \frac{169}{144}$ or $\frac{13^2}{12^2}$

Ans: D

03 $\sin A = \frac{3}{5}$

$\sec A = \frac{5}{4}$



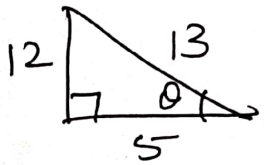
Ans: A

04 $\operatorname{cosec}^2 A - 1 = (\cot^2 A + 1) - 1 = \cot^2 A = (2)^2 = 4$

Ans B

05 $\cos\theta = \frac{5}{13}$

$\therefore \tan\theta = \frac{12}{5}$



Ans: A

06 $\frac{1}{\cos\theta - \cot\theta} + \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

$= \operatorname{cosec}\theta + \cot\theta + \operatorname{cosec}\theta - \cot\theta - 2\operatorname{cosec}\theta$

Ans: B

07 $\sec^4 A - \sec^2 A = \sec^2 A(\sec^2 A - 1)$

$= (1 + \tan^2 A)(\tan^2 A)$

$= \tan^2 A + \tan^4 A$

Ans: C

08 $(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$

put $A = 45^\circ$

$= \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2$

$= 2\left(\frac{3}{\sqrt{2}}\right)^2 = 9.$

Opt: B: $(1 + \sec A \operatorname{cosec} A)^2$

$= (1 + \sqrt{2} \cdot \sqrt{2})^2 = 9$

Ans: B

09) $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = K \quad \rightarrow (1)$

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = K \quad \rightarrow (2)$

$(1) \times (2) \Rightarrow (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C) = K^2$

$\Rightarrow K^2 = 1$

$\Rightarrow K = \pm 1$

Ans: C

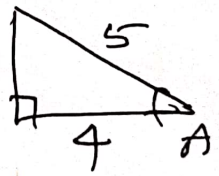
10. $\frac{1}{\sec A + \tan A} - \frac{1}{\cos A}$

$= \sec A - \tan A - \sec A = -\tan A$

Ans: C

b JEE ADVANCED

01. $\sin A = \frac{3}{5}$



$\cos A = \frac{4}{5}$, $\tan A = \frac{3}{4}$

$\sec A = \frac{5}{4}$, ~~$\cot A = \frac{3}{4}$~~

Ans: A, B, C

02 $(\sec\theta + \tan\theta)(1 - \sin\theta)$

$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)$

$= \frac{(1 + \sin\theta)(1 - \sin\theta)}{\cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta}$

$= \cos\theta$ or $\frac{1}{\sec\theta}$

Ans: A, C

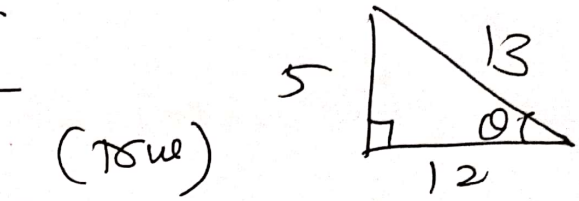
03 Statement I: $\sin^2 A + \cos^2 A = 1 \quad \forall A \in R$ (False)

Statement II: Conceptual (True)

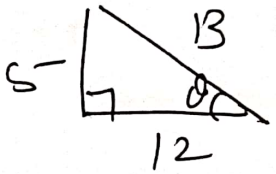
Ans: (D)

04. Statement I: $\tan\theta = \frac{5}{12}$

$\sec\theta = \frac{13}{12}$ (True)



Statement II:



(True)

Ans. A

05 Assertion: $\tan\theta = \frac{3}{4}$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$ (True)

Reason: Conceptual (True)

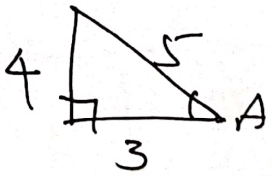
Ans. A

06 Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans. A

07 $\cot A = \frac{3}{4}$



A) $\operatorname{cosec} A = \frac{5}{4}$

$\Rightarrow \operatorname{cosec}^2 A = \frac{25}{16}$

Ans. A

08 B) $\cot 0^\circ$ = not defined

D) $\operatorname{cosec} 0$ = not defined

Ans. B, D

09. $\sin(A+B) = \frac{\sqrt{3}}{2} \Rightarrow A+B = 60^\circ \rightarrow (1)$

$\cos(A-B) = \frac{\sqrt{3}}{2} \Rightarrow A-B = 30^\circ \rightarrow (2)$

Solving (1) & (2) $\Rightarrow A = 45^\circ, \quad B = 15^\circ$

$\sin 2A + \cos 2A = \sin 2\times 45^\circ + \cos 2\times 45^\circ$

$= \sin 90^\circ + \cos 90^\circ$

$= 1 + 0 \quad = 1$

Ans. A

---

10 $\sin^2 A + \cos^2 B$

$= \sin^2 45^\circ + \cos^2 3\times 15^\circ$

$= \sin^2 45^\circ + \cos^2 45^\circ$

$= 1$

Ans: C

---

11 $\tan^2 2A + \tan^2 B$

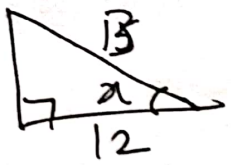
$= \tan^2 2\times 45^\circ + \tan^2 15^\circ$

$= \tan^2 90^\circ + \tan^2 15 \rightarrow$ Not defined

Ans: D

---

12 $\sec x = \frac{13}{12}$



$\tan x = \frac{5}{12}$

$144 \tan^2 x = 25$

Ans: 25

---

13 $\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \cdots \cos 90^\circ \cdots \cos 180 = 0$

$\cos 90^\circ \rightarrow 0$

Ans: 0

---

14 $3(\cos^4\alpha + \sin^4\alpha) - 2(\sin^6\alpha + \cos^6\alpha)$

put $\alpha = 0^\circ \Rightarrow 3(1+0) - 2(0+1) = 1$ Ans. 1

15 Given $\cos A + \cos^2 A = 1$

$\Rightarrow \cos A = 1 - \cos^2 A$

$\Rightarrow \cos A = \sin^2 A$

$\Rightarrow \cos^2 A = \sin^4 A$

Now, $\sin^2 A + \sin^4 A$

$= \sin^2 A + \cos^2 A = 1$ Ans. 1

16 a) Conceptual (p)

b) Conceptual (q)

c) Conceptual (r)

d) Conceptual (r) Ans. p, q, r, r

17 a) $\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha} \times \frac{1-\sin\alpha}{1-\sin\alpha}} = \frac{1-\sin\alpha}{\cos\alpha}$

$= \sec\alpha - \tan\alpha$ (q) (s)

b) $\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha} \times \frac{1+\sin\alpha}{1+\sin\alpha}} = \frac{1+\sin\alpha}{\cos\alpha}$

$= \sec\alpha + \tan\alpha$ (p)

c) $\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha} \times \frac{1+\cos\alpha}{1+\cos\alpha}} = \frac{1+\cos\alpha}{\sin\alpha} = \operatorname{cosec}\alpha + \cot\alpha$ (t)

d) $\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha} \times \frac{1-\cos\alpha}{1-\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} = \operatorname{cosec}\alpha - \cot\alpha$ (q)

Ans. s, p, t, q

ADDITIONAL PRACTICE QUESTIONS

01. $\cos^2\theta + \frac{1}{1+\cot^2\theta}$

$= \cos^2\theta + \frac{1}{\operatorname{cosec}^2\theta} = \cos^2\theta + \sin^2\theta = 1$

Ans. D

02 $\sec\theta\,(1-\sin\theta)(\sec\theta+\tan\theta)$

$= (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$

$= \sec^2\theta - \tan^2\theta = 1$

Ans : D

03 $\sin^2\theta + \frac{1}{1+\tan^2\theta}$

$= \sin^2\theta + \frac{1}{\sec^2\theta}$

$= \sin^2\theta + \cos^2\theta$

$= 1$

Ans : A

04 $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$

put $\theta = 0$

$= \frac{1+0}{1} + \frac{1}{1+0} = 2$

opt: B : $2\sec\theta = 2\sec 0 = 2\times 1 = 2$

Ans. B

05 $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$

$= \tan^2\theta \times \cot^2\theta = 1$

Ans : C

06 $(1+\cot\theta - \operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)$

put $\theta = 45^\circ$

$(1+1-\sqrt{2})(1+1+\sqrt{2}) = (2-\sqrt{2})(2+\sqrt{2})$

$= 4-2 = 2$

Ans : C

07. $\cot^2\alpha\left(\frac{\sec\alpha-1}{1+\sin\alpha}\right)+\sec^2\alpha\left(\frac{\sin\alpha-1}{1+\sec\alpha}\right)$

Let $\alpha = 45^\circ$

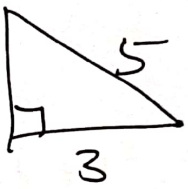
$= 1\left(\frac{\sqrt{2}-1}{1+\frac{1}{\sqrt{2}}}\right)+(\sqrt{2})^2\left(\frac{\frac{1}{\sqrt{2}}-1}{1+\sqrt{2}}\right)$

$= \sqrt{2}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)+\sqrt{2}\left(\frac{1-\sqrt{2}}{1+\sqrt{2}}\right)$

$= \sqrt{2}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)-\sqrt{2}\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right)=0$

Ans: A

08 $\tan A = -\frac{4}{3} \Rightarrow A \in Q_2$



$\sec A = -\frac{5}{3} \rightarrow \alpha$

$\operatorname{cosec} A = \frac{5}{4} \rightarrow \beta$

$\therefore x^2-\left(-\frac{5}{3}+\frac{5}{4}\right)x+\left(\frac{-5}{3}\right)\left(\frac{5}{4}\right)=0$

$\Rightarrow 12x^2+5x-25=0$

Ans. A

09. $\sec^4 A(1-\sin^4 A)-2\tan^2 A$

put $A = 0^\circ$

$(1)^4(1-0)-2(0)^2=1$

Ans: B

$\Rightarrow$ THE END $\in$