2. FUNCTIONS

Class: IX . MATHEMATICS

FOUNDATION⁺ SOLUTIONS

2025-26

TEACHING TASK
JEE MAINS LEVEL

01. $f(x) = \dfrac{1}{\sqrt{|x| - x}}$

f is defined only when $|x| - x > 0$

$\Rightarrow |x| > x$

This is possible only when $x \in (-\infty, 0)$

$\boxed{\text{Ans: A}}$

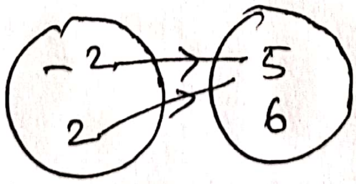
02 $n(A) = n$, $n(B) = 2$

No. of Surjections $= 2^n - 2 = 62$

$\Rightarrow n = 6$

$\boxed{\text{Ans: B}}$

03 Case (i)



$f(-2) = 5$; $f(2) = 5$

$f(x) = ax + b$

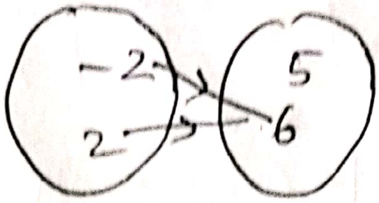
$f(-2) = 5 \Rightarrow -2a + b = 5$

$\underline{2a + b = 5}$ (subtracting)

$a = 0$

here $a=0$, this is not possible

Case(ii)

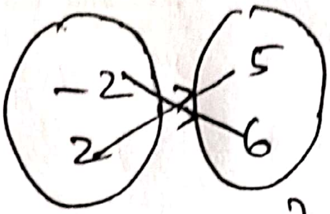


$f(-2)=6$; $f(2)=6$

$$\begin{aligned} -2a+b&=6 \\ 2a+b&=6 \\ \hline a&=0 \end{aligned}$$

here $a=0$, this is not possible

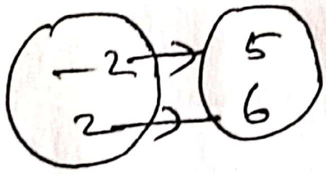
case (iii)



$f(-2)=6$; $f(2)=5$

$\left.\begin{aligned} -2a+b&=6 \\ 2a+b&=5 \end{aligned}\right\}$ we can solve for a & b

Case (iv)



$f(-2)=5$, $f(2)=6$

$\left.\begin{aligned} -2a+b&=5 \\ 2a+b&=6 \end{aligned}\right\}$ we can solve for a & b

hence, there are only 2 linear functions

Ans: opt C

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04 Let $f(x)=e^x$ or a^x

Let $x_1, x_2, x_3 \rightarrow A.P$

Let $1, 2, 3 \rightarrow A.P$

$e^1, e^2, e^3 \rightarrow G.P$

Ans: B

05 $f(x) = \cos(\log x)$

$f\left(\frac{x^2}{y^2}\right) = \cos\left[\log\left(\frac{x^2}{y^2}\right)\right]$

$= \cos(\log x^2 - \log y^2)$

$f(x^2y^2) = \cos[\log(x^2y^2)]$

$= \cos[\log x^2 + \log y^2]$

Now,

$f\left(\frac{x^2}{y^2}\right) + f(x^2y^2) = \cos(\log x^2 - \log y^2) + \cos(\log x^2 + \log y^2)$

$= 2\cos(\log x^2) \cdot \cos(\log y^2)$

$\frac{1}{2}\left[f\left(\frac{x^2}{y^2}\right) + f(x^2y^2)\right] = f(x^2) \cdot f(y^2)$

$\therefore f(x^2) \cdot f(y^2) - \frac{1}{2}\left[f\left(\frac{x^2}{y^2}\right) + f(x^2y^2)\right] = 0$

Ans: B

06 $f(x) \cdot g(y) + g(x) \cdot f(y)$

$= \frac{1}{2}(3^x + 3^{-x}) \cdot \frac{1}{2}(3^y - 3^{-y}) + \frac{1}{2}(3^x - 3^{-x}) \cdot \frac{1}{2}(3^y + 3^{-y})$

$= \frac{1}{4}\left[3^{x+y} - 3^{x-y} + 3^{y-x} - 3^{-(x+y)} + 3^{x+y} + 3^{x-y} - 3^{-x+y} - 3^{-(x+y)}\right]$

$= \frac{2}{4}\left[3^{x+y} - 3^{-(x+y)}\right] = g(x+y)$

Ans: B

07 $y = f(x) = 10x - 7$

$\therefore 10x - 7 = y \Rightarrow x = \dfrac{y+7}{10}$

$\therefore f^{-1}(x) = \dfrac{x+7}{10} = g(x)$

Ans: C

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08 $f(201) = -\left(\dfrac{201-1}{2}\right) = -100$

$\therefore f^{-1}(-100) = 201$

Ans: C

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09 Given $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x = f(x)$

We know, domain of $\sin^{-1}x = A = [-1, 1]$

domain of $\cos^{-1}x = B = [-1, 1]$

domain of $\tan^{-1}x = C = (-\infty, \infty)$

$\therefore$ Domain of $f(x) = A \cap B \cap C$

$= [-1, 1]$

Now, $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$

$= \dfrac{\pi}{2} + \tan^{-1}x$

$\therefore f'(x) = \dfrac{1}{1+x^2} > 0$

$\therefore$ f is an increasing function

$\therefore f(-1) = \dfrac{\pi}{2} + \tan^{-1}(-1) = \dfrac{\pi}{2} - \dfrac{\pi}{4} = \dfrac{\pi}{4}$

$f(1) = \dfrac{\pi}{2} + \tan^{-1}(1) = \dfrac{\pi}{2} + \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

$\therefore$ Range $= \left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right]$

Ans: B

10. We have $(a+b+c)^2 \geq 0$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca) \geq 0$

$\Rightarrow \quad ab+bc+ca \geq -\frac{1}{2} \longrightarrow (1)$

Also, $a^2+b^2+c^2-(ab+bc+ca)$

$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right] \geq 0$

$= \quad 1-(ab+bc+ca) \geq 0$

$\Rightarrow \quad ab+bc+ca \leq 1 \longrightarrow (2)$

from (1) & (2) $\Rightarrow \quad -\frac{1}{2} \leq ab+bc+ca \leq 1$ Ans: A

B JEE ADVANCED

Q1 $f(n) = 1!+2!+3!+\cdots+n! \rightarrow (1)$

$f(n+1) = 1!+2!+3!+\cdots+n!+(n+1)! \rightarrow (2)$

$f(n+2) = 1!+2!+3!+\cdots+(n+1)!+(n+2)! \rightarrow (3)$

(2) − (1) $\Rightarrow \quad f(n+1)-f(n) = (n+1)!$

(3) − (2) $\Rightarrow \quad f(n+2)-f(n+1) = (n+2)!$

$= (n+2)(n+1)!$

$= (n+2)\left[f(n+1)-f(n)\right]$

$= (n+2)f(n+1)-(n+2)f(n)$

$\Rightarrow$ ~~Ot~~

$\Rightarrow (n+3)f(n+1) - (n+2).\ f(n) = f(n+2)$

$\therefore f(n+2) = (n+3)f(n+1) - (n+2).\ f(n)$

$\therefore f(n+2) = P(n)f(n+1) + Q(n)f(n)$

$\therefore P(x) = x+3, \quad Q(x) = -x-2$ Ans: A, B

02 Consider option: C

$f(x) = x^3 - 3x$

$\Rightarrow f(x+\frac{1}{x}) = (x+\frac{1}{x})^3 - 3(x+\frac{1}{x})$

$= x^3 + \frac{1}{x^3}$

Ans: C

03 Statement I: $x^3 - 3x + 2 > 0$

$\Rightarrow (x-1)(x-2) > 0$ (True)

$x \in (-\infty, 1) \cup (2, \infty)$

Statement II: Conceptual (True) Ans: A

04 Statement I: $n(A) = 4$, $n(B) = 8$

No. of relations from A to B = $2^{4\times8} = 2^{32}$

No. of relations from A to B = $8^4 = 4096$

No. of relations which are not functions

$\therefore$ No. of relations

$= 2^{32} - 4096 \neq 4096$

Clearly, Statement I is False.

Statement II:

Many-one functions = total functions − one-one function

$= 4^3 - {}^4P_3$

$= 64 - 24 = 40$ (True)

$\boxed{\text{Ans: D}}$

05 Assertion: $f(x) = |x|$, is not one-one function,

Since $f(-1) = |-1| = 1$
$f(1) = |1| = 1$

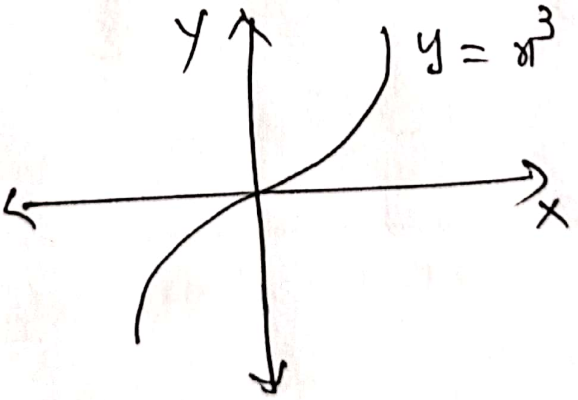
i.e Both −1 and 1 have the same Image (False)

Ans: D

Reason: Conceptual (True)

06 Assertion: $f(x) = x^3$ is one-one (True)

Reason:



(True)

Ans: A

07

07. $f(x) = \sqrt{\dfrac{x-2}{x-3}}$

f is defined only when $\dfrac{x-2}{x-3} \geq 0$

and $x-3 \neq 0 \Rightarrow x \neq 3$

Now, $\dfrac{x-2}{x-3} \geq 0 \Rightarrow (x-2)(x-3) \geq 0$

$\Rightarrow x \in (-\infty, 2] \cup [3, \infty)$

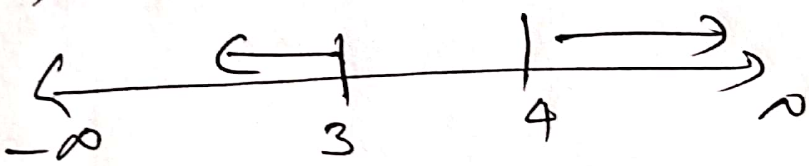
Ans: D

$\boxed{08}$ $f(x) = \sqrt{\dfrac{3-x}{4-x}}$

$\therefore \dfrac{3-x}{4-x} \geq 0$ and $4-x \neq 0 \Rightarrow x \neq 4$

$\Rightarrow (3-x)(4-x) \geq 0$

$\Rightarrow (x-3)(x-4) \geq 0$



$\therefore$ Domain $= (-\infty, 3] \cup (4, \infty)$

$\boxed{\text{Ans: D}}$

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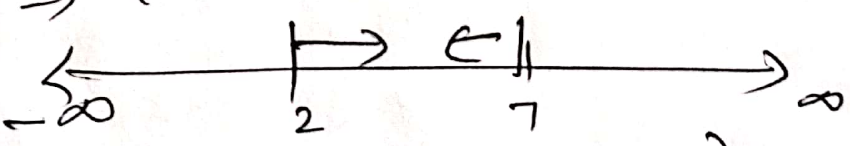
09 $\dfrac{2-x}{x-7} \geq 0$ and $x-7 \neq 0 \Rightarrow x \neq 7$

$\Rightarrow (2-x)(x-7) \geq 0$

$\Rightarrow (x-2)(x-7) \leq 0$



$\therefore$ Domain $= [2, 7)$

Ans: B

10 $x^2+x+1>0$ and $x^2+x+1 \neq 1$

$\Delta < 0$

$\forall x \in R,\ x^2+x+1>0$

$x^2+x \neq 0$

$\Rightarrow x(x+1) \neq 0$

$\Rightarrow x \neq 0$ and $x \neq -1$

Ans: B

$\therefore$ Domain $= R-\{-1,0\}$

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11 $f(x)=\log(x^2+x+1)$

$\therefore x^2+x+1>0,\ \forall x \in R$ since $\Delta<0$

Minimum value $= \dfrac{4ac-b^2}{4a} = \dfrac{3}{4}$

Minimum value of $\log(x^2+x+1) = \log\left(\frac{3}{4}\right)$.

Ans: A

Hence, Range $= \left[\log\left(\frac{3}{4}\right), \infty\right)$

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12 $f(x)=\dfrac{2^x}{x+2}$

$\Rightarrow x+2 \neq 0$

$\Rightarrow x \neq -2$

Ans: D

$\therefore$ Domain $= R-\{-2\}$

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13 $n(x)=5,\ n(y)=3$

No. of Surjections $= n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - \ldots$

here $m=5,\ n=2$

$= 3^5 - {}^3C_1(3-1)^5 + {}^3C_2(3-2)^5 - {}^3C_3(3-3)^5$

$= 243-96+3$

$= 150$

Ans: 150

14. $n(A) = 5, \quad n(B) = 2$

No. of Surjections $= 2^n - 1$

$= 2^5 - 1$

$= 31$

Ans: 31

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15 $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$ $\quad \pi = 3.14$

$= \cos[9.859]x + \cos[-9.859]x$ $\quad \pi^2 = 9.859$

$= \cos 9x + \cos(-10)x$

$= \cos 9x + \cos 10x$

a) $f(\frac{\pi}{2}) = \cos\frac{9\pi}{2} + \cos\frac{10\pi}{2} = 0 - 1 = -1$

b) $f(\frac{\pi}{4}) = \cos\frac{9\pi}{4} + \cos\frac{10\pi}{4}$

$= \cos 375^\circ + \cos 225^\circ$

$= \cos 15^\circ - \cos 45^\circ$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

c) $f(\pi) = \cos 9\pi + \cos 10\pi = \cos(2\times 2\pi - \pi) + \cos(2\times 5\pi)$

$= \cos\pi + \cos 0 = -1 + 1 = 0$

d) $f(0) = \cos 0 + \cos 0 = 1 + 1 = 2$

Ans: q, s, p, r

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16 a) $1 + x > 0 \Rightarrow x > -1 \Rightarrow$ Domain $= (-1, \infty)$

b) $|x| - x > 0 \Rightarrow |x| > x \Rightarrow$ Domain $= (-\infty, 0)$

c) $1 - |x| > 0 \Rightarrow |x| < 1 \Rightarrow$ Domain $= [-1, 1]$

d) $f(x) = \sqrt{-x^2} \Rightarrow$ Domain $= \phi$

Ans: $\boxed{p, t, r, q}$

LEARNERS TASK

CUQ'S

01	$A \times B = \{(1,x),(1,y),(2,x),(2,y),(3,x),(3,y)\}$	Ans: B
02	No. of relations $= 2^{m \times n}$	Ans: C
03	No. of functions $= n^m$	Ans: A
04	$f(x) = 3x^4 + 6x^3 - 7x^2 + 2$	Ans: C
05	Conceptual	Ans: B
06	${}^{n}P_{m}$ $(n \geq m)$	Ans: B
07	$f(A) \neq B$	Ans: A
08	No. of Bijections $= n!$	Ans: D
09.	$f(x) = 2x^2 + 3x - 5$ $f(1) = 2(1)^2 + 3(1) - 5 = 0$ Similarly, other values follows Range $= \{0, 9, 22, 39, 60\}$	Ans: A
10	(i) when x is very large and positive 10^{-x} approaches to 0. (ii) when x is very large and negative 10^{-x} approaches infinity (iii) when $x = 0$, $10^{-x} = 1$ $\therefore$ Range $= (0, \infty)$	Ans: C

JEE MAINS LEVEL QUESTIONS

01. $f(2) + f(3) = 0$ Ans. C

02. $f(1+x) = x^2+1$

Let $1+x = y \Rightarrow x = y-1$

$\therefore f(y) = (y-1)^2+1 = y^2-2y+2$

$\therefore f(x) = x^2-2x+2$

$f(2-h) = (2-h)^2 - 2(2-h)+2$

$= h^2 - 2h + 2$ Ans: B

03. $n(A) = 100,\ n(B) = 26$ Ans. D

No. of Constant functions $= n(B) = 26$

04. $n(A) = n(B) = 4$ Ans. C

No. of Bijections $= 4!$

05. $f(x) = 2x + |x|$

$$f(x) = \begin{cases} 3x & x \geq 0 \\ x & x < 0 \end{cases}$$

Now, given $f(3x) - f(-x) - 4x$

$$f(3x) = \begin{cases} 9x & x \geq 0 \\ 3x & x < 0 \end{cases}$$

$$f(-x) = \begin{cases} -3x & x \leq 0 \\ -x & x > 0 \end{cases}$$

Now, to calculate $f(3x) - f(-x) - 4x$

Case (i) $x \geq 0$

$f(3x) = 9x, \quad f(-x) = -x$

$$f(3x) - f(-x) - 4x = 9x + x - 4x$$
$$= 6x$$

Case(ii) $x < 0$

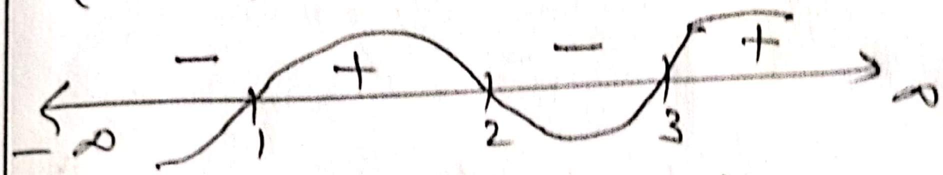
$$f(3x) - f(-x) - 4x = 3x - (-3x) - 4x$$
$$= 2x$$

$$\therefore f(3x) - f(-x) - 4x = \begin{cases} 6x & \text{if } x \geq 0 \\ 2x & \text{if } x < 0 \end{cases}$$

$$= 2|x| + 4x$$
$$= 2\left[|x| + 2x\right]$$
$$= 2f(x)$$

Ans. D

06 $(x-1)(x-2)(x-3) \geq 0$



Ans. D

Domain = $[1, 2] \cup [3, \infty)$

07 $f(-3) = (-3)^2 + 1 = 10$; $f(2) = 2(2) - 1 = 3$

$f(5) = 4(5) + 3 = 23$; $f(1) = 2(1) - 1 = 1$

$$\frac{f(-3) + f(2) + f(5)}{f(1)} = \frac{10 + 3 + 23}{1} = 36$$

Ans. B

08

$f(x) + 2f\left(\frac{1}{x}\right) = 3x \rightarrow (1)$

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x} \rightarrow (2)$

$(2) \times 2 \Rightarrow 2f\left(\frac{1}{x}\right) + 4f(x) = \frac{6}{x}$

$(1) \Rightarrow \underline{-}\, 2f\left(\frac{1}{x}\right) \underline{+}\, f(x) = \underline{3x}$

$3f(x) = \frac{6}{x} - 3x$

$\Rightarrow f(x) = \frac{2}{x} - x$

Now, $f(x) = f(-x)$

$\Rightarrow \frac{2}{x} - x = -\frac{2}{x} + x$

$\Rightarrow x = \pm\sqrt{2}$

Ans: C

09

$-1 \leq x \leq 1$

$\Rightarrow -5 \leq 5x \leq 5$

$\Rightarrow -1 \leq 5x + 4 \leq 9$

Ans: B

10

$f(x) = \sin^2 x + \cos^4 x$

$= \sin^2 x + (1 - \sin^2 x)^2$

$= \sin^4 x - \sin^2 x + 1$

$= \sin^2 x(\sin^2 x - 1) + 1$

$= -\sin^2 x \cos^2 x + 1$

$= -\frac{1}{4}(2\sin x \cos x)^2 + 1$

$= -\frac{1}{4}\sin^2 2x + 1.$

We know, $0 \le \sin^2 2x \le 1$

$$\Rightarrow 0 \ge -\frac{1}{4}\sin^2 2x \ge -\frac{1}{4}$$

$$\Rightarrow -\frac{1}{4}+1 \le \frac{-1}{4}\sin^2 x + 1 \le 1$$

$$\Rightarrow \frac{3}{4} \le f(x) \le 1$$

$\therefore$ Range $= \left[\frac{3}{4}, 1\right]$ Ans. A

ADVANCED LEVEL

01. $f(x+y) = f(x) + f(y)$

$\Rightarrow f(x) = Kx \quad K \in R - \{0\}$

$\Rightarrow f(1) = K.1 = 2 \Rightarrow K = 2$

$\therefore f(x) = 2x$ Ans. B

02 $f(x+y) = f(x) + f(y)$

$\Rightarrow f(x) = Kx$

$\Rightarrow f(1) = K.1 = 7 \Rightarrow K = 7$

$\therefore f(x) = 7x$

$$\sum_{r=1}^{n} f(r) = f(1) + f(2) + \cdots + f(n)$$

$$= 7.1 + 7.2 + \cdots + 7.n$$

$$= 7(1+2+3+\cdots+n)$$

$$= \frac{7n(n+1)}{2}$$ Ans. D

03 Statement I:

Domain of $f = R^+$

Domain of $g = N$ Also, $f(x) = g(x)$

Domains are different

hence, the given functions are not equal functions (False)

Statement II: Conceptual (True)

Ans. D

04 Statement I: $f(x) = \frac{1-x^2}{1+x^2}$

$\Rightarrow f(\tan\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \cos 2\theta$ (True)

Statement II: Conceptual (False)

Ans: C

05 Statement I: Conceptual (False)

Statement II:

No. of onto functions $= n^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m - \ldots$

$= 3^4 - {}^3C_1 (3-1)^4 + {}^3C_2 (3-2)^4$

$= 81 - 48 + 3$

$= 36$ (False)

Ans. B

06 **Assertion:** $f(x) = \frac{1}{x}$

$\therefore$ Domain $= R - \{0\}$ (False)

Reason: Conceptual (True) Ans: D

07 **Assertion:** Conceptual (False)

Reason: Conceptual (True) Ans: D

08 $f(x) = \cos 3x$ Ans: A

$f(-x) = \cos 3(-x) = \cos 3x = f(x)$

09 $g(x) = x^2 + x + 1$

$g(-x) = (-x)^2 + (-x) + 1$

$= x^2 - x + 1 \neq g(x)$

$\therefore$ $g(x)$ is neither even nor odd Ans: C

10 $gof(x) = g(f(x))$

$= g(1 - x^4)$

$= (1 - x^4)^2 + 1$

$= x^8 - 2x^4 + 2$

$\therefore gof(-x) = x^8 - 2x^4 + 2 = gof(x)$

$\therefore gof(x)$ is an Even function Ans: A

(18)

11 $A=\{a,e,i,o\} \Rightarrow n(A)=4$

$B=\{2,3,5,7\} \Rightarrow n(B)=4$

No. of Bijections $= 4! = 24$

Ans: B

12. Here $A=\{1,2,7\} \Rightarrow n(A)=3$

Total No. of functions $= 3^3 = 27$

No. of Bijections $= 3! = 6$

$\therefore$ No. of Non-Bijections $= 27-6 = 21$

Ans: A

13 Let $f(x)=x$

$f(x+f(x)) = f(x+x)$

$= f(2x)$

$\because x+f(x) = x+x$

$= 2x$

$= 2x$, which is linear.

Let $f(x) = -x$

$f(x+f(x)) = f(x-x) = f(0) = 0$

$\therefore x+f(x) = x-x = 0$

hence, there are only two linear functions satisfying the given conditions

Ans: 2

14 $n(A)=5$, $n(B)=3$

No. of many to one function = total No. of functions $-$ ~~on~~ No. of one-one functions

$= 3^5 - 0 = 243$

Ans: 243

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15 $f(x) = \dfrac{7^{1+\log_e x}}{x^{\log_e 7}} = \dfrac{7 \cdot 7^{\log_e x}}{7^{\log_e x}} = 7$

$\therefore f(2014) = 7$ Ans: 7

16 a) $|x| - x \neq 0 \Rightarrow |x| \neq x \quad \forall x \in (-\infty, 0)$

b) $|x| \Rightarrow$ Domain $= R$

c) $\sec x$, Domain $= R - \{(2n+1)\frac{\pi}{2}\}$

d) $\sqrt{x}$, Domain $= [0, \infty)$

Ans. t, s, r, q

17 a) we know $-1 \leq \cos x \leq 1$

$\Rightarrow -2 \leq 2\cos x \leq 2$

$\Rightarrow 2 \geq -2\cos x \geq -2$

$\Rightarrow -2 \leq -2\cos x \leq 2$

$\Rightarrow -1 \leq 1 - 2\cos x \leq 3$

$\Rightarrow -1 \geq \dfrac{1}{1-2\cos x} \geq \dfrac{1}{3}$

Range $= (-\infty, -1] \cup [\frac{1}{3}, \infty)$

b) $-1 \le \cos 2x \le 1$

$-3 \le 3\cos 2x \le 3$

$-2 \le 1 + 3\cos 2x \le 4$

$\therefore$ Range $= [-2, 4]$

c) $5\sin x + 12\cos x - 13$

max. value $= c + \sqrt{a^2+b^2}$

$= -13 + \sqrt{25+144}$

$= -13 + 13$

$= 0$

min. value $= -13 - 13 = -26$

$\therefore$ Range $= [-26, 0]$

d) $|x-2|$

$\therefore$ Range $= [0, \infty)$

Ans: r, q, p, t

ADDITIONAL PRACTICE QUESTIONS

01. No. of constant functions $= n(B) = n$

Ans. B

02. $n(A) = 10$, $n(B) = 20$

No. of functions $= 20^{10}$

Ans. D

03. $n(A) = 4$, $n(B) = 6$

No. of one-one function $= {}^6P_4$

Ans. C

04. $n(A) = 106$

No. of Bijection $= 106!$

Ans: C

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05 $f(x) = |x|$, $g(x) = [x-3]$

$g(f(x)) = g(|x|)$

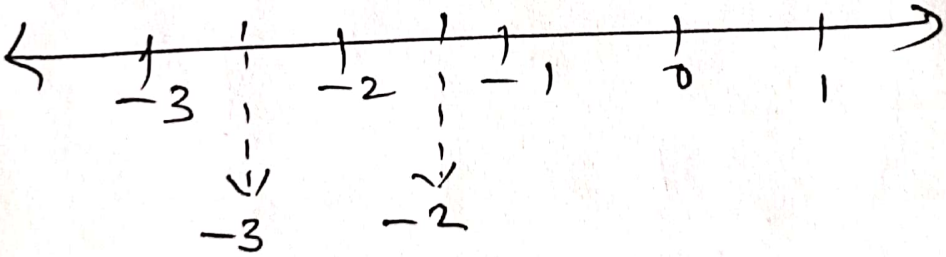
$= [|x| - 3]$

Given $-\frac{8}{5} < x < \frac{8}{5}$

$\Rightarrow \quad 0 < |x| < \frac{8}{5}$

$\Rightarrow \quad -3 < |x| - 3 < -\frac{7}{5}$

$\Rightarrow \quad -3 < |x| - 3 < -1.4$



$\therefore [|x| - 3] = \{-3, -2\}$

Ans: C

$\Longrightarrow$ THE END $\Longleftarrow$