2. GRAPHS OF POLYNOMIALS (1)

Class: IX, Mathematics

I.I.T., FOUNDATION

SOLUTIONS

TEACHING TASK
JEEMAINS LEVEL

01. $x^2+2xy+y^2-z^2$

$= (x+y)^2 - z^2$

$= (x+y+z)(x+y-z)$ Ans: B

02 $P(x) = px^2+5x+r$

$P(2) = p(2)^2 + 5(2) + r = 0$

$= 4p + 10 + r = 0 \rightarrow (1)$

$P\left(\frac{1}{2}\right) = \frac{p}{4} + \frac{5}{2} + r = 0$

$\Rightarrow p + 10 + 4r = 0 \rightarrow (2)$

solving (1) and (2), $p = -2$, $q = -2$ Ans: A

opt: A. $p^2 - q^2 = 0$

03 $a^{12}x^4 - a^4x^{12} = (a^6x^2)^2 - (a^2x^6)^2$

$= (a^6x^2 + a^2x^6)(a^6x^2 - a^2x^6)$

$= a^2x^2(a^4+x^4)(a^3x + ax^3)(a^3x - ax^3)$

$= a^2x^2(a^4+x^4)(ax)(a^2+x^2)(ax(a^2-x^2)$

$= a^4x^4(a^2+x^2)(a^4+x^4)(a+x)(a-x)$ Ans: A

04. We have $a+b+c=0 \Rightarrow a^3+b^3+c^3=3abc$

$(x-a)^3+(x-b)^3+(x-c)^3$

here $x-a+x-b+x-c$

$= 3x-(a+b+c)$

$= 3x-3x = 0.$

Hence $(x-a)^3+(x-b)^3+(x-c)^3 = 3(x-a)(x-b)(x-c)$

Ans: A

05

$$\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$$

$$= \frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)}$$

$$= \frac{(a-b)(a+b)(b+c)(b-c)(c+a)(c-a)}{(a-b)(b-c)(c-a)}$$

$= (a+b)(b+c)(c+a).$

Ans: B

06. $xy+yz+zx+y^2$ | $x^2+xy+yz+zx$

$\Rightarrow (x+y)(y+z)$ | $= (x+y)(x+z)$

L.C.M $= (x+y)(y+z)(z+x)$

Ans: B

07. $f(x) = x^2+px+q \Rightarrow p(2) = 4+2p+q = 0$

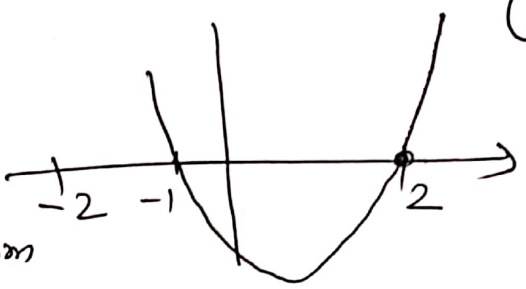
$p(1) = 4 \Rightarrow 1+p+q = 4 \Rightarrow p+q = 3$

Ans: B

08 $p(x) = x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = -1$ or 2



At $x = -2$, $p(x)$ has maximum value.

~~$\therefore p(2) = 2^2 - 2 \cdot 2 = 0$~~

$p(-2) = (-2)^2 - (-2) - 2 = 4 + 2 - 2 = 4$ Ans: ~~A~~

09. $p(x) = x^3 - 6x^2 + 11x - 6$.

Option Verification technique.

opt: A: $x = 1, 2, 3$

clearly $p(1) = 1 - 6 + 11 - 6 = 0$

$p(2) = 8 - 24 + 22 - 6 = 0$

$p(3) = 27 - 54 + 33 - 6 = 0$.

~~Ans: A~~

10. $x^2 - 1 = (x+1)(x-1)$ | $x^2 - x = x(x-1)$

L.C.M $= x(x+1)(x-1) =$ Ans: A

JEE ADVANCED LEVEL

01. ~~$p(x) = px^2 + 5x + r$~~

~~$p(-2) = 4p + 10 + r$~~

JEE ADVANCED LEVEL

01. $p(x) = px^2 + 5x + r$

If $p = r$, then $p(x)$ has reciprocal roots (or zeroes)

$\therefore (x+2)\left(x+\frac{1}{2}\right) = \frac{2}{=}x^2 + 5x + \frac{2}{=}$.

hence $(x+2)$ & $\left(x+\frac{1}{2}\right)$ are the factors

Ans: A, C

---

02. $p(x) = ax^4 + bx^3 + cx^2 + dx + e$

$\therefore x^2 - 1 = (x+1)(x-1)$ are the factors

$\therefore p(1) = a + b + c + d + e = 0$

$p(-1) = a - b + c - d + e = 0$

$\Rightarrow a + c + e = b + d$

Ans : A, C

---

03. $f(x) = x^3 - 2x^2 + x - 2$

A) $f(1) = 1 - 2 + 1 - 2 = -2$ (False)

B) $f(1) = 1 - 2 + 1 - 2 = -2$ (False)

C) $f(-2) = -8 - 8 - 2 - 2 \neq 0$ (True)

D) $f(2) = 8 - 8 + 2 - 2 = 0$ (True)

Ans: C, D

4. Statement I:

$(3x-1)^7 = a_7 x^7 + a_6 x^6 + a_5 x^5 + \dots + a_1 x + a_0$

put $x = 1$

$2^7 = a_7 + a_6 + a_5 \dots + a_1 + a_0$.

$\therefore$ Sum of the coefficients $= 128$ (True)

Statement II: Conceptual (True) Ans: A

5. Statement I:

$f(x) = x^3 - 6x^2 + 11x - 6$

$f(1) = 1 - 6 + 11 - 6 = 0$

$f(2) = 8 - 24 + 22 - 6 = 0$

$f(3) = 27 - 54 + 33 - 6 = 0$. (True)

Statement II: $(x-1)(x-2)(x-3)$

$= x^3 - 6x^2 + 11x - 6$ (True) Ans: A

6. Assertion: $f(x) = x^3 - 3x + 2$

$f(1) = 1 - 3 + 2 = 0$.

$\therefore 1$ is a real zero of $f(x)$. (True)

Reason: Conceptual (True) Ans: A

07. Assertion: $x-3$ is a factor of $f(x)$
$\Rightarrow f(3)=0$. (False)

Reason: Conceptual (True)

Ans: D

08 $=\left(x-\frac{1}{2}\right)\left(x-\sqrt{2}\right)$

$= x^2-\left(\frac{1}{2}+\sqrt{2}\right)x+\frac{1}{\sqrt{2}}$

Ans: C

09 $\left(x-\frac{m}{n}\right)\left(x+\frac{m}{n}\right)=0$

$= x^2-\frac{m^2}{n^2}=0$

$= n^2x^2-m^2=0$

Ans: C

10. product of the Zeroes $= \sqrt{2}\times\sqrt{3}=\sqrt{6}$ Ans: A

11 Synthetic Division Row
1 −6 11 −6

Ans: B

12. $f(x)=x^3-6x^2+11x-6$
$f(1)=1-6+11-6=0$.

Ans: A

13. $\sum x^2(y^2-z^2)=x^2(y^2-z^2)+y^2(z^2-x^2)+z^2(y^2-z^2)=0$.

Ans: D

14. $f(x) = x^2 - 4 = (x+2)(x-2)$ (7)

$g(x) = x^2 - x - 6 = (x-3)(x+2)$

H.C.F $= (x+2) \rightarrow$ Degree $= 1$ Ans: 1

15. a). $f(x) = x^3 - px^2 + 6x - p$

$f(p) = p^3 - p^3 + 6p - p = 5p$

b) $f(x) = x^3 - px^2 + 5x - p$

$f(-p) = -p^3 - p^3 - 5p - p = -2p^3 - 6p$

$= -2p(p^2 + 3)$

c) $f(x) = x^3 - px^2 + p^2x - 1$

$f(1) = 1 - p + p^2 - 1 = p^2 - p$

d) $f(x) = p^3x^3 - 3px^2 + 3px + 2$

$f(-1) = -p^3 - 3p - 3p + 2$

$= -p^3 - 6p + 2$

Ans: r, t, q, s

16 a) $x^2 - 4 = (x+2)(x-2)$

$x^2 - x - 6 = (x-3)(x+2)$

$\therefore$ H.C.F $= x + 2$

b) $x^2 - 1 = (x+1)(x-1)$; $x^2 - x = x(x-1)$

L.C.M $= x(x+1)(x-1)$

c) $x = 1$

	1	−6	11	−6
	0	1	−5	6
	1	−5	6	0

$x^2 - 5x + 6$; Remainder $= 0$

D) Quotient $= x^2 - 5x + 6$

Ans: −, −, p, q

LEARNERS TASK

CUQ's

01. $= x^{-2}(x^5 + 3x^4 + x^{11})$

$= x^3 + 3x^2 + x^9 \rightarrow$ Degree $= 9$

Ans: C

02 $x^3 + 2x^4y^2 - x^5y^3z + 13 \rightarrow$ Degree $= 5+3+1$

$= 9$

Ans: D

03 $A = 1, \quad B = -1 \Rightarrow \frac{A}{B} = -1$ Ans: B

04. $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$ Ans: A

05 Conceptual $(a \neq 0, a>0, a<0)$ Ans: B, C, D

06 $3x^2 + 2xy + 7.$

when $x=0, y=7 \Rightarrow 0 + 0 + 7.$

i.e $3x^2$ becomes 0, $2xy$ becomes 0

Ans: D

07. $x(x-1)(x+2)^2$

$= (x^2 - x)(x^2 + 4x + 4)$

Degree $= 4$

Ans: B

08 Conceptual Ans: B

(9)

09. $f(x) = ax + b$

$f(0) = a(0) + b = 0 \Rightarrow b = 0$ Ans: B

10. $x^2 + \sqrt{x} \rightarrow$ is not a polynomial Ans: A

JEE MAINS LEVEL

01. $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (9)^2 = a^2+b^2+c^2+2(26)$

$\Rightarrow a^2+b^2+c^2 = 81-52 = 29$ Ans: C

02. $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4.$

Let $5x - \frac{1}{x} = a$

$= a^2 + 4a + 4$

$= (a+2)^2 = \left(5x - \frac{1}{x} + 2\right)^2 = \left(\frac{5x^2+2x-1}{x}\right)^2$

Ans: A

03. $1 + 2b - (a^2 - b^2)$

$= 1 + 2b - a^2 + b^2$

$= (b^2 + 2b + 1) - a^2$

$= (b+1)^2 - a^2$

$= (b+1+a)(b+1-a)$

Ans: A

04 $a^3 - b^3 + 1 + 3ab$ (10)

$= a^3 + (-b)^3 + 1^3 - 3(a)(-b)(1)$

$= (a - b + 1)(a^2 + b^2 + 1 + ab + b - a)$

One of the factors $= a - b + 1$ Ans: B

05 $(x+y)(1-z) - (y+z)(1-x)$

$= x - xz + y - yz - y + xy - z + xz$

$= x(1+y) - y($

$= (x-z)(1+y)$

Ans: D

06. Conceptual. Ans: C

07. $f(x) = 2x^5 + kx + 16$

$f(-2) = -16 - 2k + 6 = 0$

$\Rightarrow k = -5$ Ans: —

08 $f(x) = x^2 - 4x + 3$

$f(0) = 3$; $f(3) = 9 - 12 + 3 = 0$

$\therefore$ Sum of the values $= 3 + 0 = 3$ Ans: C

09. $x^2 - 4 = (x+2)(x-2)$; $x^2 - x - 6 = (x-3)(x+2)$

H.C.F $= x + 2$ Ans: B

10 $x = 2$

2	−3	1	5	
0	4	2	6	
2	1	3	11	

Quotient $= 2x^2 + x + 3$

Ans: A

JEE ADVANCED LEVEL

01. $f(x) = 2x^3 - 5x^2 + ax + b$

$f(2) = 16 - 20 + 2a + b = 0$

$\Rightarrow 2a + b = 4$

$f(0) = b = 0$

$\therefore 2a + 0 = 4 \Rightarrow a = 2$

Ans: A, B

02. $f(x) = x^2 - 6x + 9$

$= (x-3)^2$

A) $f(3) = (3-3)^2 = 0$ (True)

B) $f(3) = 0$ (True)

C) $f(-3) \neq 0$ (False)

D) True

Ans: A, B, D

03. Statement I: $f(x) = 4x^2 - 1 = 0$

$\Rightarrow (2x+1)(2x-1) = 0$

$\Rightarrow x = \pm \frac{1}{2}$ (True)

Statement II: Conceptual (True)

Ans: A

04. Statement I: Conceptual (True)

Statement II: Conceptual (False)

Ans: C

05. Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans: A

06. Assertion: Conceptual (True)
Reason: Conceptual (True)
Ans: A

07. $f(x) = x^2 - 5x + 6 = 0$
$= (x-2)(x-3) = 0.$
$\therefore$ Now, $b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot 6$
$= 25 - 24$
$= 1 > 0$
Ans: C

08. $f(x) = x^2 + 4x + 8$
$\Delta = 4^2 - 4.1.8 = 16 - 32 = -16 < 0$
Never touches x-axis
Ans: A

09. $b^2 - 4ac = 0 \Rightarrow$ roots are equal $\Rightarrow \alpha = \beta$
Ans: C

10. $f(x) = x^3 - 2x^2 + x - 1$
$f(1) = 1 - 2 + 1 - 1 = -1$
Ans: C

11. $f(1) \neq 0$
$\Rightarrow x - 1$ is not a factor
Ans: B

12. $f(x) = ax^2 + bx + c \rightarrow$ passes through $(0, 0)$
$\Rightarrow 0 = a(0)^2 + b(0) + c$
$\Rightarrow c = 0$
Ans: D

13. clearly $a = 1$

$f(x) = x^3$

$f(b) = b^3 = 0 \Rightarrow b = 0$

$\therefore a + b = 1 + 0 = 1$

Ans: 1

14. $f(x) = x^2 - 1 = (x+1)(x-1)$

$g(x) = x^2 - x = x(x-1)$

$LCM = (x-1)(x+1)x \longrightarrow$ Degree $= 3$

Ans: 3

15 a) $x^2y^2 + 2xyz^2 - x^2z^2 - y^2z^2$

$= (y^2 - z^2)x^2 + 2xyz^2 - xy^2z^2$

b) $x^3 + y^3 - z^3 + 3xyz$

$= x^3 + y^3 + (-z)^3 - 3xy(-z)$

$= (x + y + (-z))(x^2 + y^2 + (-z)^2 - xy - y(-z) - (-z)x)$

$= (x + y - z)(x^2 + y^2 + z^2 - xy + yz + zx)$

c) $(y - z)x^3 + (z - x)y^3 + (x - y)z^3$

$= (y - z)x^3 - y^3x + z^3x + zy^3 - yz^3$

d) $x^2(y - z) + y^2(x - z) + z^2(x - y) + xyz$

$= x^2(y - z) + (y^2 + z^2 + yz)x - y^2z - z^2y$

Ans: a, -, s, q

16 a) $f(x) = x^2 - 4x + 4$

$f(2) = 2^2 - 8 + 4 = 0 \Rightarrow 2$ is zero of $f(x)$

b) $f(x) = x^3 - 1$

$f(1) = 1 - 1 = 0 \Rightarrow$ Remainder $= 0$

c) $f(x) = x^2 - 1$

$f(1) = 1 - 1 = 0 \Rightarrow$ Remainder $= 0$

d) $f(x) = x^2 + 3$

$f(-1) = 1 + 3 = 4. \Rightarrow$ Remainder $\neq 0$

Ans : q, p, s, r

ADDITIONAL QUESTIONS FOR PRACTICE

01. $x^{12} - y^{12} = (x^6 + y^6)(x^6 - y^6)$

$= \left[(x^2)^3 + (y^2)^3\right]\left[(x^2)^3 - (y^2)^3\right]$

$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)(x^2 - y^2)(x^4 + x^2y^2 + y^4)$

$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)(x + y)(x - y)$

$(x^2 + y^2 - xy)(x^2 + y^2 + xy)$

Total No. of factors $= 6$

Ans : C

---

02 G.E $= p^3(q-r)^3 + q^3(r-p)^3 + r^3(p-q)^3$

G.E $= [p(q-r)]^3 + [q(r-p)]^3 + [r(p-q)]^3$

We have $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

$\therefore$ G.E $= 3p(q-r)(q)(r-p)r(p-q)$

$= 3pqr(p-q)(q-r)(r-p)$

$\therefore$ One of the factor $= pqr$

Ans: D

03 $f(x) = x^4 - 2x^3 + 3x^2 - ax - b$

$f(1) = 1 - 2 + 3 - a - b = 6$

$\therefore a+b = -4$

Ans: C

04. put $x = 1$

We get sum of the coefficients $= 128$

Ans: C

05 $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$

$(a+b+c)(a+c-b) = (a+c)^2 - b^2$

$= a^2 + c^2 + 2ac - b^2$

$= a^2 + c^2 + 2ac - ac$

$= a^2 + c^2 + ac$

Ans: B

06. $f(x) = x^4 + x^3$

$f(-9) = (-9)^4 + (-9)^3 =$ constant $\rightarrow$ Degree $= 0$

Ans: B

07. $f(x) = x^3 + 3x^2 + 3x + 1 = (x+$

$f\left(\frac{22}{7}\right) = \left(\frac{22}{7}\right)^3 + 3\left(\frac{22}{7}\right)^2 + 3\left(\frac{22}{7}\right) + 1$

$= \left(\frac{22}{7}\right)\left[\left(\frac{22}{7}\right)^2 + 3\left(\frac{22}{7}\right) + 3\right] + 1$

$= \frac{22}{7}$

---

07 $f(x) = x^3 + 3x^2 + 3x + 1$

$= (x+1)^3$

$\therefore f\left(\frac{22}{7}\right) = \left(\frac{22}{7} + 1\right)^3 = \left(\frac{29}{7}\right)^3$ Ans: C

---

08 Divide $3x^3 + x^2 - 2x + 9$ with $3x^2 + 7x - 6$.

$18x - 3$ should be added Ans: A

---

09. $a + c + e = 0$, $b + d = 0$, $a + b + c + d + e = 0$

Ans: D

---

10. $c^2 x + d^2 = 0$

$\Rightarrow x = -\frac{d^2}{c^2} = -\left[-\left(\frac{d}{c}\right)\right]^2$ Ans: B

$\Rightarrow$ THE END $\Leftarrow$