3. POLYNOMIALS – II (1)

class: IX, MATHEMATICS

INTEGRATED SOLUTIONS

TEACHING TASK
JEE MAINS LEVEL

01) $x^4 - 4x^3 + 4x^2 - 2 \div x - 3$

NOTE: options delete
A) 2009
B) 0
C) 2007
D) 2
delete.

$x=3$	1	−4	4	0	−2
	0	3	−3	3	9
	1	−1	1	3	7

∴ Quotient: $x^3 - x^2 + x + 3$

Remainder: 7

Ans: A

02 $f(x) = ax^4 + 2x^3 - 3x^2 + bx - 4$

$f(2) = 16a + 16 - 12 + 2b - 4 = 0$

$16a + 2b = 0 \Rightarrow b = -8$ $\quad 8a + b = 0 \rightarrow (1)$

$f(-2) = 16a - 16 - 12 - 2b - 4 = 0$

$16a - 2b - 32 = 0$

$8a - b - 16 = 0 \rightarrow (2)$

Solving (1) & (2) $a = 1, \; b = -8$

Ans: A

03 $x^2 - 3x + 2 = (x-1)(x-2)$;

$x^2 + x - 6 = (x+3)(x-2)$

H.C.F. $= x - 2$

Ans: C

04. $x^{12} - 7x^6 + 2001$

$= (x^2)^6 - 7(x^2)^3 + 2001$

$= (2)^6 - 7(2)^3 + 2001$

$= 2009$

Ans. A

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05 $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$a^3+b^3+1^3-3ab.1 = (a+b+1)(a^2+b^2+1^2-ab-b-a)$

$\therefore$ Quotient $= a^2+b^2+1^2-ab-b-a$ Ans: A

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06 $x^6+y^6 = (x^2)^3+(y^2)^3$

$= (x^2+y^2)(x^4-x^2y^2+y^4)$ Ans: B

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07 $y^3-7y+6 \div y-1$

$y=1$	1	0	−7	6
	0	1	1	6
	1	1	−6	0

$\therefore$ Quotient $= y^2+y-6$

$= (y+3)(y-2)$ Ans: B

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08 $x^{100} \div x^2-3x+2$

Let $q(x)$ be the quotient & R be the remainder

$x^{100} = q(x)(x^2-3x+2) + [ax+b]$

$x^{100} = q(x)(x-1)(x-2) + (ax+b)$

put $x=1$ and $x=2$

$1 = a + b \rightarrow (1)$

$2^{100} = 2a + b \rightarrow (2)$

(2) − (1) $\Rightarrow a = (2^{100} - 1)$

(1) $\Rightarrow a + b = 1$

$2^{100} - 1 + b = 1$

$b = 2 - 2^{100}$ (3)

Remainder $= (2^{100} - 1)x + (2 - 2^{100})$

Ans: B

09 $(25x^2 - 1) + (1 + 5x)^2$

$= (5x + 1)(5x - 1) + (1 + 5x)^2$

$= (5x + 1)[5x - 1 + 1 + 5x]$

$= (5x + 1)(10x)$

Ans: D

10 $\Sigma (a+b)(b-c) = (a+b)(b-c) + (b+c)(c-a) + (c+a)(a-b)$

Ans: A

JEE ADVANCED LEVEL QUESTIONS

11. $f(x) = x^3 + x^2 - ax + b$

Divided by $x^2 - x = x(x-1)$

$f(0) = \boxed{b = 0}$

$f(1) = 1 + 1 - a + 0 = 0 \Rightarrow \boxed{a = 2}$

Ans: D

12 $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = p^2 - 2q$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{p}{q}$

Ans: B, D

13 Statement I: (4)

$(a-b-c)(a^2+b^2+c^2+ab-ac-bc)$

$=[a+(-b)+(-c)]\left[a^2+(-b)^2+(-c)^2-(a)(-b)-(-b)(-c)-(-c)a\right]$

$=(a)^3+(-b)^3+(-c)^3-3(a)(-b)(-c)$

$=a^3-b^3-c^3+3abc$ (False)

Ans:

Statement II:

~~$f(x)=ax^3+3x^2-13$~~

~~$f(2)=8a+12-13=0$~~

Conceptual (true)

Ans. D

14. Statement I: Please, change the question.

"If the polynomial ax^3+3x^2-13 is divided by $x-2$, then the value of a is 3

$f(x)=ax^3+3x^2-13$

$f(2)=8a+12-13=0 \Rightarrow a=\frac{1}{8}$ (False)

Ans: D

Statement II: Conceptual (True)

15 $(x+y+2z)(x+y)=\left[\frac{x+y+2z+x+y}{2}\right]^2-\left[\frac{x+y+2z-x-y}{2}\right]^2$

$=(x+y+z)^2-(z)^2$

Ans: A

16. $[(x+2a)(x+8a)][(x+4a)(x+6a)]+7a^4$ (5)

$=[x^2+10ax+16a^2][x^2+10ax+24a^2]+7a^4$

Let $x^2+10ax+16a^2=K$

$\therefore (K)(K+8a^2)+7a^4$

$= K^2+8Ka^2+7a^4$

$=(K+4a^2)^2-9a^4$

$=(x^2+10ax+20a^2)^2-(3a^2)^2$ Ans. A

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17 $p(x)=x^2-5x+K$.

$\alpha+\beta=5$

$\alpha-\beta=1$

$2\alpha=6$

$\Rightarrow \alpha=3$

$3+\beta=5$

$\beta=2$

$\alpha\cdot\beta=K$

$3\cdot 2=K$

$\boxed{K=6}$

Ans. 6

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18 $a^4(b^2-c^2)+b^4(c^2-a^2)+c^4(a^2-b^2)$

$=K(a^2-b^2)(b^2-c^2)(c^2-a^2)$

Let $a=1,\ b=2,\ c=0$

$1.(4-0)+16(0-1)+0(1-4)$

$=K(1-4)(4-0)(0-1)$

$\Rightarrow -12=K(12) \Rightarrow x=-1$

Ans: -1

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19) a) Conceptual (2) b) Conceptual (9)

c) Conceptual (8) d) Conceptual (5)

[9] a) Standard form of homogeneous expression of 3rd degree in x and y (6)

$\rightarrow ax^3 + bx^2y + cxy^2 + dy^3$ (S)

b) Standard form of homogeneous expression of 2nd degree in x, y, z

$\rightarrow ax^2 + by^2 + cz^2 + dxy + eyz + fzx$ (Q)

c) Standard form of symmetric expression of degree 2 in x, y, z

$\rightarrow a(x^2+y^2+z^2) + b(xy+yz+zx) + c(x+y+z) + d$ (P)

d) Homogeneous + symmetric of degree 2 in x, y, z

$\rightarrow a(x^2+y^2+z^2) + b(xy+yz+zx)$ (R)

Ans:- S, Q, P, R

LEARNERS TASK (CUQ's)

01. $(x-y)^3 + (y-z)^3 + (z-x)^3$

$(x-y)+(y-z)+(z-x) = 0$

$\therefore$ Factors are $= 3(x-y)(y-z)(z-x)$ Ans. A

02 $\frac{1}{2}[(x-y)^2 + (y-z)^2 + (z-x)^2]$ Ans: B

03 $(x-y)^3 + 4^3 = (x-y+4)(\quad\quad)$

Ans: A

04 $\left(x-\frac{1}{x}\right)^2 = \left(x+\frac{1}{x}\right)^2 - 4 \cdot x \cdot \frac{1}{x}$ (4)

$= (2)^2 - 4$

$= 0$

Ans: A

05 $f(x) = x^{505} + a$

Remainder $= f(1) = 1^{505} + a = 1 + a$

Ans: C

06 Conceptual

Ans. A

07 $\sum a^2 = a^2 + b^2 + c^2$

Ans. D

08 Conceptual

Ans. B

09 $\sum_{i=1}^{n} 1 = 1 + 1 + 1 + \cdots + 1 = 1 \times n = n$

Ans: A

10 $f(x) = 4x^3 - 5x^2 + 3x - 4$

$f(0) = -4$

Ans: C

JEE MAINS LEVEL

01. $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$= 7^3 - 3 \cdot 5 (7)$

$= 343 - 105 = 238$

Ans. D

02 $\frac{x^2+x-2}{x+2} = \frac{(x+2)(x-1)}{(x+2)} = x - 1$

Ans: C

03 $9 - a^6 + 2a^3b^3 - b^6$

$= 9 - [a^6 - 2a^3b^3 + b^6]$

$= 3^2 - (a^3 - b^3)^2$

$= (3 + a^3 - b^3)(3 - a^3 + b^3)$ Ans: B

04 $\sqrt{2a^2 - 2\sqrt{6}ab + 3b^2}$

$= \sqrt{(\sqrt{2}a)^2 - 2 \cdot \sqrt{2} \cdot \sqrt{3}ab + (\sqrt{3}b)^2}$

$= \sqrt{(\sqrt{2}a - \sqrt{3}b)^2} = \sqrt{2}a - \sqrt{3}b$ Ans: A

05 $36x^2 + 60x + 25$

$= (6x)^2 + 2 \cdot 6x \cdot 5 + (5)^2$

$= (6x + 5)^2$ ∴ Square root $= \pm(6x + 5)$ Ans: C

06. Let $x^2 - 4x = a$

$\therefore (a)(a - 1) - 20$

$a^2 - a - 20$

$= (a - 5)(a + 4)$

$= (x^2 - 4x - 5)(x^2 - 4x + 4)$

$= (x - 5)(x + 1)(x - 2)^2$ Ans: D

07 $f(x) = 3x^3 + x^2 - 20x + 12$

$f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right)^3 + \left(\frac{y}{3}\right)^2 - 20\left(\frac{y}{3}\right) + 12 \neq 0$ Ans: A

08 $f(x) = 2x^4 - 7x^3 + ax + b$ (9)

$f(3) = 162 - 189 + 3a + b = 0$

$3a + b = 27$

Ans: A

09 $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= (-5)^2 - 4.6$

$= 25 - 24 = 1$

Ans. A

10 $(1+\alpha)(1+\beta) = 1 + (\alpha + \beta) + \alpha\beta$

$= 1 + (-1) + C$

$= C$

Ans: C

JEE ADVANCED LEVEL

11 $P(z) = z^5 + 0z^4 + 0z^3 - 2z^2 + 0z + 4$

Coefficients → 1, 0, 0, −2, 0, 4

product → 0

Ans: B, C

12 $x^2-2)2x^4-3x^3-3x^2+6x-2(2x^2-3x+1$ (10)

$2x^4 \quad -4x^2$

$+ \quad +$

$-3x^3+x^2+6x$

$-3x^3 \quad +6x$

$+ \quad -$

x^2-2

x^2-2

$- \quad +$

0

$\therefore 2x^2-3x+1=(2x-1)(x-1)=0$

$\therefore x=\frac{1}{2}$ or 1

Ans: A, D

13 Conceptually all optins are correct.

Ans. A, B, C, D

14 Statement I:

$(\sqrt{3}x+2)(4x-\sqrt{3})$

$=4\sqrt{3}x^2-3x+8x-2\sqrt{3}$

$=4\sqrt{3}x^2+5x-2\sqrt{3}$ (true)

Statement II: Conceptual (true)

Ans. A

15 $\sum a^2b=a^2b+b^2c+c^2a$ Ans. A

16 $\sum(x^2y+xy^2)=\sum xy(x+y)$

$=xy(x+y)+yz(y+z)+zx(z+x)$ Ans. C

17 $\pi a^2b^2 = (a^2b^2)(b^2c^2)(c^2a^2)$ Ans. B

18 $\Sigma a^2 + 2\Sigma ab$

$= (a^2+b^2+c^2) + 2(ab+bc+ca)$

$= (a+b+c)^2$ Ans. B

19 $27x^3 - y^3 - z^3 - 9xyz$

$= (3x)^3 + (-y)^3 + (-z)^3 - 3(3x)(-y)(-z)$

$= (3x-y-z)(9x^2+y^2+z^2+3xy-yz+3xz)$ Ans: C

20 $125 + 8x^3 - 27y^3 + 90xy$

$= (5)^3 + (2x)^3 + (-3y)^3 - 3(5)(2x)(-3y)$

$= (5+2x-3y)(25+4x^2+9y^2-10x+6xy+15y)$

Ans: B

21. $(3x-5y-4)(9x^2+25y^2+15xy+12x-20y+16)$

$= [(3x)+(-5y)+(-4)][\quad\quad\quad]$

$= (3x)^3 + (-5y)^3 + (-4)^3 - 3(3x)(-5y)(-4)$

$= 27x^3 - 125y^3 - 64 - 18xy$ Ans: C

22 Quadratic equation → reciprocal roots →

$a = c \Rightarrow k^2 + 4 = 4k$

$\Rightarrow k^2 - 4k + 4 = 0$

$\Rightarrow k = 2$ Ans: 2

23 $\alpha^2 + \beta^2 = 40$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 40$

$\Rightarrow (8)^2 - 2k = 40$

$\Rightarrow k = 12$

$k = 12$

Please change List-II Accordingly

24 a) $a^3(b-c) + b^3(c-a) + c^3(a-b)$

$= -(a-b)(b-c)(c-a)(a+b+c)$

b) $a^4(b-c) + b^4(c-a) + c^4(a-b)$

$= -(a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)$

c) $x^3(y-z) + y^3(z-x) + z^3(x-y)$

$= -(x-y)(y-z)(z-x)(x+y+z)$

d) $x^4(y-z) + y^4(z-x) + z^4(x-y)$

$= -(x-y)(y-z)(z-x)(x^2+y^2+z^2+xy+yz+zx)$

ADDITIONAL PRACTICE QUESTIONS

01. Given $x = 2y + 6$

$x^3 - 8y^3 - 36xy - 216$

$= x^3 - (2y)^3 - 36xy - 216$

$= (x-2y)^3 + 3x.2y(x-2y) - 36xy - 216$

$= (6)^3 + 3x.2y(6) - 36xy - 216$

$= 216 + 36xy - 36xy - 216$

$= 0$

Ans. A

02 $x^3+y^3-12xy+64$

$=(x+y)^3-3xy(x+y)-12xy+64$

$=(-4)^3-3xy(-4)-12xy+64$

$=-64+12xy-12xy+64$

$=0$

Ans: C

03 remainder $f(-5)=-8752$

Ans: D

04 n is odd $\rightarrow$ let $n=1$

$f(x)=x^{mn}+1=x^m+1$

Ans: A

05 $p(x)=x^2+6x+K$

Let $\alpha, 2\alpha$ be the zeroes

$\therefore \alpha+2\alpha=-6$

$\Rightarrow \alpha=-2$

The zeroes are $-2, -4$

$f(-2)=(-2)^2+6(-2)+K=0$

$\Rightarrow 4-12+K=0$

$\Rightarrow K=8$

Ans: A

06 $p(x)=x^2-(a^2-3a+2)x+4$

Let the zeroes be $\alpha, -\alpha$

$\therefore \alpha+(-\alpha)=a^2-3a+2$

$\Rightarrow a^2-3a+2=0$

$\Rightarrow (a-1)(a-2)=0$

$\Rightarrow a=1$ or 2

Ans B

07 $(x-2)(x+3)=0$

$\Rightarrow x^2+x-6=0$

$x^2+(a+1)x+b=0$

$a+1=1 \quad | \quad b=-6$

$a=0$

Ans D

08 $\frac{ab+bc+ca}{abc}=\frac{4}{2}$

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=2$

Ans. D

09 $\frac{x}{y}+\frac{y}{x}=-1$

$\Rightarrow x^2+y^2+xy=0$

Now, $x^3-y^3=(x-y)(x^2+xy+y^2)$

$=(x-y)(0)=0$

Ans: C

$\Longrightarrow$ THE END $\Longleftarrow$