

WS-8
8th foundation
elastic collision
Thank

①

Given $m_1 = 25 \text{ kg} \rightarrow u_1 = 40 \text{ m/s}$

$m_2 = 15 \text{ kg} \rightarrow u_2 = 0$

After collision both are moving with same velocity

$$v_1 = v_2 = v_{\text{com}}$$

According to law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 25 \times 40 + 15(0) = 25 \times v_{\text{com}} + 15 v_{\text{com}}$$

$$\Rightarrow 25 \times 40 = 40 v_{\text{com}}$$

$$\Rightarrow v_{\text{com}} = 25 \text{ m/s} \rightarrow A$$

②

$m_1 = 20 \text{ gm} \rightarrow u_1 = 20 \text{ m/s}$

$m_2 \rightarrow u_2 = 5 \text{ cm/s}$

After collision both are moving with same velocity

$$v_1 = v_2 = v_{\text{com}} = 10 \text{ cm/s}$$

According to law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 20 \times 20 + m_2 \times 5 = 20 v_{\text{com}} \times 10 + m_2 \times 10$$

$$\Rightarrow 400 + 5m_2 = 200 + 10m_2$$

$$\Rightarrow 200 = 5m_2$$

$$\Rightarrow m_2 = 40 \text{ gm}$$

(2)

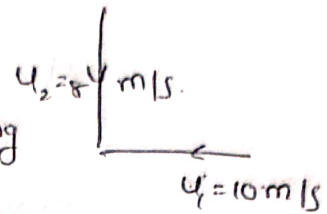
(3)

Given

$$m_1 = 3 \text{ kg} ; m_2 = 5 \text{ kg}$$

$$u_1 = 10 \text{ m/s} ; u_2 = 8 \text{ m/s}$$

After collision both are moving with same velocity $v_1 = v_2 = v_{\text{com}}$.



According to law of conservation of linear momentum

Momentum Before collision = momentum

After collision $\rightarrow$ (1)

$$\begin{aligned} \text{Momentum Before collision} &= \sqrt{p_1^2 + p_2^2} = \sqrt{(m_1 u_1)^2 + (m_2 u_2)^2} \\ &= \sqrt{(3 \times 10)^2 + (5 \times 8)^2} = \sqrt{(30)^2 + (40)^2} \\ &= 50 \text{ kg m/s} \end{aligned}$$

From (1)

$$50 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 50 = 3 v_{\text{com}} + 5 v_{\text{com}}$$

$$\Rightarrow 50 = 8 v_{\text{com}} \Rightarrow v_{\text{com}} = \frac{50}{8} = 6.25 \text{ m/s}$$

(4)

Given $u_1 = 40 \text{ m/s} ; u_2 = 0$

After collision both bodies move with same velocity

$$v_1 = v_2 = v_{\text{com}} = 30 \text{ m/s}$$

Acc to law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow m_1 \times 40 + m_2 (0) = m_1 \times 30 + m_2 \times 30$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{30}{10} = \frac{3}{1}$$

$$\Rightarrow 40 m_1 = 30 m_1 + 30 m_2$$

$$\Rightarrow 10 m_1 = 30 m_2$$



⑤

Given $m_1 = 5 \text{ kg}$; $u_1 = 2\hat{i} \text{ m/s}$

$m_2 = 10 \text{ kg}$; $u_2 = \sqrt{3}\hat{j} \text{ m/s}$

After collision both are moving with same velocity

$v_1 = v_2 = v_{\text{com}}$

Momentum Before collision = $m_1 u_1 + m_2 u_2$

$= 5(2\hat{i}) + 10(\sqrt{3}\hat{j})$

$= 10\hat{i} + 10\sqrt{3}\hat{j}$

$|P_{\text{Before}}| = \sqrt{10^2 + (10\sqrt{3})^2} = 10\sqrt{1+3} = 20 \text{ kg m/s}$

According to law of conservation of linear momentum

$\Rightarrow m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$

$\Rightarrow 5v_{\text{com}} + 10v_{\text{com}} = 20$

$\Rightarrow 15v_{\text{com}} = 20 \Rightarrow v_{\text{com}} = \frac{20}{15} = \frac{4}{3} \text{ m/s} \rightarrow c$

⑥

Given

$m_1 = 2 \text{ kg} \rightarrow u_1 = \hat{i} + 2\hat{j} - 3\hat{k} \text{ m/s}$

$m_2 = 3 \text{ kg} \rightarrow u_2 = 2\hat{i} + \hat{j} + \hat{k} \text{ m/s}$

After collision both bodies are moving with same velocity

$v_1 = v_2 = v_{\text{com}}$

According to law of conservation of linear momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow 2(\hat{i} + 2\hat{j} - 3\hat{k}) + 3(2\hat{i} + \hat{j} + \hat{k}) = 2v_{\text{com}} + 3v_{\text{com}}$

$\Rightarrow 2\hat{i} + 4\hat{j} - 6\hat{k} + 6\hat{i} + 3\hat{j} + 3\hat{k} = 5v_{\text{com}}$

$\Rightarrow 8\hat{i} + 7\hat{j} - 3\hat{k} = 5v_{\text{com}}$

$\Rightarrow v_{\text{com}} = \frac{1}{5} [8\hat{i} + 7\hat{j} - 3\hat{k}] \rightarrow A$



(7)

let mass of 2nd body is m_2

its velocity is $u_2 = 0$.

Given $m_1 = 6 \text{ kg}$ let its velocity be u_1

and after collision 1st body moves with a velocity

$$v_1 = \frac{u_1}{2}$$

$$\therefore \text{we know } v_1 = \left[\frac{m_1 - m_2}{m_1 + m_2} \right] u_1 + \frac{2m_2 u_2}{m_1 + m_2}$$

$$\Rightarrow \frac{u_1}{2} = \left[\frac{6 - m_2}{6 + m_2} \right] u_1 + \frac{2m_2(0)}{6 + m_2}$$

$$\Rightarrow \frac{1}{2} = \frac{6 - m_2}{6 + m_2}$$

$$\Rightarrow 6 + m_2 = 12 - 2m_2$$

$$\Rightarrow 3m_2 = 6 \Rightarrow m_2 = 2 \text{ kg}$$

(8)

Given $m_1 = 3 \text{ kg}$; $u_1 = 4 \text{ m/s}$

$m_2 = 4 \text{ kg}$ $u_2 = -2 \text{ m/s}$

After collision both bodies move with same velocity

Initial momentum of the body $= m_1 u_1 + m_2 u_2$

$$= 3 \times 4 + 4(-2)$$

$$= 12 - 8$$

$$= 4 \text{ kg m/s}$$

(9)

Given

$$m_1 = m \rightarrow \text{initial velocity} = u_1 = u$$

$$m_2 = 2m \rightarrow \text{initial velocity } u_2 = 0$$

velocity of 1st body after collision

$$u_1 = \left[\frac{(m_2 - m_1)u_1 + 2m_2 u_2}{m_1 + m_2} \right]$$

$$\Rightarrow u_1 = \frac{(2m - m)u}{2m + m} = \frac{m}{3m} u = \frac{1}{3} u$$

loss of kinetic energy of colliding particles

$$\Delta K.E = \frac{1}{2} m (u_1^2 - u^2)$$

$$= \frac{1}{2} m \left(\left(\frac{1}{3}u\right)^2 - u^2 \right)$$

$$= -\frac{1}{2} m u^2 \left(\frac{8}{9} \right)$$

$$\therefore \text{Fractional loss} = \frac{\Delta K.E}{K.E_{\text{original}}} = \frac{-\frac{1}{2} m u^2 \left(\frac{8}{9} \right)}{\frac{1}{2} m u^2} = \frac{8}{9}$$

've' sign shows loss of K.E

(10)

$$\text{Given } m_{\text{bullet}} = 4 \times 10^{-2} \text{ kg} \rightarrow \text{initial speed } u_1 = 802 \text{ m/s}$$

$$m_{\text{block}} = 15 \text{ kg} \rightarrow \text{initial speed } u_2 = 0$$

velocity of bullet + block after collision $u_1 = u_2 = 2 \text{ m/s}$

$$\therefore \text{Loss of K.E} = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - \left(\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \right)$$

$$\Rightarrow \frac{1}{2} m_{\text{bullet}} v_{\text{cm}}^2 + \frac{1}{2} m_{\text{block}} v_{\text{cm}}^2 - \frac{1}{2} m_{\text{bullet}} u_1^2 -$$

$$\frac{1}{2} m_{\text{block}} u_2^2$$

$$\begin{aligned}
 \Delta K.E &= \frac{1}{2} v_{cm}^2 (15 + 0.04) - \frac{1}{2} [4 \times 10^{-2} + (802)^2 + 15(0)] \\
 &= \frac{1}{2} (2)^2 [15.04] - \frac{1}{2} \times 4 \times 10^{-2} \times (802)^2 \\
 &= 2 \times 15.04 - 2 \times 10^{-2} \times 643204 \\
 &= 30.08 - 12864.08 \\
 &= 12834 \text{ mJ}
 \end{aligned}$$

Advanced level

9, 10

Given $M = 5 \text{ kg} \rightarrow \text{initial velocity} = 0$

$m_1 = 3 \text{ kg} \rightarrow \text{let its velocity be } v_1$

$m_2 = 2 \text{ kg} \rightarrow \text{its velocity } v_2 = 60 \text{ m/s}$

According to law of conservation of linear momentum

$$M u = m_1 v_1 + m_2 v_2$$

$$5(0) = 3(v_1) + 2(60)$$

$$3v_1 = -2 \times 60 \Rightarrow v_1 = -2 \times \frac{60}{3} = -40 \text{ m/s}$$

Kinetic energy of 2 kg piece is

$$K.E_2 = \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} \times 2 \times (60)^2$$

$$= (60)^2$$

$$= 3600 \text{ J}$$

$1)^2 + 15(10)]$ 5

we know velocities of two bodies after collision

$$u_1 = \frac{(m_1 - m_2)u_1 + 2m_2 u_2}{m_1 + m_2}$$

$$u_2 = \frac{2m_1 u_1 + (m_2 - m_1)u_2}{m_1 + m_2}$$

Given $m_2 \gg m_1$; $u_1 = 0$

$$\therefore u_1 = \frac{2m_2 u_2}{m_2} = 2u_2 \quad m_1 \text{ can be neglected}$$

$$u_2 = \frac{+m_1 u_2}{m_2} = +u_2$$

so lighter body moves with twice the velocity of heavier body and heavier body moves with same velocity in same direction

6

If $m_1 \gg m_2$

m_1 is heavier body, $\therefore u_1 = 0$

we know $u_1 = \frac{(m_1 - m_2)u_1 + 2m_2 u_2}{m_1 + m_2}$ m_2 can be neglected

$$u_2 = \frac{(m_1)0 + \dots}{m_1} \approx 0$$

$$u_2 = \frac{2m_1 u_1 + (m_2 - m_1)u_2}{m_1 + m_2} = 0 + \frac{(-m_1)u_2}{m_1} = -u_2$$

lighter body retraces its path, heavier body does not move.

⑦

Given $m_1 = m_2$ and $u_1 = -u_2$

After collision
$$u_1 = \frac{(m_1 - m_2)u_1 + 2m_2 u_2}{m_1 + m_2}$$

$$\Rightarrow u_1 = \frac{0 \cdot u_1 + 2m_1 u_1}{2m_1}$$

$$u_1 = u_2 = u_1$$

$$u_2 = \frac{(m_2 - m_1)u_2 + 2m_1 u_1}{m_1 + m_2} = \frac{0 \cdot u_2 + 2m_1 u_1}{2m_1}$$

$$\boxed{u_2 = u_1} = u_1$$

They move away with the same velocity in magnitude

Tee main level

①

Given $m_1 = 2 \text{ kg} \rightarrow u_1 = 5 \text{ m/s}$

$m_2 = 3 \text{ kg} \rightarrow u_2 = 0$

After collision both bodies move with same velocity

$$v_1 = v_2 = v_{\text{com}}$$

According to law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 2 \times 5 + 3(0) = 2 v_{\text{com}} + 3 v_{\text{com}}$$

$$\Rightarrow 2 \times 5 = 5 v_{\text{com}}$$

$$\Rightarrow v_{\text{com}} = 2 \text{ m/s}$$

(2)

$$u_1 = 4 \text{ m/s}$$

$$\begin{aligned} \frac{1}{2} \times 8 \times u_2^2 &= 24 \\ u_2^2 &= 6 \\ u_2 &= \sqrt{6} \text{ m/s} \end{aligned}$$

Given

$$m_1 = 2 \text{ kg} \quad ; \quad m_2 = 8 \text{ kg}$$

$$k_1 = 16 \text{ J}$$

$$k_2 = 24 \text{ J}$$

$$\text{we know that } k \cdot E = \frac{p^2}{2m}$$

$$\Rightarrow p = \sqrt{2m k \cdot E}$$

$$\begin{aligned} \text{momentum of first body } p_1 &= \sqrt{2m_1 k \cdot E_1} \\ &= \sqrt{2 \times 2 \times 16} \\ &= 8 \text{ kgm/s} \end{aligned}$$

$$\begin{aligned} \text{momentum of 2nd body } p_2 &= \sqrt{2m_2 k \cdot E_2} \\ &= \sqrt{2 \times 8 \times 24} = 19.56 \text{ kgm/s} \\ &= 8\sqrt{6} \text{ kgm/s} \end{aligned}$$

According to law of conservation of linear momentum

momentum after collision = momentum before collision

$$\begin{aligned} &= p_1 + p_2 = 8 + 19.56 \\ &= 8 + 19.56 = 27.56 \text{ N s} \end{aligned}$$

(3)

(4)

Given

masses of two bodies are m_1, m_2 let their initial velocities $u_1, u_2 = 0$ $v_1 = \frac{2}{3} u_1$; v_2 be the final velocity of 2nd body

According to law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow m_1 u_1 + m_2 (0) = m_1 \left(\frac{2}{3} u_1\right) + m_2 v_2$$

$$\Rightarrow m_1 u_1 - \frac{2m_1}{3} u_1 = m_2 v_2$$

$$\Rightarrow \frac{1}{3} m_1 u_1 = m_2 v_2 \rightarrow (1)$$

According to law of conservation of energy

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\Rightarrow m_1 u_1^2 + m_2 (0)^2 = m_1 \left(\frac{2}{3} u_1\right)^2 + m_2 v_2^2$$

$$\Rightarrow m_1 u_1^2 = \frac{4}{9} m_1 u_1^2 + m_2 v_2^2$$

$$\Rightarrow \frac{5}{9} m_1 u_1^2 = m_2 v_2^2 \quad \text{From (1) } m_2 v_2 = \frac{1}{3} m_1 u_1$$

$$\Rightarrow \frac{5}{9} m_1 u_1^2 = (m_2 v_2) v_2$$

$$\Rightarrow \frac{5}{9} m_1 u_1^2 = \frac{1}{3} m_1 u_1 v_2$$

$$\Rightarrow v_2 = \frac{5}{3} u_1$$

Substitute v_2 in (1) we get

$$\Rightarrow \frac{1}{3} m_1 u_1 = m_2 \frac{5}{3} u_1$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{5}{1} \rightarrow B$$

(4)

Given

mass of bullet $m_1 = 500 \text{ gm} = 0.5 \text{ kg}$ mass of bag $m_2 = 4.5 \text{ kg}$ initial velocity of bullet $u_1 = 200 \text{ m/s}$ initial velocity of bag $u_2 = 0$ After collision both sand and bag move with same velocity $v_1 = v_2 = v$

According to law of conservation of linear momentum

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 0.5 \times 200 + 4.5(0) = 0.5v + 4.5v$$

$$\Rightarrow 100 + 0 = 5v$$

$$\Rightarrow v = \frac{100}{5} = 20 \text{ m/s} \rightarrow C$$

(5)

let $m_1 = 6 \text{ kg} \rightarrow u_2 = 0$ let $u_1 \rightarrow$ initial velocity of first bodylet m_2 be the mass of 2nd body.Given After collision both bodies travel with a velocity $= \frac{1}{3} u_1 = v_1 = v_2$ $\therefore$ According to law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 6 \times u_1 + m_2(0) = 6 \frac{u_1}{3} + m_2 \frac{u_1}{3}$$

5th continuation

$$\Rightarrow 6m_2 u_f = \frac{m_1 u_1}{3} + m_2 \frac{u_1}{3}$$

$$\Rightarrow 6m_2 u_1 - \frac{6m_2 u_1}{3} = m_2 \frac{u_1}{3}$$

$$\Rightarrow \frac{18u_1 - 6u_1}{3} = m_2 \frac{u_1}{3}$$

$$\Rightarrow \frac{12u_1}{3} = m_2 \frac{u_1}{3}$$

$$\Rightarrow m_2 = 12 \text{ kg} \rightarrow C$$

(6)

let $m_1 = 2 \text{ kg}$ be the mass of 1st body

$$u_1 = 40 \text{ m/s}$$

m_2 be the mass of 2nd body which is at rest; $u_2 = 0$

After collision two bodies move with a common velocity

$$v_1 = v_2 = 25 \text{ m/s}$$

According to law of conservation of momentum

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 2 \times 40 + m_2(0) = 2 \times 25 + m_2(25)$$

$$\Rightarrow 80 = 50 + 25m_2 \Rightarrow 25m_2 = 30$$

$$\Rightarrow m_2 = \frac{30}{25} = \frac{6}{5} = 1.2 \text{ kg} \rightarrow C$$

(7)

let $m_1 = 5 \text{ kg} \rightarrow$ initial velocity $u_1 = 10 \text{ m/s}$

$m_2 = 20 \text{ kg} \rightarrow u_2 = 0$. Given After collision 5 kg

comes to rest (i.e.) $v_1 = 0$

and velocity of 20 kg mass is $v_2 = v$.

According to law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 5 \times 10 + 20(0) = 5(0) + 20v$$

$$\Rightarrow 50 = 20v$$

$$\Rightarrow v = \frac{50}{20} = \frac{5}{2} = 2.5 \text{ m/s}$$

⑧

let $m_1 = 6 \text{ kg} \Rightarrow u_1 = 10 \text{ m/s}$

$m_2 = 4 \text{ kg} \rightarrow u_2 = -5 \text{ m/s}$

Given After collision 1st body comes to rest i.e. $v_1 = 0$

According to law of conservation of linear momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 6 \times 10 + 4 \times (-5) = 6(0) + 4(v_2)$$

$$\Rightarrow 60 - 20 = 4v_2$$

$$\Rightarrow 40 = 4v_2 \Rightarrow v_2 = 10 \text{ m/s} \rightarrow D$$

⑨

let $m_1 = 20 \text{ kg} ; u_1 = 20 \text{ m/s}$

$m_2 = 40 \text{ kg} ; u_2 = 10 \text{ m/s}$

Given After collision both bodies move with same velocity

i.e. $v_1 = v_2 = v_{\text{com}}$

According to law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 20 \times 20 + 40 \times 10 = 20v_{\text{com}} + 40v_{\text{com}}$$

$$\Rightarrow 400 + 400 = 60v_{\text{com}}$$

$$\Rightarrow 60 \text{ } V_{com} = \frac{80}{40}$$

$$\Rightarrow V_{com} = \frac{40}{3} \text{ m/s}$$

(10)

$$\text{let } m_1 = 2m \quad ; \quad m_2 = 3m$$

$$u_1 = u \quad u_2 = 0$$

clearly after collision both spheres move with same velocity $v_1 = v_2 = V_{com}$.

According to law of conservation of linear momentum

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 2m u + 3m(0) = 2m V_{com} + 3m V_{com}$$

$$\Rightarrow 2m u = 5m V_{com}$$

$$\Rightarrow V_{com} = \frac{2u}{5}$$

$$\therefore \text{Loss of k.E} = \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right) - \left(\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \right)$$

$$\Rightarrow \frac{1}{2} \left[(2m V_{com}^2 + 3m V_{com}^2) - (2m u^2 + 3m(0)) \right]$$

$$= \frac{1}{2} \left[5m V_{com}^2 - 2m u^2 \right]$$

$$= \frac{1}{2} \left[5m \times \left(\frac{2u}{5} \right)^2 - 2m u^2 \right]$$

$$= \frac{1}{2} \left[5m \times \frac{4u^2}{25} - 2m u^2 \right]$$

$$\Rightarrow \frac{1}{2} \left[\frac{4m u^2}{5} - 2m u^2 \right]$$

$$\Rightarrow \frac{1}{2} \left[-\frac{6m u^2}{5} \right] = -\frac{6}{5} \left[\frac{1}{2} m u^2 \right]$$

% Loss of k.E

$$= \frac{-\frac{6}{5} \left[\frac{1}{2} m u^2 \right]}{\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2} \times 100\%$$

$$= \frac{-\frac{6}{5} \times \frac{1}{2} m u^2}{\frac{1}{2} (2m) u^2} \times 100\%$$



10th continuation

(9)

$$\text{Loss of K.E} = -\frac{6}{8} \times \frac{1000}{2} = -60\%$$

Advanced

(9), (8)

$$\text{let } M_1 = 10 \text{ kg}, u_1 = 10 \text{ m/s}$$

$$m_1 = 6 \text{ kg} \rightarrow v_1 = 18 \text{ m/s}$$

$$m_2 = \text{mass of 2nd piece} = 4 \text{ kg}$$

$$v_2 = ?$$

According to law of conservation of linear momentum

$$Mu = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 10 \times 10 = 6 \times 18 + 4 v_2$$

$$\Rightarrow 100 = 108 + 4 v_2 \Rightarrow 4 v_2 = -8 \text{ m/s}$$

$$\Rightarrow v_2 = -2 \text{ m/s}$$

K.E of 2nd piece after collision

$$K.E_2 = \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} \times 4 \times (-2)^2$$

$$= 2 \times 4 = 8 \text{ J}$$