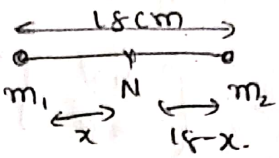
WS-10 · Gravitation   8th foundation

Task

①

(1)

let $m_1 = 8\,kg$: $m_2 = 2\,kg$



let N - be the point where field is zero.

(ie) Gravitational field strength due to m_1 and m_2 are equal and opposite. $[r_1 = x ; r_2 = 18 - x]$

$$\therefore \frac{m_1}{r_1^2} = \frac{m_2}{r_2^2}$$

$$\Rightarrow \frac{\cancel{8}^{4}}{x^2} = \frac{\cancel{2}}{(18-x)^2} \Rightarrow 4 = \left(\frac{x}{18-x}\right)^2$$

$$\Rightarrow \frac{x}{18-x} = \sqrt{4} = 2$$

$$\Rightarrow x = 2(18-x)$$

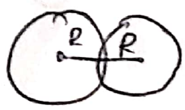
$$\Rightarrow x = 36 - 2x$$

$$\Rightarrow 3x = 36$$

$$\Rightarrow x = 12\,cm = 0.12\,m \text{ from } m_1 = 8\,kg$$

(2)

let 'R' be the radius of each sphere.. when two spheres are kept in contact distance between ~~two~~ the centres of two sphere's is $r_1 = 2R$



$F_1 = F$.

If Radius is doubled $r_2 = 4R$. then the force is

F_2

2nd continuation

According to newton's law of gravitation

$$F \propto \frac{1}{r^2}$$

$$\Rightarrow \frac{F_1}{F_2} = \left[\frac{r_2}{r_1}\right]^2$$

$$\Rightarrow \frac{F}{F_2} = \left(\frac{\cancel{4}^{2} \cancel{R}}{\cancel{2} \cancel{R}}\right)^2 = 2^2 = 4$$

$$\Rightarrow F_2 = \frac{F}{4}. \rightarrow A$$

(3) Given weight of the body on surface of Earth is

$w = mg = 64$ N.

when the body is at a height $h = \frac{R}{3}$.

Acceleration due to gravity $g' = \frac{gR^2}{(R+h)^2}$

$$g' = \frac{gR^2}{(R+\frac{R}{3})^2} = \frac{9gR^2}{16R^2}$$

$$g' = \frac{9}{16} g$$

$\therefore$ weight of the body $w' = mg' = mg \times \frac{9}{16}$

$$= \cancel{64}^{4} \times \frac{9}{\cancel{16}}$$

$$= 36 \text{ N} \rightarrow D$$

(4) Given $T_{planet} = 8 T_{earth}$

Distance of planet from sun is $R_p = r$, then $R_e = ?$

According to kepler's law of planetary motion

$$T^2 \propto r^3$$

$$\left[\frac{T_p}{T_e}\right] = \left[\frac{r_p}{r_e}\right]^{3/2}$$

$$\Rightarrow \frac{8\,\cancel{T_e}}{\cancel{T_e}} = \left[\frac{r}{r_e}\right]^{3/2}$$

$$\Rightarrow 2^{3/2} = \left(\frac{r}{r_e}\right)^{3/2}$$

$$\Rightarrow 2^2 = \frac{r}{r_e} \Rightarrow r_e = \frac{r}{4} \rightarrow C.$$

(5)

let 'd' be the depth and $h = 64\,km$ is height

Given $g_{depth} = g_{height}$

$$\Rightarrow \cancel{g}\left[1-\frac{d}{R}\right] = \cancel{g}\left[1-\frac{2h}{R}\right]$$

$$\Rightarrow \cancel{1}-\frac{d}{R} = \cancel{1}-\frac{2h}{R}$$

$$\Rightarrow \frac{d}{\cancel{R}} = \frac{2h}{\cancel{R}} \Rightarrow d = 2h = d = 2\times 64 = 128\,km$$

(6)

Given

g at a height $= 64\%\, g$ [36% Reduced]

$$\Rightarrow \frac{\cancel{g}R^2}{(R+h)^2} = \frac{64}{100}\cancel{g}$$

$$\Rightarrow \frac{R}{R+h} = \sqrt{\frac{64}{100}} \Rightarrow \frac{R}{R+h} = \frac{8}{10} = \frac{4}{5}$$

$$\Rightarrow 5R = 4R + 4h \Rightarrow h = \frac{R}{4} = \frac{6400}{4} = 1600\,km$$

(7)

$H_{max} = 0.5m$

Given velocity of Projection is same on earth and moon, $g_{moon} = \frac{1}{6} g_{earth}$

Maximum height reached by vertically projected body $H_{max} = \frac{u^2}{2g} \Rightarrow H_{max} \propto \frac{1}{g}$

$$\Rightarrow \frac{H_e}{H_m} = \frac{g_m}{g_e} \Rightarrow \frac{0.5}{H_m} = \frac{\frac{1}{6} \cancel{g_e}}{\cancel{g_e}}$$

$$\Rightarrow \frac{0.5}{H_m} = \frac{1}{6} \Rightarrow H_m = 3m \rightarrow D$$

(8)

depth to which the body taken $d = 16km$

$$\% \text{ change in } g = \frac{\Delta g}{g} \times 100 = \frac{d}{R} \times 100$$

$$\Rightarrow \frac{\cancel{16}}{\cancel{64}00} \times 100 = \frac{100}{400} = \frac{1}{4} = 0.25\% \rightarrow B$$

(9)

Given

$M_e : M_m = 3 : 2$. $R_e : R_m = \frac{6}{9} : 1$

we know weight $w = \underset{\text{mass of body}}{m} g$: $\left[g = \frac{G \overset{\text{mass of planet}}{M}}{R^2} \right]$

$$\Rightarrow W = \frac{GM}{R^2} \Rightarrow \frac{W_e}{W_m} = \frac{M_e}{M_m} \times \left[\frac{R_m}{R_e}\right]^2$$

$$\Rightarrow \frac{W_e}{W_m} = \frac{3}{2} \times \left[\frac{1}{6}\right]^2 = \frac{\cancel{3}}{2} \times \frac{1}{\cancel{36}_{12}} = \frac{1}{24} \rightarrow B$$

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10

If Earth is shrinking to half of its radius means the body is now at a depth $d = \frac{R}{2}$ from centre of Earth.

$$w = mg = \frac{GMm}{R^2} \quad \left[g = \frac{GM}{R} \right]$$

At depth d' $\quad w' = mg'$

$$= \cancel{mg'\left(1 - \frac{d}{R}\right)} = \cancel{mg\left(1 - \frac{R}{2R}\right)} = \cancel{2mg}$$

$$w = \frac{GMm}{d'^2} \qquad g = \frac{Gm}{R^2}$$

$$w = \frac{GMm}{\left(\frac{R}{2}\right)^2} = 4\frac{GMm}{R^2} = 4w$$

weight of the body changes by 4 times

11

comet does not have elliptical orbit. Hence does not obey kepler's law of Planetary action. they move along the paths which are hyperbolic or almost Parabola

12

According to kepler's 2nd law. Angular momentum is conserved. This suggests that for the planet, radial and tangential acceleration is zero.

(14)

We know that According to kepler's 3rd law

$$T^2 \propto r^3.$$

As the distance of the planet from the sun increases, the length of the semi-major axis of it's orbit will also increase and therefore the period of revolution will also increase

(15)

The Gravitational acceleration on the earth's surface is $g = \frac{GM}{R_e^2}$; we know density $= \frac{\text{Mass}}{\text{volume}}$

$$\Rightarrow \rho = \frac{M}{V}$$

$$\Rightarrow M = \rho V = \rho \frac{4\pi}{3} R_e^3$$

$$\therefore g = \frac{G\rho \frac{4\pi}{3} R_e^{\cancel{3}}}{\cancel{R_e^2}} = \frac{4\pi}{3} G\rho R_e$$

$$\therefore G = \frac{3g}{4\pi\rho R_e} \quad \text{(or)} \quad G = \frac{g R_e^2}{M}$$

(16)

we know $g = \frac{GM}{r^2}$. $r \to$ distance of the body from centre of Earth

when $r > R$, -moving away from centre. $g \to$ decreases because r increases.

At the centre of Earth $r = 0$ and hence $g = \frac{GMr}{R^3}$

$$\therefore g = 0.$$

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16^{th} continuation

If Earth stops rotating, the gravitational force will increase since there will be no centrifugal force.

(18)

(i) If Earth shrinks to half of it's Present value then Radius $R' = \frac{R}{2}$.

we know that $g = \frac{GM}{R^2}$ [M = constant]

$\Rightarrow g \propto \frac{1}{R^2} \Rightarrow \frac{g'}{g} = \left[\frac{R}{R'}\right]^2$

$\Rightarrow \frac{g'}{g} = \left[\frac{R'}{\frac{R}{2}}\right]^2 = 4$

$\Rightarrow g' = 4g$

(ii) If Earth shrinks to $\frac{1}{3}$rd of it's Present value

then $R' = \frac{R}{3}$.

We know $g \propto \frac{1}{R^2}$ [M = constant]

$\Rightarrow \frac{g'}{g} = \left[\frac{R}{R'}\right]^2$

$\Rightarrow \frac{g'}{g} = \left[\frac{R'}{\frac{R}{3}}\right]^2 = 3^2 = 9$

$\Rightarrow g' = 9g$

(19)

Given $r_1 = 10^{12}\,m$; $r_2 = 10^{10}\,m$

According to kepler's law of planetary motion

$$T^2 \propto r^3$$

$$\Rightarrow \left[\frac{T_1}{T_2}\right]^2 = \left[\frac{r_1}{r_2}\right]^3$$

$$\Rightarrow \frac{T_1}{T_2} = \left[\frac{10^{12}}{10^{10}}\right]^{3/2} = \left[10^{\cancel{2}}\right]^{3/\cancel{2}} = 10^3 = 1000.$$

(20)

Given

$g_m = \frac{1}{5} g_e$; $R_m = \frac{1}{4} R_e$. then $\frac{M_e}{M_m} = ?$

From $g = \frac{GM}{R^2}$

$$\Rightarrow \frac{g_m}{g_e} = \frac{M_m}{M_e} \times \left[\frac{R_e}{R_m}\right]^2$$

$$\Rightarrow \frac{1}{5} = \frac{M_m}{M_e} \left[4\right]^2$$

$$\Rightarrow \frac{1}{5} = \frac{M_m}{M_e} 16$$

$$\Rightarrow \frac{M_e}{M_m} = 16 \times 5 = 80.$$

LTask
CUQ's

1

when body is taken from equator to poles
r ~~increases~~ decreases since $g \propto \frac{1}{r^2}$

As r increases g decreases. vice versa

So $W = mg$ = ~~decreases~~ increases.

2

we know that $T^2 \propto r^3$

If the distance between earth and sun is
$R' = \frac{1}{4}$ (present distance R)

$$\therefore \frac{T'}{T} = \left(\frac{R'}{R}\right)^{3/2} \Rightarrow \frac{T'}{T} = \left(\frac{1}{4}\right)^{3/2}$$

$$\Rightarrow \frac{T'}{T} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$\Rightarrow \boxed{T' = \frac{T}{8}}$$

3

Since

$$g_{height} = g\left[1 - \frac{2h}{R}\right]; \quad g_{depth} = g\left[1 - \frac{d}{R}\right]$$

As height and depth are increasing
Acceleration due to gravity decreases

(4)

If Earth shrinks to half of it's radius

i.e. $R' = \frac{R}{2}$.

g becomes $g = \frac{GM}{R^2}$ $g \propto \frac{1}{R^2}$

$\Rightarrow \frac{g'}{g} = \left[\frac{R}{R'}\right]^2 = \left[\frac{R}{\frac{R}{2}}\right]^2 = 2^2 = 4$

$\Rightarrow g' = 4g$

$\therefore$ weight $w = mg' = 4mg$ = increases 4 times

(5)

If the earth stops rotating, the weight of a body at the equator will increase. However, there is no influence on the weight at the poles. The impact of the earth's rotating on gravity acceleration is to reduce its value. As a result, if the earth stops rotating, the value of 'g' will rise.

(10)

we know that Areal velocity $A = \frac{L}{2M}$.

$\Rightarrow L = 2MA$

$= 7.5 \times 10^5$ pascal

$\Rightarrow \frac{\Delta g}{g} =$

Jee main level

(1)

According to given question $\frac{\Delta g}{g} = 4\%$

$\Rightarrow$ we know $\frac{\Delta g}{g} = \frac{2h}{R}$

$\therefore \frac{2h}{R} = 4\% \Rightarrow \frac{\cancel{2}h}{R} = \frac{\cancel{4}2}{100}$

$\Rightarrow h = \frac{2}{\cancel{2}00} \times R = \frac{1}{50} \times 6400$

(2)

weight of the body on surface of Earth $w = 360N$

g ~~at~~ at a depth 'd' : $g' = g\left(1 - \frac{d}{R}\right)$

Given $R = 800$ km

$$g' = g\left(1 - \frac{800}{\cancel{6400}_{8}}\right)$$

$$g' = g\left(1 - \frac{1}{8}\right) = \frac{7g}{8}$$

$$\text{weight} = mg' = m\frac{7g}{8} = \frac{7}{8} \times mg = \frac{7}{8} \times 360$$

$$= 315N$$

(3)

we know that $R_{max} = \frac{u^2}{2g} \Rightarrow R \propto \frac{1}{g}$

$$\frac{R}{R'_p} = \frac{g_p}{g_e} \Rightarrow \frac{R}{\frac{5R}{4}} = \frac{g_p}{g_e} \Rightarrow \frac{g_p}{g_e} = \frac{4}{5}$$

$$\Rightarrow \frac{g_e}{g_p} = \frac{5}{4} \Rightarrow \frac{g_p}{g_e} = \frac{4}{5}.$$

(4)

$M_e = 80 M_p \qquad D_e = 2D_p$

we know $\quad g = \frac{GM}{R^2}$

$\Rightarrow \frac{g_p}{g_e} = \frac{M_p}{M_e}\left[\frac{R_e}{R_p}\right]^2$

$\Rightarrow \frac{g_p}{9.8} = \frac{\cancel{M_p}}{80\cancel{M_p}}\left[\frac{2\cancel{D_p}}{\cancel{D_p}}\right]^2$

$\Rightarrow \frac{g_p}{9.8} = \frac{1}{\cancel{80}_{20}} \times \cancel{4} \Rightarrow g_p = \frac{9.8}{20} = 0.49 m/s^2$

(5)

Given $M_p = \frac{1}{9} M_e \; : \; R_p = \frac{1}{2} R_e \qquad w_e = mg_e = 9N.$

$\frac{w_e}{w_p} = \frac{g_e}{g_m} \qquad$ Since $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{M}{R^2}$

$\Rightarrow \frac{w_e}{w_p} = \frac{M_e}{M_p} \times \left[\frac{R_p}{R_e}\right]^2 \qquad \Rightarrow \frac{1}{w_p} = \frac{1}{2^2}$

$\Rightarrow \frac{\cancel{9}}{w_p} = \cancel{9} \times \left[\frac{1}{2}\right]^2 \qquad \Rightarrow w_p = 4 N.$

(6)

Given $\quad w_{height} = \frac{1}{4} w_{surface}.$

$\Rightarrow mg_h = \frac{1}{4} mg$

$\Rightarrow \cancel{mg}\left[1 + \frac{2h}{R}\right]^2 = \frac{1}{4}\cancel{mg}$

$\Rightarrow$ ~~$1 - \frac{1}{4} = \frac{2h}{R} \Rightarrow \frac{3}{4} = \frac{2h}{R}$~~

$\left(1 + \frac{h}{R}\right)^2 = 4 \qquad \Rightarrow 1 + \frac{h}{R} = 2$

$\Rightarrow \frac{h}{R} = 2 - 1 = 1$

$\Rightarrow h = R$

(7)

let $m_1 = xm$: $m_2 = (1-x)m$

we know that Gravitational force

$$F = \frac{G m_1 m_2}{r^2} = \frac{G\, xM(1-x)M}{r^2}$$

$$F = \frac{GM^2}{r^2} x(1-x)$$

For maximum value of force $\frac{dF}{dx} = 0$

$$\Rightarrow \frac{d}{dx}\left[\frac{GM^2}{r^2}(x-x^2)\right] = 0$$

$$\Rightarrow \frac{d}{dx}(x-x^2) = 0 \qquad \left[\text{use } \frac{d}{dx}x^n = nx^{n-1}\right]$$

$$\Rightarrow 1-2x = 0 \Rightarrow x = \frac{1}{2}$$

(8)

let $m_A = m$: $r_A = r$
$m_B = 2m$ $r_B = 4r$

we know that $T^2 \propto r^3$

$$\Rightarrow T \propto r^{3/2}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{r_A}{r_B}\right)^{3/2} = \left(\frac{r}{4r}\right)^{3/2}$$

$$= \left(\frac{1}{2^2}\right)^{3/2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

(9)

let $r_1 = 10^{13}$: $r_2 = 10^{12}$ we know that $T \propto (r)^{3/2}$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2} = 10\sqrt{10}$$

(11)

If $m_1' = 2m$; $m_2' = 2m$. $r' = 2r$

we know $F = \frac{G m_1 m_2}{r^2} = \frac{G m m}{r^2} = \frac{G m^2}{r^2}$

$\Rightarrow F' = \frac{G m_1' m_2'}{r'^2}$

$\Rightarrow F' = \frac{G (2m)(2m)}{(2r)^2} = \frac{4 G m^2}{4 r^2} = F$

(13)

As the depth increased the mass of the earth decreases. At the surface of earth this value will be maximum because radius will be maximum. when radius becomes less this value also decreases. Hence acceleration due to gravity decrease with increase in the depth.

(14)

Due to the equatorial bulge, i.e. the equator being more bulged towards the outside, the poles are near to the centre of the earth. as compared to the equitorial region. And as the acceleration due to gravity is inversely proportional to the distance from the centre of the earth, the distance is less on poles, g is more.

(15)

If G starts decreasing, then gravitational force and earth will start to decrease. therefore earth will try to follow a spiral path of decreasing radius. Hence, it's period of revolution around the sun will increase. ∴ Duration of the year will increase.

~~Since revolution of~~

But rotational motion of earth about it's own axis will remain unchanged, hence period of rotation will remain unchanged or length of the day will remain unchanged.

Since, the radius of circular path of earth will increase or the earth will follow spiral path.

∴ therefore it's P.E will increase but P.E is always -ve so the magnitude of K.E decreases.

(16)

$g_m = \frac{1}{6} g_e$.

we know $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$

$\Rightarrow \frac{g_m}{g_e} = \left(\frac{R_e}{R_m}\right)^2 \Rightarrow$

$\Rightarrow \frac{1}{6} = \left(\frac{R_e}{R_m}\right)^2 \Rightarrow \frac{R_e}{R_m} = \frac{1}{\sqrt{6}}$

If density same $g \propto R$. $\frac{R_e}{R_m} = \frac{g_e}{g_m} = \frac{6}{1} \Rightarrow \frac{g_m}{g_e} = \frac{1}{6}$

If $\frac{\rho_m}{\rho_e} = \frac{5}{3}$ $\quad \frac{R_e}{R_m} = \frac{\rho_e}{\rho_m} \times \frac{g_m}{g_e} = \frac{5}{3} \times \frac{1}{6} = \frac{5}{18}$

17

(i) $g_h = g$ Given $h = 2R$. $g_h = g_0$.

we know $g_h = \frac{g}{(1+\frac{h}{R})^2} = \frac{g}{(1+\frac{2\cancel{R}}{\cancel{R}})^2}$

$\Rightarrow g_0 = \frac{g}{(1+2)^2} = \frac{g}{9}$

$\Rightarrow g = 9g_0$

(ii)

$\frac{\Delta g}{g} = \frac{75}{100} = \frac{3}{4}$

$\Rightarrow \frac{d}{R} = \frac{3}{4} \Rightarrow d = \frac{3}{4}R$

18

Given $m_p = 2m$: $R_p = 2R$.

we know $W = mg$ and $g = \frac{GM}{R^2}$ $g \propto \frac{m}{R^2}$

$\frac{g_p}{g_e} = \frac{M_p}{m_e}\left(\frac{R_e}{R_p}\right)^2$

$= \frac{2\cancel{m}}{\cancel{m}}\left(\frac{\cancel{R}}{2\cancel{R}}\right)^2$

$\Rightarrow \frac{g_p}{g_e} = 2 \times \frac{1}{4} \Rightarrow g_p = \frac{g_e}{2}$

$W_p = mg_p = \frac{mg_e}{2} = \frac{W}{2}$

(19)

let masses are m_1 & m_2 distance $= r$.

$F = \frac{G m_1 m_2}{r^2}$ $\quad m_1' = 2m_1 \ ; \ m_2' = 2m_2$

$r' = \frac{r}{2}$

$F' = \frac{G m_1' m_2'}{r'^2} = \frac{G (2m_1)(2m_2)}{(\frac{r}{2})^2}$

$\Rightarrow F' = \frac{16 \, G m_1 m_2}{r^2} = 16F$

(20)

use $F = \frac{G m_1 m_2}{r^2}$

(a) $m_1 = 20kg : m_2 = 30kg ; r = 1m \rightarrow F = G \times 20 \times 30 = 600GN$

(b) $m_1 = 40kg : m_2 = 60kg : r = 0.5m \Rightarrow F = \frac{G \times 40 \times 60}{(\frac{1}{2})^2} = 9600GN$

(c) $m_1 = 30kg : m_2 = 50kg ; r = 2m \Rightarrow F = \frac{G \times 30 \times 50}{2^2} = 375GN$

(d) $m_1 = 10kg : m_2 = 40kg \rightarrow r = 2.5m \Rightarrow F = \frac{G \, 10 \times 40}{(2.5)^2} = 64GN.$