

FOUNDATION

class: VII. MATHEMATICS

4. FUNDAMENTALS OF LOGARITHMS

TEACHING TASK (JEE MAINS)

01. $\log_2 8 = x \Rightarrow 2^x = 8 \Rightarrow x = 3$ Ans: C

02. $\log_5 125 = m \Rightarrow 5^m = 125 \Rightarrow m = 3$ Ans: B

03. $\log_2 2$ Ans: B

04. $\log_3 81 = p \Rightarrow 3^p = 81 \Rightarrow p = 4$ Ans: C

05. $\frac{\log_{10} 1000}{\log_{10} 10} = \frac{\log_{10} 10^3}{1} = 3$ Ans: D

06. $\log_{\left(\frac{1}{a}\right)} a = b \Rightarrow a = \left(\frac{1}{a}\right)^b$ Ans: B

07. $\log_{\left(\frac{1}{y}\right)} \left(\frac{1}{x}\right) = \frac{1}{z} \Rightarrow \frac{1}{x} = \left(\frac{1}{y}\right)^{\frac{1}{z}}$
 option A: $\left(\frac{1}{x}\right)^{\frac{1}{y}} = \frac{1}{z}$ is not true Ans: A

08. $\log_{10} 0.001 = x \Rightarrow 0.001 = 10^x \Rightarrow 10^{-3} = 10^x \Rightarrow x = -3$ Ans: A

09. $\log_{a^2-b^2} (a+b)(a-b) = \log_{a^2-b^2} a^2-b^2 = 1$ Ans: A

10. $\log_{10} 0.0001 = x \Rightarrow 0.0001 = 10^x \Rightarrow 10^{-4} = 10^x \Rightarrow x = -4$ Ans: D

$$11. \log_{10} x = -2 \Rightarrow x = 10^{-2} \Rightarrow x = \frac{1}{100}$$

$$\text{Also } \log_{10} x = -2 \Rightarrow \frac{\log x}{\log 10} = -2$$

Ans: A, C

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$$\log_4 6 = \frac{\log_{10} 6}{\log_{10} 4} \text{ or } \frac{\log 6}{\log 4}$$

Ans: C, D

$$13. \log_{\sqrt{5}} x^2 + 1 = \log_{\sqrt{x^2+1}} 5 \Rightarrow x^2 + 1 = 5 \Rightarrow x = \pm 2$$

Ans: B, D

$$14. \text{Statement I: } \log_5 \left(\frac{1}{25} \right) = y \Rightarrow \frac{1}{25} = 5^y$$

$$\Rightarrow 5^{-2} = 5^y \Rightarrow y = -2$$

$$\text{Statement II: } 5^{-2} = \frac{1}{25} \text{ (True)}$$

(True)
Ans: A

$$15. \text{Statement I: } a^N = x \Rightarrow N = \log_a x \text{ (False)}$$

$$\text{Statement II: } N = \log_a x, a > 0, a \neq 1, x > 0, N \in \mathbb{R} \text{ (False)}$$

Ans: B

$$16. \text{Assertion: Conceptual (True)}$$

$$\text{Reason: Conceptual (True)}$$

Ans: A

$$17. \text{Assertion: } \log_a b = c \Rightarrow a^c = b \text{ (True)}$$

$$\text{Reason: Conceptual (True)}$$

Ans: A

$$18. \ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2} \cdot \ln e = \frac{1}{2} \times 1 = \frac{1}{2}$$

Ans: A

$$19. \ln e = 1$$

Ans: B

$$20. \log \sqrt[5]{\sqrt[6]{x}} = \log \sqrt[30]{x} = \log x^{1/30} = \frac{1}{30} \log x \text{ Ans: B}$$

$$21. \log_{10} \sqrt[10]{y} = 0 \Rightarrow \sqrt[10]{y} = 1 \Rightarrow y = 1 \text{ Ans. A}$$

$$22. \log_2 \sqrt{1+y} = 0 \Rightarrow \sqrt{1+y} = 1 \Rightarrow 1+y = 1 \rightarrow (1)$$

$$\log_2 \sqrt[3]{1-y} = 1 \Rightarrow \sqrt[3]{1-y} = 2 \Rightarrow 1-y = 8 \rightarrow (2)$$

solving (1) & (2) $\Rightarrow x = \frac{9}{2}, y = -\frac{7}{2}$

$$\therefore \frac{x}{y} = -\frac{9}{7} \text{ Ans. D}$$

$$23. \log_2 x = 10 \Rightarrow \log_x 2 = \frac{1}{10} = 0.1 \text{ Ans: 0}$$

$$24. \log_x \frac{1}{\sqrt[3]{2}} = -\frac{1}{3} \Rightarrow \frac{1}{\sqrt[3]{2}} = x^{-1/3} \Rightarrow 2^{-1/3} = x^{-1/3}$$

$$\Rightarrow x = 2 \text{ Ans: 2}$$

$$25. a) \log_{10} 0.01 = \log_{10} 10^{-2} = -2 \text{ (q)}$$

$$b) \log_2 1 = 0 \text{ (p)}$$

$$c) \log_5 125 = \log_5 5^3 = 3 \cdot \log_5 5 = 3 \times 1 = 3 \text{ (r)}$$

$$d) \log_7 \left(\frac{1}{343} \right) = \log_7 7^{-3} = -3 \cdot \log_7 7 = -3 \text{ (s)}$$

Ans. q, p, r, s

$$26. a) (a^2) \log b = b^2 \log a \text{ (s)}$$

$$b) (b^2) \log a = a^2 \log b \text{ (t)}$$

$$c) (a^2)^{\log b^2} = a^{2 \log b^2} = a^{\log b^4} \text{ (r)}$$

$$d) (b^2)^{\log b^2} = b^{2 \log b^2} = b^{\log b^4} \text{ (u)}$$

LEARNERS TASK (CUU'S)

(4)

01.	$\log_a a = 1$	Ans: A
02.	$\log_a M \Rightarrow a > 0, a \neq 1, M > 0$	Ans: A
03.	$\log_a M \cdot \log_M a = 1$	Ans: B
04.	$a^{\frac{1}{n}} = M$	Ans: B
05.	$a^x = N \Rightarrow x = \log_a N$	Ans: A
06.	${}_7 \log_7 6^{-1} = 6^{-1} = \frac{1}{6}$	Ans: B
07.	$2 < e < 3$	Ans: D
08.	Conceptual	Ans: B
09.	$\log_a 1 = 0 \Rightarrow 1 = a^0 \Rightarrow a > 0, a \neq 1$	Ans: C
10.	Conceptual	Ans: D

JEE MAINS LEVEL QUESTIONS

01.	We know, $\log_a b \times \log_c a \times \log_{ab} c = 1$ $\Rightarrow 2 \times \log_c a \times 3 = 1$ $\Rightarrow \log_c a = \frac{1}{6}$	Ans: A
02.	$\log_5 (x^2 + 1) = 2 \Rightarrow x^2 + 1 = 5^2 = 25$ $\Rightarrow x^2 = 24$	Ans: A
03.	We know, $\log_a b \times \log_c b \times \log_a c = 1$ $\Rightarrow 2 \times 3 \times \log_a c = 1 \Rightarrow \log_a c = \frac{1}{6}$	Ans: A



04. $\log_2 3 = p \Rightarrow 3 = 2^p$ (5)
 $\log_4 9 = \log_{2^2} 3^2 = \frac{2}{2} \cdot \log_2 3 = \log_2 3 = p$ Ans: A

05. We know, $\log_a b \times \log_b c \times \log_c a = 1$
 $\Rightarrow p \times \log_c a \times q = 1$
 $\Rightarrow \log_c a = \frac{1}{pq} \Rightarrow \log_a c = pq$ Ans: A

06. $\log_2 A = x \rightarrow \textcircled{1}$ | $\Rightarrow \frac{1}{2} \cdot \log_2 A = y$ | $\Rightarrow x = 2y$
 $\log_4 A = y$ | $\Rightarrow \frac{x}{2} = y$ |
 $\Rightarrow \log_{2^2} A = y$ | } Ans: B

07. We know, $\log_5 x \times \log_x 5 = 1$ | $\Rightarrow \log_x 5 = \frac{1}{3}$
 $\Rightarrow 3 \times \log_x 5 = 1$ | } Ans: A

08. $\log_{10} x = a \Rightarrow x = 10^a$
 Now, $10^{3a-2} = \frac{(10^a)^3}{10^2} = \frac{x^3}{100}$ Ans: C

09. $\log_{10} x = k \Rightarrow x = 10^k$
 Now, $10^{2k-3} = \frac{(10^k)^2}{10^3} = \frac{x^2}{1000}$ Ans: D

10. $m = 100^n$ | $m \times n = 100^{m+n}$
 $n = 100^m$ | $\log_{100} mn = m+n$ Ans: D

11. $\log_x 64 = 6 \Rightarrow 64 = x^6 \Rightarrow 2^6 = x^6 \Rightarrow x = 2$

Ans: A, B, C

12. $\log_2 128 = \frac{\log_{10} 128}{\log_{10} 2}$ ~~$\frac{\ln 128}{\ln 2}$~~

Also $\log_2 128 = \log_2 2^7 = 7 \cdot \log_2 2 = 7 \times 1 = 7$

Ans: A, B, D

13 A) $\log_a 1 = 0$ (True) B) $\log_a a = 1$ (True)

C) $\log_a b^n = n \cdot \log_a b$ (True) D) $\log_a b = \frac{1}{\log_b a}$ (True)

Ans: A, B, C, D

14. Statement I: $\log_4 2 = \log_{2^2} 2 = \frac{1}{2} \cdot \log_2 2 = \frac{1}{2} \times 1 = \frac{1}{2}$

(True)

Statement II: $4^{\frac{1}{2}} = 2$ (True)

Ans: A

15. Statement I: $\sqrt[5]{a^x} = y \Rightarrow a^{\frac{x}{5}} = y$

$\Rightarrow \frac{x}{5} \log a = \log y \Rightarrow x \log a = 5 \log y$ (True)

Statement II: $\log a^m = m \cdot \log a$

$\log \frac{x^2}{y^3} = \frac{2}{3} \cdot \log \frac{x}{y} = \frac{2 \log x}{3 \log y}$ (True)

Ans: A

16 Assertion: $\ln e^5 = 5 \ln e = 5$ (True)

Reason: Conceptual (True)

Ans: A

17. Assertion: $\log_b^2 \cdot \log_4^3 \cdot \log_5^4$ ⑦
 $= \log_b^2 \cdot \log_5^3 \neq \log_5^2$ (False)
Reason: $\log_a b \times \log_b c = \log_a c$ (True)
Ans: D

18. $\log_2 64 = \log_2 2^6 = 6 \cdot \log_2 2 = 6 \times 1 = 6$ (A)

Ans: A

19. $\log_2 1 = 0$

20. $\log_{10} x = p \Rightarrow x = 10^p$

Ans: B

$10^{p+1} = 10^p \times 10 = 10x$

Ans: B

21. $10^{p+3} = 10^p \times 10^3 = 1000x$

22. $10^{p-2} = \frac{10^p}{10^2} = \frac{x}{100}$

Ans: B

23. $\log_a 729 = 6$, Now $\log_3 729 = \log_3 3^6 = 6 \times \log_3 3 = 6$
Ans: 6

24. $\log_2 a^2 = 2 \Rightarrow a^2 = 2^2 \Rightarrow a = \pm 2$

Now, $\log_{a^2} b^2 = 1 \Rightarrow b^2 = a^2 \Rightarrow b^2 = 4 \Rightarrow b = \pm 2$
Ans: ± 2

25. a) $\log_{10} 1000 = 3 \Rightarrow 1000 = 10^3$ (q)

b) $\log_2 64 = 6 \Rightarrow 64 = 2^6$ (p)

c) $\log_e e^4 = 4 \Rightarrow e^4 = e^4$ (r)

d) $\log_5 \frac{1}{25} = -2 \Rightarrow 5^{-2} = \frac{1}{25}$ (s)
Ans: q, p, r, s

$$\boxed{26} \text{ a) } \log_{\sqrt{x}} 3\sqrt{x} = a \Rightarrow \sqrt[3]{x} = x^a \quad \left| \begin{array}{l} a = \frac{1}{3} \\ \Rightarrow x^{\frac{1}{3}} = x^a \end{array} \right. \Rightarrow a^{\frac{1}{2}} = \sqrt{\frac{1}{3}} (r) \quad (8)$$

$$\boxed{b) } \log_{\sqrt{a}} 4\sqrt{a} = x \Rightarrow 4\sqrt{a} = a^x \quad \left| \begin{array}{l} x = \frac{1}{4} \\ \Rightarrow a^{\frac{1}{4}} = a^x \end{array} \right. \Rightarrow x^{\frac{1}{3}} = \sqrt[3]{\frac{1}{4}}$$

$$\text{c) } \log_{\sqrt{a}} 3\sqrt{a} = m \quad \left| \begin{array}{l} m = \frac{1}{3} \\ m^2 = \frac{1}{9} (s) \end{array} \right. \Rightarrow \log_a a^{\frac{1}{3}} = m$$

$$\text{d) } \log_{x^2} \sqrt[5]{x} = a \Rightarrow \log_{x^2} x^{\frac{1}{5}} = \frac{(\frac{1}{5})}{2} = a \Rightarrow a = \frac{1}{10} \\ \Rightarrow a^2 = \frac{1}{100} (p)$$

Ans: r, -, s, p

$\Rightarrow$ THE END $\Leftarrow$