WS-11 vectors - sub, add, multiplication

6th foundation +

Tbank

①

Given $\vec{P} = \hat{i} - 2\hat{j} - 2\hat{k}$

It is a 3D vector so it's magnitude

$= \sqrt{x^2 + y^2 + z^2}$

$= \sqrt{1^2 + (-2)^2 + (-2)^2}$

$= \sqrt{1+4+4} = \sqrt{9} = 3$

②

let $\vec{A} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

$\vec{B} = -3\hat{j} + 6\hat{k}$

Given that $2\vec{A} + 7B + 4\vec{C} = 0$

$\Rightarrow 2(3\hat{i} + 5\hat{j} - 2\hat{k}) + 7(-3\hat{j} + 6\hat{k}) + 4C = 0$

$\Rightarrow 6\hat{i} + 10\hat{j} - 4\hat{k} - 21\hat{j} + 42\hat{k} + 4C = 0$

$\Rightarrow 6\hat{i} - 11\hat{j} + 38\hat{k} + 4C = 0$

$\Rightarrow 4C = -6\hat{i} + 11\hat{j} - 38\hat{k}$

$\Rightarrow C = -\frac{6}{4}\hat{i} + \frac{11}{4}\hat{j} - \frac{38}{4}\hat{k}$

$\vec{C} = -1.5\hat{i} + 2.75\hat{j} - 9.5\hat{k} \rightarrow$ C.

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Given $\vec{A} = 6\hat{i} - 2\hat{k}$

Given $\vec{A} - 2\vec{B} = -3(\vec{A} + \vec{B})$

$\Rightarrow \vec{A} - 2\vec{B} = -3\vec{A} - 3\vec{B}$

$\Rightarrow 4\vec{A} = +2\vec{B} - 3\vec{B}$

$\Rightarrow 4\vec{A} = -\vec{B}$

$\Rightarrow \vec{B} = -4\vec{A} = -4(6\hat{i} - 2\hat{k}) = -24\hat{i} + 8\hat{k} \rightarrow A$

(4)

Given vector is $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$ is a unit vect

$\Rightarrow \sqrt{(0.5)^2 + (0.8)^2 + c^2} = 1$

$\Rightarrow 0.25 + 0.64 + c^2 = 1$

$\Rightarrow c^2 = 1 - 0.89 = 0.11$

$\Rightarrow c = \sqrt{0.11} \rightarrow D$

(5)

Given $\vec{P} = \hat{i} + 3\hat{j} - 7\hat{k}$ and $\vec{Q} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

length of vector $PQ = 5\hat{i} - 2\hat{j} + 4\hat{k} - (\hat{i} + 3\hat{j} - 7\hat{k})$

$= 5\hat{i} - 2\hat{j} + 4\hat{k} - \hat{i} - 3\hat{j} + 7\hat{k}$

$\Rightarrow 4\hat{i} - 5\hat{j} + 11\hat{k}$

$|\vec{PQ}| = \sqrt{4^2 + (-5)^2 + 11^2} = \sqrt{16 + 25 + 121}$

$= \sqrt{162} \rightarrow B$

(6)

Given

$$\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k} \; ; \; \vec{b} = 5\hat{i} + 3\hat{j} - 2\hat{k}$$

Given that $3\vec{a} + 2\vec{b} - \vec{c} = 0$

$$\Rightarrow \vec{c} = 3\vec{a} + 2\vec{b}$$

$$= 3(2\hat{i} + \hat{j} - 3\hat{k}) + 2(5\hat{i} + 3\hat{j} - 2\hat{k})$$

$$\Rightarrow 6\hat{i} + 3\hat{j} - 9\hat{k} + 10\hat{i} + 6\hat{j} - 4\hat{k}$$

$$= 16\hat{i} + 9\hat{j} - 13\hat{k} \rightarrow C.$$

(7)

Given vector $\vec{A} = 6\hat{i} + 8\hat{j} + 10\hat{k}$.

unit vector Parallel to $\vec{A} = \dfrac{\vec{A}}{|\vec{A}|}$

Magnitude of $\vec{A} = |\vec{A}| = \sqrt{6^2 + 8^2 + 10^2}$

$$= \sqrt{36 + 64 + 100}$$

$$= 10\sqrt{2}.$$

unit vector $\hat{A} = \dfrac{6\hat{i} + 8\hat{j} + 10\hat{k}}{10\sqrt{2}} = \dfrac{2(3\hat{i} + 4\hat{j} + 5\hat{k})}{\cancel{10}_{5}\sqrt{2}}$

$$= \frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{5\sqrt{2}.} \rightarrow A.$$

(8)

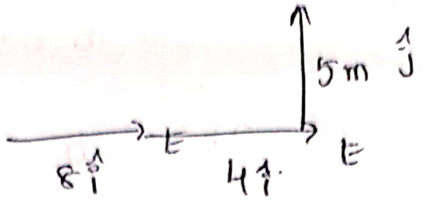
Given $F_1 = 3\hat{i} - 4\hat{j} + 2\hat{k}$; $F_2 = 2\hat{i} + 3\hat{j} - \hat{k}$; $F_3 = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\therefore$ Resultant force $F = F_1 + F_2 + F_3 = 3\hat{i} - 4\hat{j} + 2\hat{k} + 2\hat{i} + 3\hat{j} - \hat{k} + 2\hat{i} + 4\hat{j} - 5\hat{k}$

$$= 7\hat{i} + 3\hat{j} - 4\hat{k} \rightarrow C$$

(9)

(6)



Displacement $= 12\hat{i} + 5\hat{j}$.

$|\text{Displacement}| = \sqrt{12^2 + 5^2} = \sqrt{144+25} = \sqrt{169} = 13\,m.$

(13)

Electric current is a scalar. because it does not obey's laws of vector addition.

(14), (15), (16)

Given $\vec{a} = 4\hat{i} - 3\hat{j}$; $\vec{b} = 8\hat{i} - 6\hat{j}$

magnitude of $\vec{a} + \vec{b} = 4\hat{i} - 3\hat{j} + 8\hat{i} - 6\hat{j}$

$= 12\hat{i} - 9\hat{j}$

$|\vec{a} + \vec{b}| = \sqrt{12^2 + (-9)^2} = \sqrt{144+81} = \sqrt{225} = 15 \rightarrow C$

$\Rightarrow \vec{a} - \vec{b} = 4\hat{i} - 3\hat{j} - 8\hat{i} + 6\hat{j} = -4\hat{i} + 3\hat{j}$

$|\vec{a} - \vec{b}| = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5 \rightarrow A$

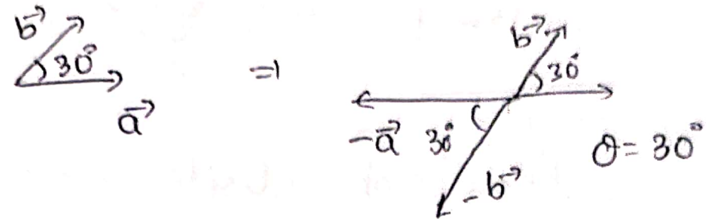
$\Rightarrow \vec{b} - \vec{a} = 8\hat{i} - 6\hat{j} - 4\hat{i} + 3\hat{j} = 4\hat{i} - 3\hat{j}$

$|\vec{b} - \vec{a}| = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5 \rightarrow A$

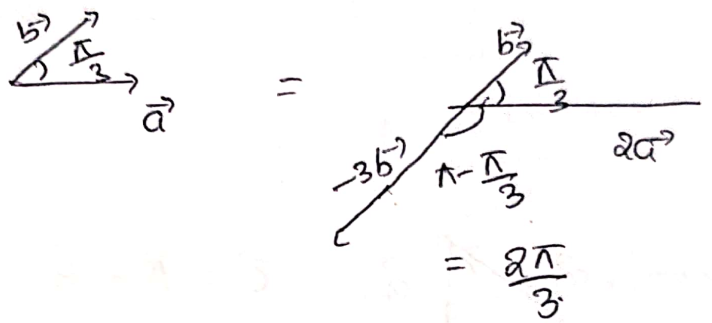
LTank

CUQ

(6)



(7)



Jee Mains level

(11)

Given $\vec{A} = 2\hat{i} + \hat{j}$; $\vec{B} = \hat{j} - \hat{k}$

Then $2\vec{A} + 3\vec{B} = 2(2\hat{i} + \hat{j}) + 3(\hat{j} - \hat{k})$

$= 4\hat{i} + 2\hat{j} + 3\hat{j} - 3\hat{k}$

$= 4\hat{i} + 5\hat{j} - 3\hat{k}$

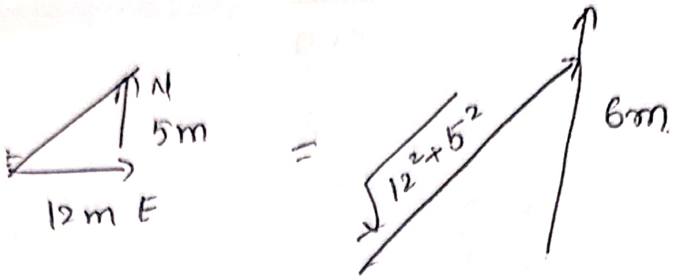
Magnitude of $2\vec{A} + 3\vec{B} = \sqrt{4^2 + 5^2 + (-3)^2}$

$= \sqrt{16 + 25 + 9}$

$= \sqrt{50}$

$\Rightarrow 5\sqrt{2} \rightarrow c$

(12)



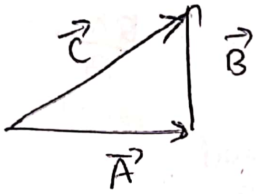
(13)

Resultant displacement $= \sqrt{12^2+5^2+6^2}$

$= \sqrt{144+25+36}$

$= \sqrt{205} = 14.32\,m.$

(14)



$\vec{C} = \vec{A} + \vec{B}$

$= 2\hat{i} - \hat{j} - 3\hat{k} + \hat{i} + 2\hat{j} - \hat{k}$

$= 3\hat{i} + \hat{j} - 4\hat{k}$

so $\vec{C}$ is a closing side for a triangle of ~~so~~ two adjustant sides $\vec{A}$ & $\vec{B}$.

$\therefore$ $\vec{A}$, $\vec{B}$ and $\vec{C}$ form a triangle $\rightarrow$ B

(15)

let P = initial point $= (1, 2, -1)$

Q = Terminal point $= (3, 2, 1)$

$\therefore$ The vector $PQ = Q - P = (3-1)\hat{i} + (2-2)\hat{j} + (1-(-1))\hat{k}$

$= 2\hat{i} + 0\hat{j} + 2\hat{k}$

$= 2\hat{i} + 2\hat{k} \rightarrow$ B

Given position vector $R = -6\hat{i} - 4\hat{j}$ ...

magnitude of $\vec{R}$ is $= \sqrt{(-6)^2 + (-4)^2 + (-12)^2}$

$= \sqrt{36 + 16 + 144}$

$= \sqrt{196} = 14$

unit vector parallel to $\vec{R}$ is $\hat{n} = \frac{\vec{R}}{|\vec{R}|}$

$\Rightarrow \hat{n} = \frac{-6\hat{i} - 4\hat{j} - 12\hat{k}}{\pm 14}$

$\Rightarrow \frac{-\cancel{2}(3\hat{i} + 2\hat{j} + 6\hat{k})}{\pm \cancel{14}\, 7}$

$\Rightarrow \pm \frac{1}{7}[3\hat{i} + 2\hat{j} + 6\hat{k}] \rightarrow C$

(18)

let the initial position $r_1 = (1, 2, 3)$ m.

Final position $r_2 = (5, 4, 2)$ m.

$\therefore$ Displacement vector $= (r_2 - r_1)$

$\Rightarrow (5-1)\hat{i} + (4-2)\hat{j} + (2-3)\hat{k}$

$\Rightarrow 4\hat{i} + 2\hat{j} - \hat{k}. \rightarrow D$

(19)

let $\vec{r}_1 = 2\hat{i} + 3\hat{j} + 5\hat{k}$ m. ; $\vec{r}_2 = \hat{i} - 2\hat{j} + \hat{k}$ m

Displacement $= \vec{r}_2 - \vec{r}_1 = \hat{i} - 2\hat{j} + \hat{k} - (2\hat{i} + 3\hat{j} + 5\hat{k})$

$= \hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k}$

$\Rightarrow -\hat{i} - 5\hat{j} - 4\hat{k}.$

(24), (25), (26)

Given $\vec{F_1} = 8\hat{i} + 6\hat{j}$ N ; $\vec{F_2} = 4\hat{i} - 8\hat{j}$ N.

then $\vec{F_1} + \vec{F_2} = 8\hat{i} + 6\hat{j} + 4\hat{i} - 8\hat{j}$

$= 12\hat{i} - 2\hat{j} \rightarrow A$

and $\vec{F_1} - \vec{F_2} = 8\hat{i} + 6\hat{j} - 4\hat{i} + 8\hat{j}$

$= 4\hat{i} + 14\hat{j} \rightarrow C$

Magnitude of Resultant of $\vec{F_1}$ and $\vec{F_2}$ is

$|\vec{F_1} + \vec{F_2}| = \sqrt{12^2 + (-2)^2}$

$= \sqrt{144 + 4} = \sqrt{148}$ N $\rightarrow B$

(29)

let $\vec{A} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

Magnitude of $\vec{A} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{\frac{2}{2}}$

$= \sqrt{1} = 1$

so $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ is a unit vector. — B

(15)

let initial point $P = (1, 1, 2)$;
Terminal point $Q = (2, 1, 3)$

length of vector $PQ = (2-1)\hat{i} + (1-1)\hat{j} + (3-2)\hat{k}$

$= \hat{i} + \hat{k}$

. length of the vector $PQ = \sqrt{1^2 + 1^2} = \sqrt{2}$ — A