INVERSE TRIGONOMETRIC FUNCTIONS

Class: IX, Mathematics.

I.I.T. ADVANCED.

SOLUTIONS

TEACHING TASK.

01. $\sin^{-1}x - \cos^{-1}x$

$= \frac{\pi}{2} - \cos^{-1}x - \cos^{-1}x$ Since $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$= \frac{\pi}{2} - 2\cos^{-1}x$

We know, $0 \le \cos^{-1}x \le \pi$

$0 \ge -2\cos^{-1}x \ge -2\pi$

$\frac{\pi}{2} \ge \frac{\pi}{2} - 2\cos^{-1}x \ge \frac{\pi}{2} - 2\pi$

$\therefore -\frac{3\pi}{2} \le f(x) \le \frac{\pi}{2}$

Hence, Range $= \left[-\frac{3\pi}{2}, \frac{\pi}{2}\right]$ Ans. A

02. domain of $\tan^{-1}(5x)$ is $\mathbb{R}$ Ans. A

03. $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$

$= \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$

$= \sin^{-1}\left[\cos\frac{3\pi}{5}\right]$

$= \sin^{-1}\left[\sin\left(-\frac{\pi}{10}\right)\right]$

$= -\frac{\pi}{10}$

Ans: D

$$\cos^{-1}\left(\cos\frac{10\pi}{7}\right)+\sin^{-1}\left(\sin\frac{35\pi}{7}\right)+\tan^{-1}\left(\tan\frac{24\pi}{13}\right)+\cot^{-1}\left(\cot\frac{26\pi}{5}\right) \quad (2)$$

$$\cancel{=\cos^{-1}\left[\cos\frac{10\pi}{7}\right]=\cos^{-1}\left[\cos\left(2\pi-\frac{4\pi}{7}\right)\right]}$$

$$=$$

$$\cos^{-1}\left[\cos\left(2\pi-\frac{4\pi}{7}\right)\right]+\sin^{-1}\left[\sin\left(3\pi+\frac{2\pi}{11}\right)\right]+\tan^{-1}\left(\tan\left(2\pi-\frac{2\pi}{13}\right)\right]+\cot^{-1}\left[\cot\left(5\pi+\frac{\pi}{5}\right)\right]$$

$$=\frac{4\pi}{7}-\frac{2\pi}{11}+\frac{2\pi}{13}+\frac{\pi}{5}$$

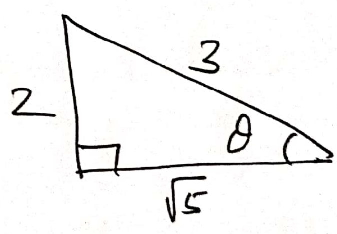
$$=\frac{4\pi}{7}-\frac{2\pi}{11}+\frac{\pi}{5}+\frac{2\pi}{13}$$

$$=\frac{4\pi}{7}-\left(\frac{2\pi}{11}-\frac{\pi}{5}\right)+\frac{2\pi}{13}$$

$$=\frac{4\pi}{7}+\frac{\pi}{55}+\frac{2\pi}{13}$$

Ans: A

$$\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$$



Let $\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)=\theta$

$$\Rightarrow \cos\theta=\frac{\sqrt{5}}{3}$$

$$\therefore \tan\left(\frac{\theta}{2}\right)=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\sqrt{\frac{1-\frac{\sqrt{5}}{3}}{1+\frac{\sqrt{5}}{3}}}$$

$$=\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}=\sqrt{\frac{(3-\sqrt{5})^2}{4}}$$

$$=\frac{3-\sqrt{5}}{2}$$

opt: C

3

06 $\tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{24}{25}\right) = \cos^{-1} x$

$\Rightarrow$

07. $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{x+1+x-1}{1-(x+1)(x-1)}\right) = \tan^{-1}\left(\frac{8}{31}\right)$$

$$\Rightarrow \frac{2x}{1-x^2+1} = \frac{8}{31}$$

$$\Rightarrow \frac{x}{2-x^2} = \frac{4}{31}$$

$\Rightarrow$ $x = \frac{1}{4}$ satisfies the above equations

Ans. D

08 $\tan^{-1}\frac{5}{6} + \frac{1}{2}\cdot\tan^{-1}\frac{11}{60}$

$$= \frac{1}{2}\left[2\tan^{-1}\frac{5}{6} + \tan^{-1}\frac{11}{60}\right]$$

$$= \frac{1}{2}\left[\tan^{-1}\frac{60}{11} + \tan^{-1}\frac{11}{60}\right]$$

$$= \frac{1}{2}\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi}{4}$$

Ans. C

09

Scanned with OKEN Scanner

09 $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \frac{5\pi}{12} \longrightarrow ①$ ④

$\sin^{-1}\frac{x}{a} - \sin^{-1}\frac{y}{b} = \frac{\pi}{12}$

$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1}\frac{x}{a}\right) - \left(\frac{\pi}{2} - \cos^{-1}\frac{y}{b}\right) = \frac{\pi}{12}$

$\Rightarrow \cos^{-1}\frac{y}{b} - \cos^{-1}\frac{x}{a} = \frac{\pi}{12} \longrightarrow ②$

$① + ② \Rightarrow \cos^{-1}\frac{y}{b} = \frac{\pi}{4} \Rightarrow \frac{y}{b} = \frac{1}{\sqrt{2}}$

$① - ② \Rightarrow \cos^{-1}\frac{x}{a} = \frac{\pi}{6} \Rightarrow \frac{x}{a} = \frac{\sqrt{3}}{2}$

Now $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{5}{4}$ Ans: D

---

10 put $x = y = z = \frac{1}{\sqrt{3}}$

these values satisfies the equation

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$ also

$(x-y)^2 + (y-z)^2 + (z-x)^2 = 0$

hence $x^2 + y^2 + z^2 = \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2$

$= 1$ Ans: C

---

11. $\tan^{-1}\left(\dfrac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right) = \frac{\pi}{4}$

$\Rightarrow \dfrac{(x-1)(x+2) + (x+1)(x-2)}{(x-2)(x+2) - (x-1)(x+1)} = 1$

$\Rightarrow$ upon solving this equation we get

$x = \pm\frac{1}{\sqrt{2}}$

Ans: A, C

Scanned with OKEN Scanner

12. $\cot^{-1}\left(\frac{x}{x+1}\right)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}3x-\tan^{-1}x$ (5)

$\Rightarrow \tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}3x-\tan^{-1}x$

$\Rightarrow \tan^{-1}\left(\frac{x+1+x-1}{1-(x+1)(x-1)}\right)=\tan^{-1}\left(\frac{3x-x}{1+3x\cdot x}\right)$

$\Rightarrow \frac{2x}{2-x^2}=\frac{2x}{1+3x^2}$

upon solving this epuation, we get

$x=\pm\frac{1}{2}$ Ans. B, C

---

13 **Statement I:**

$\cos^{-1}x+\cos^{-1}\left(\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)$

$=\cos^{-1}x+\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)-\cos^{-1}x$

$=\frac{\pi}{4}$ (True)

Statement II: Conceptual (True) Ans: A

---

14 **Statement I:**

$\cos^{-1}(\cos 12)-\sin^{-1}(\sin 12)$

here 12 means 12 radians

clearly $\frac{7\pi}{2}<12<4\pi$

$\cos^{-1}12=4\pi-12$ and $\sin^{-1}12=12-4\pi$

hence $\cos^{-1}12-\sin^{-1}12=(4\pi-12)-(12-4\pi)=8\pi-24$ (True)

Statement II: Conceptual (True)

Ans: A