PROPERTIES OF TRIANGLES

Class: IX, Advanced.

MATHEMATICS

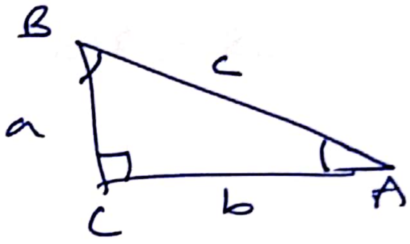
SOLUTIONS

TEACHING TASK

01. $A : B : C = 2:3:5$

$\Rightarrow A = \frac{2}{10} \times 180^\circ = 36^\circ, \quad B = \frac{3}{10} \times 180^\circ = 54^\circ$

$C = \frac{5}{10} \times 180^\circ = 90^\circ$



greatest side : smallest side

$= c : a$

$= \frac{c}{a} = \frac{2R\sin C}{2R\sin A} = \frac{\sin 90^\circ}{\sin 36^\circ} = \frac{1}{\left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)}$

$\therefore 4 : \sqrt{10-2\sqrt{5}}$ Ans. B

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02. $A + C = 2B$

$\Rightarrow A + B + C = 3B$

$\Rightarrow 180^\circ = 3B$

$\Rightarrow B = 60^\circ$

$\cos B = \frac{c^2+a^2-b^2}{2ac}$

$\Rightarrow \cos 60^\circ = \frac{c^2+a^2-b^2}{2ac}$

$\Rightarrow \frac{1}{\cancel{2}} = \frac{c^2+a^2-b^2}{\cancel{2}ac}$

$ac = c^2+a^2-b^2$

$b^2 = a^2-ac+c^2$

Now $\frac{a+c}{\sqrt{a^2-ac+c^2}} = \frac{a+c}{b} = \frac{2R\sin A + 2R\sin C}{2R\sin B} = \frac{\sin A + \sin C}{\sin \cancel{60}^\circ}$

$= \frac{2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)}{\sin B} = \frac{2\sin\left(\cancel{\frac{2B}{2}}\right)\cos\left(\frac{A-C}{2}\right)}{\cancel{\sin B}}$

$2\cos\left(\frac{A-C}{\ }\right)$ Ans: A

03 $c^4 - 2c^2(a^2+b^2) + a^4 + a^2b^2 + b^4 = 0$

$\Rightarrow (a^2)^2 + (b^2)^2 + (-c^2)^2 + a^2b^2 + 2a^2(-c^2) + 2b^2(-c^2) = 0$

Adding a^2b^2 on b/s to make it perfect square

$\Rightarrow (a^2+b^2-c^2)^2 = (ab)^2$

$\Rightarrow a^2+b^2-c^2 = ab$

Now $\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{\cancel{ab}}{2\cancel{ab}} = \frac{1}{2} \Rightarrow C = 60^\circ$

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04 Given $x^2 - 2\sqrt{3}x + 2 = 0$.

let a & b be the roots

We have $a+b = 2\sqrt{3}$; $ab = 2$, $\angle C = \frac{\pi}{3}$

$\cos C = \frac{a^2+b^2-c^2}{2ab} \Rightarrow \cos 60^\circ = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \frac{1}{\cancel{2}} = \frac{a^2+b^2-c^2}{\cancel{2}ab} \Rightarrow ab = a^2+b^2-c^2$

$\Rightarrow a^2+b^2-ab = c^2$

$\Rightarrow (a+b)^2 - 3ab = c^2$

$\Rightarrow (2\sqrt{3})^2 - 3(2) = c^2$

$\Rightarrow c^2 = 6 \Rightarrow c = \sqrt{6}$

Ans : A

Perimeter $= a+b+c = 2\sqrt{3} + \sqrt{6}$

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05 In $\triangle ABC$, $(a+b+c)(b+c-a) = \lambda bc$

$\Rightarrow (2s)(2s-2a) = \lambda bc$

$\Rightarrow \frac{s(s-a)}{bc} = \frac{\lambda}{4}$

$\Rightarrow \cos^2\frac{A}{2} = \frac{\lambda}{4}$

We know $0 \le \cos^2\frac{A}{2} \le 1$

$0 \le \frac{\lambda}{4} \le 1$

$\Rightarrow 0 \le \lambda \le 4$

Ans : C

06 $\tan\frac{A}{2}\cdot\tan\frac{B}{2}+\tan\frac{B}{2}\cdot\tan\frac{C}{2}=\frac{2}{3}$

$\Rightarrow \frac{\Delta}{s(s-a)}\cdot\frac{\Delta}{s(s-b)}+\frac{\Delta}{s(s-b)}\cdot\frac{\Delta}{s(s-c)}=\frac{2}{3}$

$\Rightarrow \frac{\Delta^2}{s^2(s-b)}\left[\frac{1}{s-a}+\frac{1}{s-c}\right]=\frac{2}{3}$

$\Rightarrow \frac{\Delta^2}{s^2(s-b)}\left(\frac{2s-a-c}{(s-a)(s-c)}\right)=\frac{2}{3}$

$\Rightarrow \frac{\Delta^2}{s^2(s-b)}\times\frac{b}{(s-a)(s-c)}=\frac{2}{3}$

$\Rightarrow \frac{b}{s}=\frac{2}{3} \longrightarrow 3b=2s$

$\Rightarrow 3b=a+b+c$

$\Rightarrow a+c=2b$

Ans: B

07 This problem can be done in shortcut way.
Let us consider an equilateral $\triangle$le.
with sides $a=b=c=1$ unit
Now, altitudes $=\frac{\sqrt{3}}{2}a=\frac{\sqrt{3}}{2}$; Area $=\frac{\sqrt{3}}{4}a^2=\frac{\sqrt{3}}{4}$.

Now $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}-\frac{2ab}{(a+b+c)\Delta}\cdot\cos^2\frac{C}{2}$

$=\frac{2}{\sqrt{3}}+\frac{2}{\sqrt{3}}+\frac{2}{\sqrt{3}}-\frac{2.1.1}{(1+1+1)\times\frac{\sqrt{3}}{4}}\times\cos^2 30^\circ$

$=\frac{6}{\sqrt{3}}-\frac{8}{3\sqrt{3}}\times\frac{3}{4}$

$=\frac{6}{\sqrt{3}}-\frac{2}{\sqrt{3}}=0$

Ans: A

08 $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{2R\sin A}{2R\cos A}+\frac{2R\sin B}{2R\cos B}+\frac{2R\sin C}{2R\cos C}$

$=\tan A+\tan B+\tan C$

$=\tan A\cdot\tan B\cdot\tan C$ ($\because$ in $\triangle ABC$)

$=\frac{a}{x}\cdot\frac{b}{y}\cdot\frac{c}{z}$

Ans: A

10 We have $a^2 = b^2+c^2-2bc\cos A$

$\Rightarrow c^2-2b\cos A\, c + b^2-a^2=0$

This is a Q. E. in c

let c_1 & c_2 be the roots.

$c_1+c_2 = 2b\cos A$; $c_1 . c_2 = b^2-c^2$

Now, $(c_1-c_2)^2 = (c_1+c_2)^2-4c_1c_2$

$= 4b^2\cos^2 A - 4(b^2-a^2)$

$= \cancel{4b^2}$

$= 4a^2 - 4b^2(1-\cos^2 A)$

$= 4a^2 - 4b^2\sin^2 A$

$= 4a^2 - \left(\frac{c_1+c_2}{\cos A}\right)^2 . \sin^2 A$

$= 4a^2 - (c_1+c_2)^2 . \tan^2 A$

$\therefore c_1-c_2 = \sqrt{4a^2-(c_1+c_2)^2 . \tan^2 A}$ Ans: A

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11 $c^2x^2 - c(a+b)x + ab=0$.

$\sin A$ and $\sin B$ are the roots.

$\therefore \sin A + \sin B = \frac{a+b}{c}$

$\Rightarrow \frac{a}{2R} + \frac{b}{2R} = \frac{a+b}{c}$

$\sin A \cdot \sin B = \frac{ab}{c^2}$

$\frac{a}{2R} \cdot \frac{b}{2R} = \frac{ab}{c^2}$

$\Rightarrow c = 2R$

$\Rightarrow 2R\sin C = 2R \Rightarrow \sin C = 1$

$\Rightarrow C = \frac{\pi}{2}$

Now, $A+B+C=\pi$

$\Rightarrow A+B = \pi - C = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

Now $\sin A + \sin B = \frac{a+b}{c} \Rightarrow \sin A + \cos A = \frac{a+b}{c}$

Ans: B, D

12 In $\triangle ABC$, $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$

$$\Rightarrow \sqrt{\frac{b+c}{2c}} = \sqrt{\frac{s(s-a)}{bc}}$$

$$\Rightarrow \frac{b+c}{2c} = \frac{s(s-a)}{bc}$$

$$\Rightarrow 2s(s-a) = b^2+bc$$

$$\Rightarrow (a+b+c)\left(\frac{a+b+c}{2} - a\right) = b^2+bc$$

$$\Rightarrow a^2+b^2=c^2$$

Area $= \frac{1}{2}ab$

Circum radius $= \frac{1}{2}c$

Ans : A, B

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13 $\frac{2a}{b+c-a} = \frac{2a}{a+b+c-2a} = \frac{2a}{2s-2a} = \frac{a}{s-a}$

$\|^{ly}$ $\frac{2b}{c+a-b} = \frac{b}{s-b}$, $\frac{2c}{a+b-c} = \frac{c}{s-c}$

Now A·M $\geq$ H·M.

$$\frac{\frac{a}{s-a} + \frac{b}{s-b} + \frac{c}{s-c}}{3} \geq \frac{3}{\frac{s-a}{a} + \frac{s-b}{b} + \frac{s-c}{s}}$$

$\Rightarrow$

13 statement I

Let $A = \dfrac{a}{b+c-a} + \dfrac{b}{c+a-b} + \dfrac{c}{a+b-c}$

$$\Rightarrow 2A = \frac{2a}{b+c-a} + \frac{2b}{c+a-b} + \frac{2c}{a+b-c}$$

$$\Rightarrow 2A = \left(\frac{2a}{b+c-a}+1\right) + \left(\frac{2b}{c+a-b}+1\right) + \left(\frac{2c}{a+b-c}+1\right) - 3$$

$$\Rightarrow 2A = (a+b+c)\left(\frac{1}{b+c-a} + \frac{1}{a+c-b} + \frac{1}{a+b-c}\right) - 3$$

Now AM $\geq$ HM.

$$\frac{\left[\frac{1}{b+c-a} + \frac{1}{a+c-b} + \frac{1}{a+b-c}\right]}{3} \geq \frac{3}{\left(\frac{b+c-a}{\cancel{b+c-a}}\right) + \left(\frac{a+c-b}{\cancel{a+c-b}}\right) + \left(\frac{a+b-c}{\cancel{a+b-c}}\right)}$$

$$\frac{1}{b+c-a} + \frac{1}{a+c-b} + \frac{1}{a+b-c} \geq \frac{9}{a+b+c}$$

$$(a+b+c)\left(\frac{1}{b+c-a} + \frac{1}{a+c-b} + \frac{1}{a+b-c}\right) \geq 9$$

$$\Rightarrow 2A+3 \geq 9$$

$$\Rightarrow A \geq 3$$

$\therefore$ The minimum value $= 3$ (statement I is wrong)

Statement II: (Conceptual) True   Ans: D

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14 $2\Delta = ax = by = cz$

$$\therefore \frac{b^2 xa + c^2 yb + a^2 zc}{abc} = \frac{a^2+b^2+c^2}{K}$$

$$\Rightarrow \frac{b^2(2\Delta) + c^2(2\Delta) + a^2(2\Delta)}{abc} = \frac{a^2+b^2+c^2}{K}$$

$$\Rightarrow 2\Delta/abc = \frac{1}{K} \Rightarrow K = \frac{abc}{2\Delta} = 2R$$

Ans: C

15 $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2} = \frac{a^2}{4\Delta^2}+\frac{b^2}{4\Delta^2}+\frac{c^2}{4\Delta^2}$

$$= \frac{a^2+b^2+c^2}{4\Delta^2}$$

Now $\cot A + \cot B + \cot C = \frac{R}{abc}$

$b^2+c^2-a^2+c^2+a^2-b^2+a^2+b^2-c^2$

$$= \frac{R}{abc}(a^2+b^2+c^2)$$

$$= \frac{R}{abc}\left(\frac{4\Delta^2}{x^2}+\frac{4\Delta^2}{y^2}+\frac{4\Delta^2}{z^2}\right)$$

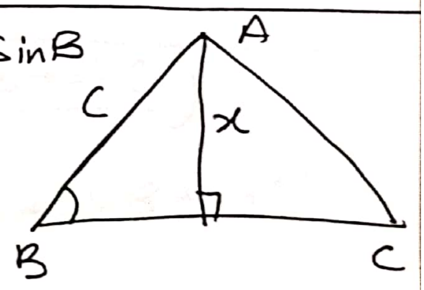
$$= \frac{4\Delta^2 R}{abc}\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)$$

$$= \frac{4\Delta R}{abc}\cdot\Delta\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)$$

$$= \Delta\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)$$

$\therefore\ K = \Delta$

16 In $\triangle ABC$, $\sin B = \frac{x}{c} \Rightarrow x = c\sin B$



$\sin C = \frac{x}{b} \Rightarrow x = b\sin C$

Hence $\frac{c\sin B + b\sin C}{x} = \frac{x+x}{x} = 2$

Therefore Given problem $= 2+2+2$
$= 6$

Ans: D

17 let a, b, c be the sides of a triangle

$$\therefore s = \frac{a+b+c}{2}$$

Also, $s, s-a, s-b, s-c$ will be positive

$$AM \geq G.M$$

$$\frac{s+(s-a)+(s-b)+(s-c)}{4} \geq \left[s(s-a)(s-b)(s-c)\right]^{\frac{1}{4}}$$

$$\Rightarrow \frac{4s-(a+b+c)}{4} \geq \left[\Delta^2\right]^{\frac{1}{4}}$$

$$\Rightarrow \frac{2s}{4} \geq \Delta^{\frac{1}{2}}$$

$$\Rightarrow \frac{s}{2} \geq \Delta^{1/2} \Rightarrow \frac{s^2}{4} \geq \Delta$$

Ans : C

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18 Given $a+b+c = 6\left(\frac{\sin A + \sin B + \sin C}{3}\right)$

$k(\sin A + \sin B + \sin C) = 2(\sin A + \sin B + \sin C)$

$$\therefore k = 2$$

Now $\frac{a}{\sin A} = 2$

$\Rightarrow \sin A =$

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18 Given $a+b+c = 6\left(\frac{\sin A + \sin B + \sin C}{3}\right)$

$$\Rightarrow 2R(\sin A + \sin B + \sin C) = 2(\sin A + \sin B + \sin C)$$

$$\Rightarrow R = 1$$

Now $2R \sin A = a$

$$\Rightarrow 2 \times 1 \times \sin A = 1$$

$$\Rightarrow \sin A = \frac{1}{2} \Rightarrow A = \frac{\pi}{6}$$

Ans: A

19 $A:B:C = 2:3:7$

$\therefore A = \frac{2}{12} \times 180 = 30^\circ$

$B = \frac{3}{12} \times 180^\circ = 45^\circ$

$C = \frac{7}{12} \times 180^\circ = 105^\circ$

Now $\sin 30^\circ : \sin 45^\circ : \sin 105^\circ$

$= \frac{1}{2} : \frac{1}{\sqrt{2}} : \frac{\sqrt{3}+1}{2\sqrt{2}}$

$= \sqrt{2} : 2 : \sqrt{3}+1$

Ans : A

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20 a) $a = 5, \quad A = 30$

We know, $a = 2R \sin A$

$\Rightarrow 5 = 2R \sin 30^\circ$

$\Rightarrow 5 = 2R \times \frac{1}{2}$

$\Rightarrow R = 5$

Ans : 5

b) $a = \sqrt{3}+1, \quad B = 30^\circ, \quad C = 45^\circ \Rightarrow A = 105^\circ$

$a = 2R \sin A$

$\Rightarrow \sqrt{3}+1 = 2R \sin 105^\circ$

$\Rightarrow \cancel{\sqrt{3}+1} = \cancel{2}R \cdot \left(\frac{\cancel{\sqrt{3}+1}}{\cancel{2}\sqrt{2}}\right)$

$\Rightarrow R = \sqrt{2}$

Now $c = 2R \sin C$

$= 2\sqrt{2} \sin 45^\circ$

$= 2\cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}}$

$= 2$

Ans : 2

21 Let x, y be any two positive numbers (10)

$\therefore$ x, a, y are in A.P.

$\Rightarrow$ $a = \dfrac{x+y}{2}$ $\rightarrow (1)$

Also x, b, c, y are in G.P

so $b^2 = cx$, $c^2 = by$ $\rightarrow (2)$

Now $\cancel{\dfrac{b^3+c^3}{abc}}$ $\dfrac{\sin^3 B + \sin^3 C}{\sin A \cdot \sin B \cdot \sin C}$

$$= \frac{b^3 + c^3}{abc} = \frac{bc(x+y)}{\left(\frac{bc(x+y)}{2}\right)} = 2$$

22 a) $(a+b+c)(b+c-a)=\lambda bc$

$\Rightarrow (2s)(2s-2a)=\lambda bc$

$\Rightarrow 4\left(\frac{s(s-a)}{bc}\right)=\lambda$

$\Rightarrow 4\cos^2\frac{A}{2}=\lambda \Rightarrow \cos^2\frac{A}{2}=\frac{\lambda}{4}$

We know $0\le \cos^2\frac{A}{2}\le 1$

$\Rightarrow 0\le \frac{\lambda}{4}\le 1 \Rightarrow 0\le \lambda\le 4.$

Greatest value of $\lambda=4.$

b) In $\triangle ABC$, $\tan A+\tan B+\tan C=\tan A\cdot\tan B\cdot\tan C$

By Cauch's Inequality.

$(t_1+t_2+t_3)\le\sqrt{t_1^2+t_2^2+t_3^2}\,(\sqrt{3})$

$\Rightarrow K<27 \Rightarrow 9.3^{1/3}\le K$

By $GM\le AM$

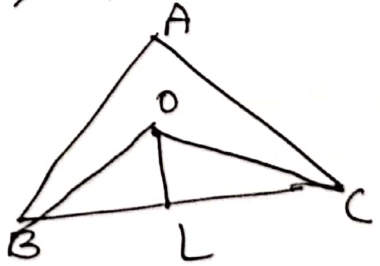
(OR) $\tan A+\tan B+\tan C=\tan A\cdot\tan B\cdot\tan C$

$AM\ge GM$

$\Rightarrow \frac{\tan^2A+\tan^2B+\tan^2C}{3}\ge(\tan^2A\cdot\tan^2B\cdot\tan^2C)^{1/3}$

$\Rightarrow \frac{K}{3}\ge(9)^{2/3} \Rightarrow K\ge 3.9^{2/3}$

$K\ge 9.3^{1/3}$



c) In $\triangle OBL$, $\cos A=\frac{OL}{OB}=\frac{r}{R}$

$\Rightarrow R\cos A=r$

$\Rightarrow R\cos A=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow \cos A=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow \cos A=\cos A+\cos B+\cos C-1$

$\Rightarrow \cos A+\cos B=1$

d) Given $a=5$, $b=4$, $\cos(A-B)=\frac{31}{32}$

$$\tan\left(\frac{A-B}{2}\right)=\sqrt{\frac{1-\cos(A-B)}{1+\cos(A+B)}}=\sqrt{\frac{1}{63}}$$

$$\Rightarrow \frac{a-b}{a+b}\cot\frac{C}{2}=\frac{1}{\sqrt{63}} \Rightarrow \frac{5-4}{5+4}\cot\frac{C}{2}=\frac{1}{\sqrt{63}}$$

$$\Rightarrow \cancel{\cot}\ \tan\frac{C}{2}=\frac{\sqrt{7}}{3}$$

Now $\cos C=\dfrac{1-\tan^2\frac{C}{2}}{1+\tan^2 C/2}=\dfrac{1-\frac{7}{9}}{1+\frac{7}{9}}=\dfrac{1}{8}$

Now, $c^2=a^2+b^2-2ab\cos C$

$$\Rightarrow c^2=25+16-40\times\frac{1}{8}=36 \Rightarrow c=6.$$

23 a) $A=60^\circ \Rightarrow B+C=120^\circ \Rightarrow \frac{B}{2}+\frac{C}{2}=60^\circ$

Now $\dfrac{b}{c+a}+\dfrac{c}{a+b}=\dfrac{\sin B}{\sin A+\sin C}+\dfrac{\sin C}{\sin A+\sin B}$

$$=\frac{\sin B}{2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)}+\frac{\sin C}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$

$$\cancel{\Rightarrow}\ \frac{\sin\frac{B}{2}}{\cos\left(\frac{A-C}{2}\right)}+\frac{\sin\frac{C}{2}}{\cos\left(\frac{A-B}{2}\right)}$$

$$=\frac{\sin\frac{B}{2}}{\cos\left(30^\circ-\frac{C}{2}\right)}+\frac{\sin\frac{C}{2}}{\cos\left(30^\circ-\frac{B}{2}\right)}$$

$$=\frac{\sin B/2}{\cos\left(30^\circ-\frac{B}{2}\right)}+\frac{\sin\frac{C}{2}}{\cos\left(30^\circ-\frac{B}{2}\right)}$$

$$=\frac{2\sin 30^\circ\cos\left(\frac{B-C}{4}\right)}{\cos\left(30^\circ-\frac{B}{2}\right)}=$$

$$=2\times\frac{1}{2}\times\frac{\cos\left(\frac{B}{2}-30^\circ\right)}{\cos\left(30^\circ-\frac{B}{2}\right)}$$

$$=1$$

b) $c^2 = a^2 + b^2 - 2ab\cos C$, $\angle C = 60^\circ$

$\Rightarrow c^2 = a^2 + b^2 - 2ab \times \frac{1}{2}$

$\Rightarrow c^2 = a^2 + b^2 - ab \rightarrow$ 1

$\boxed{\Rightarrow \cancel{\frac{1}{a+c} + \frac{1}{b+c}} = \cancel{\frac{a+b+2c}{a+b+2c(a+c)(b+c)}}}$

$\Rightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$

$\Rightarrow b(b+c) + a(a+c) = (a+c)(b+c)$

$\div$ by $(a+c)(b+c)$ and add 2 on b/s

$1 + \frac{b}{a+c} + 1 + \frac{a}{b+c} = 3$

$\Rightarrow \frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$

c) $b^2 = c^2 + a^2 - 2ac\cos B$ since $B = 60^\circ$

$\Rightarrow b^2 = c^2 + a^2 - ac$

$\boxed{\text{Now } \cancel{\frac{1}{a+b} + \frac{1}{a+c} = \frac{a+c+a+b}{(a+b)(a+c)} = \frac{2a+b+c}{a^2+ac+ba+bc}} = \frac{2a+b+c}{}}$

$\Rightarrow b^2 + ac = c^2 + a^2$

$\Rightarrow b^2 + ac + bc + ab = c^2 + cb + a^2 + ab$

$\Rightarrow b(b+c) + a(c+b) = c(c+b) + a(a+b)$

$\div$ by $(b+c)(a+b)$ and add 2. on b/s

$\Rightarrow \frac{1}{a+b} + \frac{1}{a+c} = \frac{3}{a+b+c}$

d) $C = 60^\circ \Rightarrow c^2 = a^2 + b^2 - 2ab\cos C$ (14)

$\Rightarrow c^2 = a^2 + b^2 - ab$

$\Rightarrow ab = a^2 + b^2 - c^2 \longrightarrow (1)$

Also $c^2 - a^2 = b^2 - ab$ | $\cancel{c^2 - a^2 = b^2 - a^2 - b^2 + c^2}$

$\Rightarrow c^2 - a^2 = b(b-a)$ | $\cancel{c^2 - a^2 = c^2 - a^2}$

Now $\dfrac{b}{c^2 - a^2} + \dfrac{a}{c^2 - b^2} = \dfrac{b}{b(b-a)} + \dfrac{a}{a(a-b)}$

$= \dfrac{1}{b-a} + \dfrac{1}{-(b-a)} = 0$

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24 a) $(a-b)^2\cos^2\frac{C}{2} + (a+b)^2\sin^2\frac{C}{2}$

$= (a^2 - 2ab + b^2)\cos^2\frac{C}{2} + (a^2 + 2ab + b^2)\sin^2\frac{C}{2}$

$= (a^2+b^2)(\cos^2\frac{C}{2} + \sin^2\frac{C}{2}) - 2ab(\cos^2\frac{C}{2} - \sin^2\frac{C}{2})$

$= a^2 + b^2 - 2ab\cos C = c^2$

b) $\dfrac{b^2 - c^2}{a^2}\sin^2 A + \dfrac{c^2 - a^2}{b^2}\sin^2 B$

$= \dfrac{b^2 - c^2}{a^2} \times \dfrac{a^2}{4R^2} + \dfrac{c^2 - a^2}{b^2} \times \dfrac{b^2}{4R^2}$

$= \dfrac{b^2 - c^2 + c^2 - a^2}{4R^2} = \dfrac{b^2 - a^2}{4R^2}$

c) Let $a = b = c = 1$ & $A = B = C = 60^\circ$

$\dfrac{bc\cos A + ca\cos B + ab\cos C}{\cot A + \cot B + \cot C}$

$= \dfrac{1\cdot1\cdot\frac{1}{2} + 1\cdot1\cdot\frac{1}{2} + 1\cdot1\cdot\frac{1}{2}}{\frac{1}{\sqrt3} + \frac{1}{\sqrt3} + \frac{1}{\sqrt3}} = \dfrac{\left(\frac{3}{2}\right)}{\left(\frac{3}{\sqrt3}\right)} = \cancel{3}\,\dfrac{\sqrt3}{2}$

$= 2 \times \dfrac{\sqrt3}{4}$

$= 2\Delta$. $\cancel{A}$

d) Let $a=b=c=1$ & $A=B=C=60^\circ$

$$\frac{\cos C+\cos A}{c+a}+\frac{\cos B}{b}$$

$$=\frac{\frac{1}{2}+\frac{1}{2}}{1+1}+\frac{\frac{1}{2}}{1} \Rightarrow \frac{1}{2}+\frac{1}{2}=1 \Rightarrow \frac{1}{b}$$

Ans : p, s, r, q

LEARNERS TASK

01. $a=3 \Rightarrow a=2R\sin A$

$\Rightarrow 1=2R\sin 60^\circ$

$\Rightarrow R=\frac{1}{\sqrt{3}}$

Ans : A

02 $a=2R\sin A$

$\Rightarrow 10=2\times 5\times \sin A \Rightarrow \sin A=1 \Rightarrow A=90^\circ$

Ans : D

03 $A=30^\circ,\ C=90^\circ \Rightarrow B=60^\circ$

$c=2R\sin C \Rightarrow 7\sqrt{3}=2R\sin 90^\circ$

$\Rightarrow R=\frac{7\sqrt{3}}{2}$

$a=2R\sin A$

$\Rightarrow a=2\times\frac{7\sqrt{3}}{2}\times\sin 30^\circ=\cancel{2}\times\frac{7\sqrt{3}}{\cancel{2}}\times\frac{1}{2}=\frac{7\sqrt{3}}{2}$

Ans : C

04 $\cos C=\frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \frac{4}{5}=\frac{20^2+21^2-c^2}{2\times 20\times 21}$

$\Rightarrow c=13$

Ans : B

05 $\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{3^2+4^2-2^2}{2\times 3\times 4} = \frac{7}{8}$ (16)

Ans: A

06 $\tan\frac{A}{2} = \frac{(s-b)(s-c)}{\Delta}$ OR) $\Delta^2 = s(s-a)(s-b)(s-c)$

$= \frac{\Delta}{s(s-a)}$

$= \frac{30}{15(15-5)} = \frac{1}{5}$

Ans: D

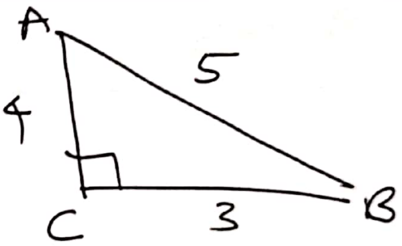
07 $\cot\frac{B}{2} = \frac{s(s-b)}{\Delta}$

$= \frac{66(66-60)}{330} = \frac{6}{5}$

Ans: B

08 $C = 90^\circ \Rightarrow \cos\frac{C}{2} = \cos 45^\circ$

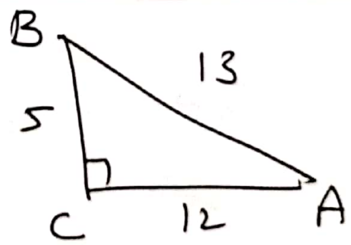
$= \frac{1}{\sqrt{2}}$



Ans: D

09 $C = 90^\circ \Rightarrow \sin\frac{C}{2} = \sin 45^\circ$

$= \frac{1}{\sqrt{2}}$



Ans. A

10 let $a=b=c=1$, $A=B=C=60^\circ$

$abc \sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2} = 1.1.1.\sin 30^\circ.\sin 30^\circ.\sin 30^\circ = \frac{1}{8}$

Now $\frac{\Delta^2}{s} = \frac{\left(\frac{\sqrt{3}}{4}\right)^2}{\left(\frac{3}{2}\right)} = \frac{3}{16}\times\frac{2}{3} = \frac{1}{8}$

$\Delta = \frac{\sqrt{3}}{4}$

$s = 3/2$

Ans : B

JEEMAIN LEVEL

01. Let $A=B=C=60^\circ$ and $a=b=c=1$

$$\sum \frac{\sin(B-C)}{bc} = \sum \frac{\sin(60^\circ-60^\circ)}{1\cdot 1} = \sum 0 = 3\times 0 = 0$$

Ans: C

02. Let $A=B=C=60^\circ$ and $a=b=c=1$

i.e. Consider an equilateral triangle.

$b^2\sin 2C + c^2\sin 2B$

$= 1^2 \cdot \sin 120^\circ + 1^2 \cdot \sin 120^\circ \Rightarrow \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}$

opt: $B = 2\Delta = 2\times\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$C = 3\Delta = 3\times\frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$

opt. $A = 4\Delta = 4\times\frac{\sqrt{3}}{4} = \sqrt{3}$

Ans. A

03. $a\cos A = b\cos B$

$\Rightarrow 2R\sin A\cos A = 2R\sin B\cos B$

$\Rightarrow \sin 2A = \sin 2B$

$\Rightarrow A = B$ (or) $\sin 2A = \sin(180^\circ - 2B)$

$\Rightarrow 2A = 180^\circ - 2B$

$\Rightarrow A + B = 90^\circ \Rightarrow C = 90^\circ$

Ans: C

∴ Right angled isosceles

04. $\frac{b}{c+a} + \frac{c}{a+b} - 1 = 0$

$\Rightarrow b^2 + c^2 - a^2 - bc = 0$

$\Rightarrow \frac{b^2+c^2-a^2}{bc} = 0$

$\cos A = 0$

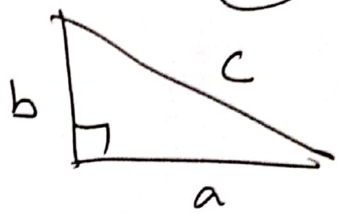
$A = 90^\circ$

Ans: D

05 let $a = \sqrt{2}$, $b = \sqrt{6}$, $c = \sqrt{8}$

$a^2 = 2$, $b^2 = 6$, $c^2 = 8$

$a^2 + b^2 = c^2 \Rightarrow \angle C = 90^\circ$



By trial and Error method.

Opt: C, $30^\circ, 60^\circ, 90^\circ$ is Correct Answer

since $45^\circ, 45^\circ, 90^\circ$ Can not be the answer

Since $a \neq b$

Ans: C

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06 $aqr + brp + cpq$

$= (2R)^2 [a\cos B\cos C + b\cos A\cos C + c\cos A\cos B]$

$= 4R^2 [\cos C (a\cos B + b\cos C) + c\cos A\cos B]$

$= 4R^2 (c\cos C + c\cos A\cos B]$

$= 4R^2 \cdot c\,[-\cos(A+B) + \cos A\cos B]$

$= 4R^2 \cdot c\,[\sin A \cdot \sin B]$

$= 4R^2 \cdot 2R\sin C \cdot \sin A \cdot \sin B$

$= (2R\sin A)(2R\sin B)(2R\sin C) = abc$

Ans: B

---

07 $AD = 4$, $BD = DC$

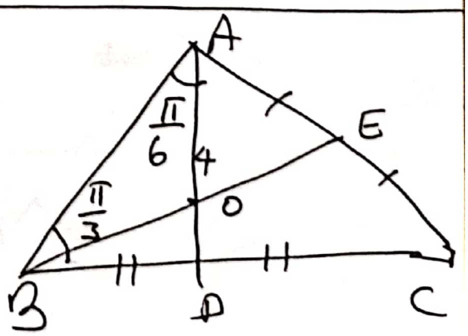
Since centroid O divides AD in the ratio 2 : 1

$AO = \frac{8}{3}$, $OD = \frac{4}{3}$

$\Rightarrow BO = AO \cot\frac{\pi}{3}$

$\Rightarrow BO = \frac{8}{3\sqrt{3}}$,

Ar $\triangle ADB = \frac{1}{2} \times 4 \times \frac{8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}}$



Ar $\triangle ABC = 2 \times$ Ar $\triangle ADB$

$= \frac{32}{3\sqrt{3}}$

Ans C

08 $(a+b+c)(b+c-a) = 3bc$

$\Rightarrow (2s)(2s-2a) = 3bc$

$\Rightarrow \dfrac{s(s-a)}{bc} = \dfrac{3}{4}$

$\Rightarrow \cos^2\dfrac{A}{2} = \dfrac{3}{4} \Rightarrow \cos\dfrac{A}{2} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \dfrac{A}{2} = 30^\circ \Rightarrow A = 60^\circ$

Ans : A

09 $1 - \tan\dfrac{A}{2} \cdot \tan\dfrac{B}{2}$

$= 1 - \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)} \times \dfrac{(s-a)(s-c)}{s(s-b)}}$

$= 1 - \dfrac{s-c}{s} \Rightarrow \dfrac{c}{s} = \dfrac{2c}{a+b+c}$

Ans : C

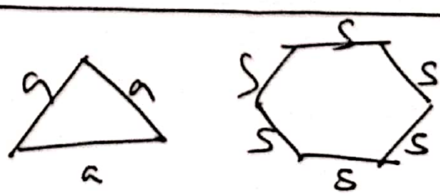
10 a, b, c AP $\Rightarrow 2b = a+c$

$\dfrac{2\sin\frac{A}{2}\sin\frac{C}{2}}{\sin\frac{B}{2}} = \dfrac{2\sqrt{\dfrac{(s-b)(s-c)}{bc}} \cdot \sqrt{\dfrac{(s-a)(s-b)}{ab}}}{\sqrt{\dfrac{(s-a)(s-c)}{ac}}}$

$= 2\left(\dfrac{s-b}{b}\right)$

$= \dfrac{2s-2b}{b} = \dfrac{\cancel{a}+b+\cancel{c}-\cancel{a}-\cancel{c}}{b} = \dfrac{b}{b} = 1$

11. $3a = 6s \Rightarrow a = 2s$



3

$\dfrac{\sqrt{3}}{4}a^2 : 6 \times \dfrac{\sqrt{3}}{4}s^2$

$\Rightarrow 4s^2 : 6s^2$

$\Rightarrow 2:3$

Ans : C