PROGRESSIONS-III ①

Class: VIII, Mathematics

SOLUTIONS

TEACHING TASK

01. $\frac{a}{r}, a, ar \rightarrow G.P$

$\Rightarrow \frac{a}{r}, 2a, ar \rightarrow A.P$

$\Rightarrow 2 \times 2a = \frac{a}{r} + ar$

$\Rightarrow 4 = \frac{1}{r} + r$

$\Rightarrow 4r = 1 + r^2$

$\Rightarrow r^2 - 4r + 1 = 0$

$r = 2 \pm \sqrt{3}$

Since G.P. is increasing r should be positive hence $r = 2+\sqrt{3}$ and $r = 2-\sqrt{3}$

Ans: A, B

02. $a^x = b^y = c^z$

$\Rightarrow \log a^x = \log b^y = \log c^z = K$

$\Rightarrow x \log a = K, \; y \log b = K, \; z \cdot \log c = K$

$\Rightarrow \log a = \frac{K}{x} \rightarrow ①, \quad \log b = \frac{K}{y} \rightarrow ②, \quad \log c = \frac{K}{z} \rightarrow ③$

$\frac{①}{②} \Rightarrow \frac{\log a}{\log b} = \frac{\left(\frac{K}{x}\right)}{\left(\frac{K}{y}\right)} = \frac{y}{x}$

$\frac{②}{③} \Rightarrow \frac{\log b}{\log c} = \frac{\left(\frac{K}{y}\right)}{\left(\frac{K}{z}\right)} = \frac{z}{y}$

x, y, z AP

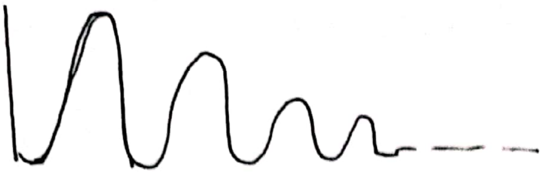
since $\Rightarrow \frac{y}{x} = \frac{z}{y}$

$\therefore \frac{\log a}{\log b} = \frac{\log b}{\log c}$

$\Rightarrow \log_b^a = \log_c^b$

Ans C

03

120m 

$$\text{Total distance} = 120 + 2\times 120\times \frac{4}{5} + 2\times 120\times \frac{4}{5}\times\frac{4}{5} + \cdots$$
$$\cdots + 2\times 120\times \frac{4}{5}\times\frac{4}{5}\times\frac{4}{5} + \cdots$$

$$= 120 + 2\times 120\times\frac{4}{5}\left[1 + \frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \cdots\right]$$

$$= 120 + 2\times 120\times \frac{4}{5}\times \frac{1}{1-\frac{4}{5}}$$

$$= 120 + 2\times 120\times\frac{4}{\cancel{5}}\times\frac{\cancel{5}}{1}$$

$$= 120 + 960$$
$$= 1080.$$

Ans: C

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04 $a, ar, ar^2, ar^3, \ldots$ | $a^3, (ar)^3, (ar^2)^3, \ldots$

$$\frac{a}{1-r} = 57 \rightarrow (1) \qquad \frac{a^3}{1-r^3} = 9747 \rightarrow (2)$$

$$\frac{a^3}{(1-r)^3} = (57)^3 \rightarrow (1)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\left[\frac{\cancel{a^3}}{(1-r)^3}\right]}{\left[\frac{\cancel{a^3}}{1-r^3}\right]} = \frac{(57)^3}{9747}$$

$$\Rightarrow \frac{1-r^3}{(1-r)^3} = 19$$

$$\Rightarrow \frac{\cancel{(1-r)}(1+r+r^2)}{\cancel{(1-r)}(1-r)^2} = 19$$

$$\frac{1+r+r^2}{1-2r+r^2} = 19$$

$$\Rightarrow 1+r+r^2 = 19-18r+19r^2$$

$$\Rightarrow 18r^2 - 39r + 18 = 0$$

$$\Rightarrow r = \frac{2}{3} \text{ or } \frac{3}{2}$$

Ans: B

(3)

05 $a^2+b^2,\ ab+bc,\ b^2+c^2 \longrightarrow G.P$

$\Rightarrow (ab+bc)^2 = (a^2+b^2)(b^2+c^2)$

$\Rightarrow b^4 - 2ab^2c + a^2c^2 = 0$

$\Rightarrow (b^2-ac)^2 = 0$

$\Rightarrow b^2 = ac$

$\Rightarrow$ a, b, c one in G.P.

06 $ar, ar^2, ar^3, \ldots ar^{10}$

$$\frac{ar(1-r^{10})}{1-r} = 18 \longrightarrow (1)$$

$\frac{1}{ar}, \frac{1}{ar^2}, \frac{1}{ar^3} \ldots \frac{1}{ar^{10}}$

$$\frac{\frac{1}{ar}\left(1-\frac{1}{r^{10}}\right)}{1-\frac{1}{r}} = 6$$

$$\Rightarrow \frac{1}{ar^{11}}\left(\frac{r^{10}-1}{r-1}\right) r = 6 \rightarrow (2)$$

$$\Rightarrow \frac{1}{a^2r^{11}} \times 18 = 6 \quad (\text{from } (1))$$

$$\Rightarrow a^2r^{11} = 3$$

produc $= ar \cdot ar^2 \cdot ar^3 \cdots ar^{10}$

$= a^{10} \cdot r^{1+2+3+\cdots+10}$

$= a^{10} \cdot r^{55}$

$= (a^2 \cdot r^{11})^5$

$= (3)^5$

$= 243$

Ans : B

07. $a_p = a+(p-1)d = A \longrightarrow (1)$

$a_q = a+(q-1)d = Ar \quad$ G.P. $\longrightarrow (2)$

$a_r = a+(r-1)d = Ar^2 \longrightarrow (3)$

(4)

$(2)-(1) \Rightarrow (q-p)d = (r-1)A \longrightarrow (4)$

$(3)-(2) \Rightarrow (r-q)d = (r-1)Ar \longrightarrow (5)$

$\frac{(5)}{(4)} \Rightarrow \frac{(r-q)\cancel{d}}{(q-p)\cancel{d}} = \frac{\cancel{(r-1)}\cancel{A}r}{\cancel{(r-1)}\cancel{A}}$

$\therefore r = \frac{r-q}{q-p} = \frac{q-r}{p-q}$

Ans: D

08 ~~$a_p = a+(p-1)d$; $a_q = a+(q-1)d$~~; a

08 $a_p = a+(p-1)d \rightarrow A \longrightarrow (1)$

$a_q = a+(q-1)d \rightarrow AR \longrightarrow (2)$

$a_r = a+(r-1)d \rightarrow AR^2 \longrightarrow (3)$

$a_s = a(s-1)d \longrightarrow AR^3 \longrightarrow (4)$

$(1)-(2) \Rightarrow (p-q) = \frac{A-AR}{d}$

$(2)-(3) \Rightarrow q-r = \frac{AR-AR^2}{d} = \frac{R(A-AR)}{d}$

$(3)-(4) \Rightarrow r-s = \frac{AR^2-AR^3}{d} = \frac{R^2(A-AR)}{d}$

$\therefore$ clearly $p-q,\ q-r,\ r-s \longrightarrow$ G.P.

Ans: B

09. $f(x), f(2x), f(4x) \rightarrow G.P.$ (5)

$2x+1,\ 2(2x)+1,\ 2(4x)+1 \rightarrow G.P$

$2x+1,\ 4x+1,\ 8x+1 \rightarrow G.P.$

$\Rightarrow (4x+1)^2 = (2x+1)(8x+1)$

$\Rightarrow x = 0$

Ans: C

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10. $\frac{t_4}{t_6} = \frac{1}{4} \Rightarrow \frac{ar^3}{ar^5} = \frac{1}{4} \Rightarrow r = \pm 2$

$t_2 + t_5 = 216 \Rightarrow ar + ar^4 = 216$

$\Rightarrow a(2+16) = 216$

$\Rightarrow a = 12$

Ans: A

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11. $p, 1, \frac{1}{p}, \frac{1}{p^2}, \ldots$

$S_\infty = \frac{a}{1-r}$

$\Rightarrow \frac{9}{2} = \frac{p}{1-\frac{1}{p}}$

$\Rightarrow \frac{9}{2} = \frac{p^2}{p-1}$

$\Rightarrow 9p - 9 = 2p^2$

$\Rightarrow 2p^2 - 9p + 9 = 0$

$p = 3, \frac{3}{2}$

Ans: B, C

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12. $\cancel{3a_1 + 7a_2 + 3a_3 - 4a_5 = 0}$

$\cancel{3a + 7ar + 3ar^2 - 4ar^4 = 0}$

$\cancel{\Rightarrow 3a + 7ar + 3r^2 - 4r^4 = 0}$

$\cancel{\Rightarrow 3 + 7r + 3r^2 - 4r^4 = 0}$

$\cancel{\Rightarrow 4r^4 - 3r^2 - 7r - 3 = 0}$

12 $\cancel{3a_1 + 7a_2 + 3a_3 - 4a_5 = 0}$

$\cancel{\Rightarrow 7a_1 + 7a_2 + 7a_3 = 4a_1 + 4a_3 + 4a_5}$

$\cancel{\Rightarrow 7(a + ar + ar^2) = 4(a + ar^2 + ar^4)}$

$\cancel{\Rightarrow 7(1 + r + r^2) = 4(1 + r^2 + r^4)}$

$\Rightarrow$

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12 $3a_1 + 7a_2 + 3a_3 - 4a_5 = 0$

$\Rightarrow 3a + 7(ar) + 3(ar^2) - 4(ar^4) = 0$

$\Rightarrow 3 + 7r + 3r^2 - 4r^4 = 0$

$\Rightarrow 4r^4 - 3r^2 - 7r - 3 = 0$

By trial & Error method $r = \frac{3}{2}, \frac{-1}{2}$ are the solutions.

Ans: B, D

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13 $ar^{m+n-1} = p \rightarrow (1)$; $ar^{m-n-1} = q \rightarrow (2)$

(1)×(2) $\Rightarrow a^2 \cdot r^{m+\cancel{n}-1+m-\cancel{n}-1} = pq$

$\Rightarrow a^2 \cdot r^{2m-2} = pq$

$\Rightarrow (ar^{m-1}) \Rightarrow (ar^{m-1})^2 = pq$

$\Rightarrow ar^{m-1} = \sqrt{pq}$

$\Rightarrow a_m = \sqrt{pq}$

Statement II: Conceptual (true).

Ans: A

14. Statement I:

$a \times ar \times ar^2 \times \dots \times ar^{n-1} =$

$= a^n \cdot r^{1+2+\dots+n-1}$

$= a^n \cdot r^{\frac{(n-1)(n)}{2}} = \left(a^2 \cdot r^{n-1}\right)^{\frac{n}{2}}$ (True)

Statement II: In a G.P. ith term from the begining $= ar^{i-1}$.

ith term from the end $= \frac{l}{r^{i-1}}$

product $= a\cancel{r^{i-1}} \times \frac{l}{\cancel{r^{i-1}}} = al$, which is independent of i

Ans: A

15 $1, -\frac{1}{2}, \frac{1}{4}, \frac{-1}{8}, \dots$

$a=1,\ r=\frac{-1}{2},\ n=20$ here $r<1$

$\therefore S_n = \frac{a(1-r^n)}{1-r} = \frac{1\left(1-\left(\frac{-1}{2}\right)^{20}\right)}{1-\left(-\frac{1}{2}\right)} = \frac{2}{3}\left(1-\left(\frac{1}{2}\right)^{20}\right)$

Ans: C

16. $3, 3, 3, \dots$ 30 term

$S_{30} = na$

$= 30 \times 3$

$= 90$.

Ans: D

17. $G.M = \sqrt{(-36)(-25)}$

$= \sqrt{-1} \times \sqrt{-1} \times \sqrt{36 \times 25}$

$= i \times i \times 6 \times 5$

$= -30$ since $i^2 = -1$

Ans: B

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18 $G.M = \sqrt{\frac{1}{2} \cdot \frac{1}{8}}$

$= \pm \frac{1}{4}$

Ans: C

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19 $\sum_{i=1}^{201} a_i = 625$

$\Rightarrow a_1 + a_2 + \cdots + a_{201} = 625$

$\Rightarrow \frac{a(1-r^{201})}{1-r} = 625 \longrightarrow (1)$

Now $\sum_{i=1}^{201} \frac{1}{a_i} = \frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_{201}} =$

$= \frac{\frac{1}{a}\left(\left(\frac{1}{r}\right)^{201} - 1\right)}{\frac{1}{r} - 1}$

$= \frac{1}{a}\left(\frac{1-r^{201}}{1-r}\right) \cdot \frac{1}{r^{200}}$

$= \frac{1}{a} \cdot \frac{625}{a} \cdot \frac{1}{r^{200}}$

$= \frac{625}{(a \cdot r^{100})^2} = \frac{625}{(25)^2} = 1$

Since $ar^{100} = 25$

Ans: 1

20 $\frac{a}{r}, a, ar \rightarrow G.P.$

(9)

$\Rightarrow$ product $\Rightarrow \frac{a}{r} \cdot a \cdot ar = -216$

$\Rightarrow a^3 = -216$

$\Rightarrow a^3 = (-6)^3$

$\Rightarrow \boxed{a = -6}$

$\Rightarrow \cancel{a}\left(\frac{1}{r} + 1 + r\right) = -\cancel{a} \Rightarrow \frac{1}{r} + 1 + r = -1$

$\Rightarrow$ ~~&~~ Again $\frac{a}{r} \cdot a + a \cdot ar + \frac{a}{r} \cdot ar = b$

$\Rightarrow a^2\left(\frac{1}{r} + r + 1\right) = b$

$\Rightarrow 36(-1) = b \Rightarrow \boxed{b = -36}$

$\therefore \frac{b}{a} = 6$

21 a) a, b, c AP $\Rightarrow 2b = a + c$

a, x, b G.P $\Rightarrow x^2 = ab$

b, y, c G.P $\Rightarrow y^2 = bc$

Now $x^2 + y^2 = ab + bc$

$= b(a+c)$

$= b(2b) = 2b^2$

$\therefore x^2, b^2, y^2$ are in ~~AP~~.

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b) x, y, z G.P $\Rightarrow y^2 = xz \rightarrow (1)$

$\Rightarrow a^x = b^y = c^z = K$

$\Rightarrow x = \log_a K = \frac{\log K}{\log a}$

$y = \frac{\log K}{\log b}, \quad z = \frac{\log K}{\log c}$

Now (1) $\Rightarrow \left(\frac{\log K}{\log b}\right)^2 = \frac{\log K}{\log a} \cdot \frac{\log K}{\log c}$

$\Rightarrow (\log b)^2 = \log a \cdot \log c$

$\Rightarrow \log a, \log b, \log c \rightarrow$ G.P.

c) $a = 3, \quad ar^{n-1} = 96, \quad \frac{a(r^n - 1)}{r-1} = 189.$

$\Rightarrow r^{n-1} = 32 \qquad \Rightarrow \frac{r^n - 1}{r-1} = 63$

$\Rightarrow r^n = 32r$

$\Rightarrow \frac{32r - 1}{r-1} = 63 \Rightarrow r = 2$

Now $2^n = 32 \times 2 = 64 \Rightarrow n = 6$

d) $S_\infty = \frac{a}{1-r} = 2 \Rightarrow \frac{1}{1-r} = 2 \Rightarrow r = \frac{1}{2}$

Ans: t, r, q, s

22 a) $(\sqrt{2}+1) + 1 + (\sqrt{2}-1) + \cdots \infty$

$$S_\infty = \frac{\sqrt{2}+1}{1-\left(\frac{1}{\sqrt{2}+1}\right)} = \frac{(\sqrt{2}+1)^2}{\sqrt{2}} = \frac{3+2\sqrt{2}}{\sqrt{2}}$$

$$= \frac{3\sqrt{2}+4}{2}$$

b) $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \frac{1}{2^5} + \frac{1}{3^6} + \cdots \infty$

$$= \left(\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \cdots\right) + \left(\frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \cdots \infty\right)$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}} + \frac{\frac{1}{3^2}}{1-\frac{1}{9}}$$

$$= 1 + \frac{1}{8} = \cancel{0} = \frac{9}{8}$$

c) $6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times \cdots =$

$$= 6^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots}$$

$$= 6^{\frac{\frac{1}{2}}{1-\frac{1}{2}}} = 6$$

d) $\frac{a}{1-r} = 20 \rightarrow (1)$

$\frac{a^2}{1-r^2} = 100 \rightarrow \cancel{(2)}$

$\Rightarrow \frac{a}{1-r} \cdot \frac{a}{1+r} = 100$

$\Rightarrow 20 \times \frac{a}{1+r} = 100$

$\Rightarrow \frac{a}{1+r} = 5 \rightarrow (2)$

$\frac{(1)}{(2)} \Rightarrow \frac{1+r}{1-r} = 4$

$\Rightarrow r = \frac{3}{5}$

~~Ans~~: S, ~~R~~, P, Q

LEARNER'S TASK

CUQ'S

01. $t_{n+1} = ar^n$ — Ans: B

02. $r = \frac{t_n}{t_{n-1}} = \frac{t_{n+1}}{t_n} = \frac{t_{n-2}}{t_{n-3}}$ — Ans: D

03. Conceptual (all the terms are in G.P. — Ans: D

04. Conceptual — Ans: A

05. $a, b, c \rightarrow$ G.P

$\Rightarrow b^2 = ac, \quad \frac{b}{a} = \frac{c}{b}, \quad \frac{a}{b} = \frac{b}{c}$ — Ans: D

06. Conceptual — Ans: D

~~07~~. $\cancel{S_n = \frac{a - lr}{1-r}}$ — Ans: B

~~08~~

07. $S_n = \frac{a(1-r^n)}{1-r}$ — Ans: B

08. $S_n = \frac{a - lr}{1-r}$ — Ans: B

09. $S_\infty = \frac{a}{1-r}$ — Ans: A

10. $S_n = na$ — Ans: A

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JEE MAINS LEVEL

B

01. $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2 \rightarrow$ 5 terms of G.P.

$a = 4$, product $= \frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 =$

$= a^5$

$= 4^5$

$= 1024$

Ans. A

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02. $\frac{a}{r}, a, ar \rightarrow$ G.P

$\frac{a}{r}, 2a, ar \rightarrow$ A.P

$\Rightarrow 2 \times 2a = \frac{a}{r} + ar$

$\Rightarrow 4 = \frac{1}{r} + r$

$\Rightarrow 4r = 1 + r^2$

$\Rightarrow r^2 - 4r + 1 = 0$

$r = 2 \pm \sqrt{3}$

Ans: A, B

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03 $\frac{a}{r} + a + ar = 70 \rightarrow (1)$

$\frac{4a}{r}, 5a, 4ar \rightarrow$ A.P.

$\Rightarrow 10a = \frac{4a}{r} + 4ar$

$\Rightarrow 10 = \frac{4}{r} + 4r$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow \boxed{r = \frac{1}{2}, \cancel{2}}$

sub in (1) $\Rightarrow \boxed{a = 20}$

Sum

$\frac{4a}{r} + 5a + 4ar$

$\frac{4(20)}{2} + 5(20) + 4(20)($

$= 40 + 100 + 160$

$= 300.$

04. $3, \cancel{ar}, —, 81$, G.P.

option verification.

let $9, 27$ be the numbers

hence, $3, 9, 27, 81$ are G.P.

Ans: D

05 $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$.

let $n = -\frac{1}{2}$

$= \dfrac{a^{-\frac{1}{2}+1}+b^{-\frac{1}{2}+1}}{a^{-\frac{1}{2}}+b^{-\frac{1}{2}}}$

$= \dfrac{\sqrt{a}+\sqrt{b}}{\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)}$

$= \sqrt{ab}$, which is G.M between a and b

Ans: D

06 $ar^5 = \frac{-1}{32} \to ①$, $ar^8 = \frac{1}{256} \to ②$

$\frac{②}{①} \Rightarrow \dfrac{ar^8}{ar^5} = \dfrac{\left(\frac{1}{256}\right)}{\left(\frac{-1}{32}\right)} = \frac{-1}{8}$

$\Rightarrow r^3 = \left(\frac{-1}{2}\right)^3 \Rightarrow \boxed{r = \frac{-1}{2}}$

$\cancel{a_1} = \cancel{A}$ also $ar^5 = \frac{-1}{32}$

$\Rightarrow a\left(\frac{-1}{2}\right)^5 = \frac{-1}{32}$

$\Rightarrow \boxed{a = 1}$

Now $a_{11} = ar^{10}$

$= 1 \cdot \left(\frac{-1}{2}\right)^{10}$

$= \frac{1}{1024}$

Ans: B

07. Sum of three numbers $= 26$

i.e. $a_1 + a_2 + a_3 = 26$

Also $a_1 a_2 + a_2 a_3 + a_1 a_3 = 156$

option verification. let us take the numbers $2, 6, 18$

Now, $2 + 6 + 18 = 26$

Also $2 \cdot 6 + 6 \cdot 18 + 18 \cdot 2 = 156$ Ans: A

~~08 Given~~ $\cancel{\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \text{up to } \infty = \frac{\pi^2}{6}}$

~~Now,~~ $\cancel{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \text{up to } \infty}$

$\cancel{= \frac{1}{1^2} + 1}$

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08 $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \text{up to } \infty = \frac{\pi^2}{6}$

Now

$$S = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

$$= \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \infty\right) - \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots \infty\right)$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \infty\right)$$

$$= \frac{\pi^2}{6} - \frac{1}{4} \times \left(\frac{\pi^2}{6}\right)$$

$$= \frac{\pi^2}{6}\left(1 - \frac{1}{4}\right)$$

$$= \frac{\pi^2}{\cancel{6}_2} \times \frac{\cancel{3}^1}{4} = \frac{\pi^2}{8}$$

Ans: C

09. $S_\infty = 4$, $ar = \frac{3}{4}$

$\frac{a}{1-r} = 4$

$\Rightarrow \frac{ar}{r-r^2} = 4$

$\Rightarrow \frac{\left(\frac{3}{4}\right)}{r-r^2} = 4$

$\frac{3}{r-r^2} = 16$

$\Rightarrow 16r - 16r^2 = 3$

$\Rightarrow 16r^2 - 16r + 3 = 0$

$\Rightarrow 16r^2 - 12r - 4r + 3 = 0$

$\Rightarrow 4r(4r-3) - 1(4r-3) = 0$

$\Rightarrow (4r-3)(4r-1) = 0$

$\Rightarrow r = \frac{3}{4}, \boxed{\frac{1}{4}}$

Now $ar = \frac{3}{4}$

$\Rightarrow a \cdot \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \boxed{a = 3}$

Ans: D

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10. $a + ar + ar^2, \ldots = 20$

$\frac{a}{1-r} = 20$

$\Rightarrow r = \frac{20-a}{20}$

Now, $a^2, a^2r^2, a^2r^4, \ldots \infty$

$S_\infty = \frac{a^2}{1-r^2} = 100$

$\Rightarrow a^2 = 100\left(1 - \left(\frac{20-a}{20}\right)^2\right)$

$\Rightarrow a^2 = 100\left(\frac{\cancel{400} - \cancel{400} + 40a - a^2}{400}\right)$

$\Rightarrow 4a^2 = 40a - a^2$

$\Rightarrow 5a^2 = 400 \Rightarrow a = 8$

Ans: D

11. $a = x^{-4}, \quad ar = x^{n}, \quad ar^{7} = x^{52}$

$ar = x^{n}$

$\Rightarrow x^{-4} r = x^{n}$

$\Rightarrow r = x^{n+4}$

Now $ar^{7} = x^{52}$

$\Rightarrow x^{-4} \cdot (x^{n+4})^{7} = x^{52}$

$\Rightarrow x^{-4} \cdot x^{7n+28} = x^{52}$

$\Rightarrow x^{7n+24} = x^{52}$

$\Rightarrow 7n+24 = 52$

$\Rightarrow n = 4$ or $\frac{8}{2}$

Ans: B, C

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12. $1+3+9+ \dots$

$a = 1, \quad r = 3$

$$\frac{1(3^{n}-1)}{3-1} = 364$$

$\Rightarrow 3^{n}-1 = 728$

$\Rightarrow 3^{n} = 729$

$\Rightarrow n = 6$ or $\frac{12}{2}$

Ans: A, B

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13. Statement I: a, b, c are in G.P.

Let $a = 1, \ b = 2, \ c = 4.$

Now $\frac{b-a}{b-c} + \frac{b+a}{b+c} = \frac{2-1}{2-4} + \frac{2+1}{2+4}$

$= \frac{1}{-2} + \frac{3}{6} = 0.$ (true)

Statement II: $S_n = \frac{a(1-r^{n})}{1-r}$ when $r<1$ (False)

Ans: C

14. Statement I: $6, x, y, z, 54$ are in G.P.

$a=6, \; ar^4 = 54$

$\Rightarrow 6.r^4 = 54$

$\Rightarrow r^4 = 9$

$\Rightarrow r = (3^2)^{\frac{1}{4}} = \sqrt{3}$

Now $z = ar^3$

$= 6.(\sqrt{3})^3 = 6 \times 3\sqrt{3} = 18\sqrt{3}$ (True)

Ans: A

Statement II: Conceptual (True)

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15 $S_\infty = \frac{a}{1-r}$

$= \frac{1}{1-\frac{1}{2}} = 2$

Ans: C

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16. $S_\infty = \frac{1}{3}$

$\Rightarrow \frac{a}{1-r} = \frac{1}{3}$

$\Rightarrow \frac{a}{1+\frac{1}{2}} = \frac{1}{3}$

$\Rightarrow \frac{2a}{3} = \frac{1}{3}$

$\Rightarrow a = \frac{1}{2}$

Ans: C

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17 $S_n = \frac{lr-a}{r-1} = \frac{32.2-2}{2-1} = 62$

Ans: C

18 $S_n = \frac{13}{27}$

$\Rightarrow \frac{a - l r}{1-r} = \frac{13}{27}$

$\Rightarrow \frac{a - \frac{1}{27}\cdot\frac{1}{3}}{1-\frac{1}{3}} = \frac{13}{27}$

$\Rightarrow a = \frac{1}{3}$ Ans: B

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19 $a, ar, ar^2, \dots ar^{n-1}$

Given $t_n = 2(t_{n+1} + t_{n+2} + \dots)$

$t_n = 2(ar^n + ar^{n+1} + \dots)$

$\Rightarrow t_n = 2ar^n(1 + r + r^2 + \dots \infty)$

$\Rightarrow t_n = 2ar^n \times \frac{1}{1-r}$

$\Rightarrow ar^{n-1} = \frac{2ar^n}{1-r}$

$\Rightarrow \frac{r}{1-r} = \frac{1}{2}$

$\Rightarrow 2r = 1-r$

$\Rightarrow 3r = 1$

$\Rightarrow r = \frac{1}{3}$

$\Rightarrow r = 0.33$

$\Rightarrow$ Integral part $= 0$. Ans: 0

20. $\frac{a+b}{2} = 13$ & $\sqrt{ab} = 12$

$\Rightarrow a+b = 26$ & $ab = 144$

Now $(a-b)^2 = (a+b)^2 - 4ab$

$= (26)^2 - 4 \times 144$

$= 100$

$\Rightarrow a-b = 10.$

Ans: 10

21 a) $t_n = ar^{n-1}$

b) $r = \frac{t_n}{t_{n-1}}$

c) $S_\infty = \frac{a}{1-r}$

d) $S_n = \frac{a(r^n - 1)}{r-1}$

Ans: t, r, a, s

22 a) $ar^3 = x$, $ar^9 = y$, $ar^{15} = z$ (21)

$y^2 = (ar^9)^2 = a^2 \cdot r^{18}$

$xz = ar^3 \cdot ar^{15} = a^2 \cdot r^{18}$

$\therefore$ $y^2 = xz \Rightarrow x, y, z$ are in G.P.

b) $2+x$, $14+x$, $62+x \longrightarrow$ G.P

$\Rightarrow (14+x)^2 = (2+x)(62+x)$

$\Rightarrow x = 2$

c) $x, 2y, 3z$ are in AP

$\Rightarrow 4y = x + 3z \longrightarrow (1)$

x, y, z are in G.P

$\Rightarrow y^2 = xz$

Now, $\frac{y}{x} = \frac{z}{y} = r$ Also $\frac{z}{x} = r^2$

Now $4 \cdot \frac{y}{x} = \frac{x}{x} + 3 \cdot \frac{z}{x}$

$\Rightarrow 4r = 1 + 3r^2$

$\Rightarrow 3r^2 - 4r + 1 = 0$

$r = \frac{1}{3}$ or 1

d) let $a=1$, $b=2$, $c=4$, $d=8$ are in G.P

$a^n + b^n$, $b^n + c^n$, $c^n + d^n$

let $n=1$ $a+b$, $b+c$, $c+d$

$1+2$, $2+4$, $4+8$

$3, 6, 12 \longrightarrow$ G.P

Ans: S, P, T, S

$\Rightarrow$ THE END $\Leftarrow$

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