

6. FUNDAMENTALS OF LOGARITHMS (2025/26)

Class: VII, MATHEMATICS
FOUNDATION. SOLUTIONS

TEACHING TASK JEE MAINS LEVEL

01 $\log_4 x = 2 \Rightarrow x = 4^2 = 16$ Ans. A

02 $\log_3 x = 9 \Rightarrow x = 3^9 = 19683$ Ans. D

03 $\log_2 9 = 2 \Rightarrow a = 2^2 = 4.$

$\log_4 9 = \log_4 4 = 1$

Ans. B

04 10 Ans. B

05 $3^{-2} = \frac{1}{9}$

$\log_3 \left(\frac{1}{9}\right) = -2$

Ans. D

06 $a = \left(\frac{1}{a}\right)^b$

Ans. B

07 $\left(\frac{1}{x}\right) = \left(\frac{1}{y}\right)^{\frac{1}{2}}$ (or) $x^{-1} = \left(\frac{1}{y}\right)^{\frac{1}{2}}$ (or) $\frac{1}{x} = \left(y^{-1}\right)^{\frac{1}{2}}$ Ans. B, C, D

08 $\log_{10} 0.001 = \log_{10} 10^{-3} = -3$

Ans. A

09. $\log \frac{(a^2-b^2)}{(a^2-b^2)} = 1$

Ans: A

10. $0.0001 = 10^x \Rightarrow 10^{-4} = 10^x$

Ans: A

JEE ADVANCED LEVEL

01. B) $8^2 = 64 \Leftrightarrow 2 = \log_8 64$

C) $4^3 = 64 \Leftrightarrow \log_4 64 = 3$

D) $64^1 = 64 \Leftrightarrow \log_{64} 64 = 1$

Ans: B, C, D

02. $x^2 + 9 = 5^2 \Rightarrow x = \pm 4$

Ans: A, C

03. option verification.
 $x = \pm 2$ satisfies the given equation

Ans: B, D

04. Statement I: Conceptual (True)

Statement II: Conceptual (True)

Ans: A

05. SI: $a^N = x \Rightarrow \log_a x = N$ (False)

Statement II: Conceptual (True)

Ans: D

06. Assertion: Conceptual (True)

Reason: Conceptual (False)

Ans: C

67

Assertion: Conceptual (True)Reason: Conceptual (True)

(3)

Ans: A

68

$$\log_3 [\log_2 x] = 1$$

$$\Rightarrow \log_2 x = 3 \Rightarrow x = 8$$

Ans: A

69

$$\log_3 [\log_3 (\log_5 b)] = 0$$

$$\Rightarrow \log_3 (\log_5 b) = 3^0 = 1$$

$$\Rightarrow \log_5 b = 3^1 = 3$$

$$\Rightarrow b = 5^3 = 125$$

Ans: B

10

$$\log_{10} [\log_2 (x^2 - 3x)] = 0$$

$$\Rightarrow \log_2 (x^2 - 3x) = 10^0 = 1$$

$$\Rightarrow x^2 - 3x = 2$$

$$\Rightarrow x^2 - 3x - 2 = 0$$

$$\Rightarrow (x-3)(x+1) = 0$$

$$\Rightarrow x = 3 \text{ or } -1$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

Ans: A

11.

$$\log \sqrt[30]{x} = \log x^{1/30} = \frac{1}{30} \log x$$

Ans: B

12

$$\sqrt[7]{y} = 1$$

$$y = 1$$

Ans: A

(4)

$$13 \quad \sqrt[3]{x+y} = 1 \quad | \quad \sqrt[3]{x-y} = 2$$

$$\Rightarrow x+y=1 \quad | \quad x-y=8$$

$$\text{Solving } x = \frac{9}{2}, y = -\frac{7}{2} \Rightarrow \frac{x}{y} = -\frac{9}{7} \quad \text{Ans: 1)$$

$$14 \quad \frac{1}{\sqrt[5]{2}} = x^{-\frac{1}{5}}$$

$$\Rightarrow 2^{-\frac{1}{5}} = x^{-\frac{1}{5}}$$

$$\Rightarrow x = 2$$

Ans: 2

$$15 \quad \frac{1}{\sqrt[3]{2}} = x^{-\frac{1}{3}}$$

$$\Rightarrow 2^{-\frac{1}{3}} = x^{-\frac{1}{3}} \Rightarrow x = 2$$

Ans: 2

$$16 \quad a) \log_a(an) = \log_a a + \log_a n = 1 + x \quad (s)$$

$$b) \log_n(A_n^2) = \log_n A^2 + \log_n n = 2 \cdot \log_n A + 1 = 2x+1 \quad (t)$$

$$d) \log_n a = \frac{1}{\log_a n} = \frac{1}{x} \quad (r)$$

$$c) \log_n(xn) = \log_n x + \log_n n = a+1 \quad (v)$$

Ans: s, t, r, v

$$17 \quad a) (a^2)^{\log b} = (a^{\log b})^2 = (b^{\log a})^2 = \underline{b^{2 \log a}} \quad (s)$$

$$b) (b^2)^{\log a} = (b^{\log a})^2 = (a^{\log b})^2 = a^{2 \log b} \quad (t)$$

$$c) (a^2)^{\log b^2} = a^{\log b^4} = b^{\log a^4} \quad (v)$$



$$d) (b^2)^{\log b^2} = b^{\log b^4} \underline{(\quad)} \quad (5)$$

Ans.. S, t, q, —

Q1

LEARNERS TASK

CUQ'S

01

$$2^4 = 16 \Rightarrow \log_2 16 = 4$$

Ans.. B

02

$$x = \left(\frac{1}{2}\right)^5 \Rightarrow x = \frac{1}{32}$$

Ans.. B

03

$$\log_5 25 = \log_5 5^2 = 2 \cdot \log_5 5 = 2 \times 1 = 2$$

Ans.. B

04

$$\log_{0.01} 0.01 = 1$$

Ans.. —

05

Conceptual

Ans.. C

06

$$\frac{1}{6} \quad \text{since} \quad a^{\log_a N} = N$$

Ans.. B

07

Conceptual.

$$e = 2.71828 \text{ (approx)}$$

Ans.. D

08

Conceptual

Ans.. B

09

Conceptual

Ans.. C

10

Conceptual

Ans.. D

JEE MAINS LEVEL

01. $\log \sqrt[3]{ab \cdot bc \cdot ca} = \log \sqrt[3]{a^2 b^2 c^2} = \log abc^{\frac{2}{3}} = \log (abc)^{\frac{2}{3}} = 1$
 Ans: A

02. $\log \left(\frac{64 \times 27}{256 \times 729} \right)$
 $= \log \left(\frac{1}{27 \times 2^2} \right)$
 Ans: B

03. $\log_a b \times \log_c b \times \log_a c = xyz$
 $\Rightarrow xyz = 1$
 Ans: A

04. $\log_{0.01} 0.0001 = 2$
 Ans: A

05. $5^k = 5^{3x} \Rightarrow 3x = \frac{1}{5} \Rightarrow x = \frac{1}{15}$
 Ans: A

06. $\sqrt[7]{7} = (49)^p \Rightarrow p = \frac{1}{14}$
 $\Rightarrow 7^{\frac{1}{7}} = 7^{2p}$
 Ans: B

07. $x = 10^a$
 Now $10^{3a-2} = \frac{(10^a)^3}{10^2} = \frac{x^3}{100}$
 Ans: B

08. Repeated (as above)

09. $x = 10^k$ | $10^{2k-2} = 10^{2k-3} = \frac{(10^k)^2}{10^3} = \frac{x^2}{1000}$
 Ans: A

(7)

10

$$\begin{aligned} m &= 100^n \\ n &= 100^m \end{aligned}$$

$$m+n = 100^m + 100^m$$

$$(2) \quad mn = 100^n \cdot 100^m = 100^{m+n}$$

$$\Rightarrow m+n = \log_{100} mn$$

Ans: D

11

$$1296 = 6^{x+2}$$

$$\Rightarrow 6^4 = 6^{x+2}$$

$$\Rightarrow x = 2$$

$$\Rightarrow x-2 = 0$$

$$\Rightarrow (x-2)^n = 0$$

Ans: C

12

$$(m+n)^2 = mn$$

$$\Rightarrow \left(\frac{m+n}{\sqrt{mn}} \right)^2 = 1$$

$$\Rightarrow \frac{m+n}{\sqrt{mn}} = \pm 1$$

$$\Rightarrow \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \pm 1$$

Ans: C

13

$$a^{\log_b \sqrt{k}} = b$$

$$\Rightarrow b^{\log_a \sqrt{k}} = b$$

$$\Rightarrow \log_{\sqrt{k}} a = 1$$

$$\Rightarrow a = \sqrt{k}$$

$$\text{Now } a + \sqrt{k} = \sqrt{k} + \sqrt{k} = 2\sqrt{k}$$

$$(2) \quad a + \sqrt{k} = a + a = 2a$$

Ans: D



14 $\left(x^2\right)^{\log_y z^2} = z^2 \quad \Rightarrow \left(z^{\log_y x}\right)^2 = z^2 \quad (2)$

$\Rightarrow \left(x^2\right)^{\log_y z} = z^2 \quad \Rightarrow z^{2 \log_y x} = z^2$

$\Rightarrow \left(x^{\log_y z}\right)^2 = z^2 \quad \Rightarrow \log_y z = 1$

$\Rightarrow x = y$

Now, $x^2 + y^2 + xy = x^2 + x^2 + x \cdot x = 3x^2$ (or)

$= y^2 + y^2 + y \cdot y = 3y^2$

Ans: D

15 $x^{y \log_y z} = (x^y)^{\log_y z}$

$\Rightarrow x = x^{y \log_y z}$ Since $a^{\log_a N} = N$

$\Rightarrow z = y \cdot \log_y z$

$\Rightarrow \frac{z}{y} = \log_y z$

Now, $\log_y z^2 = 2 \cdot \log_y z = \frac{2z}{y}$

Ans: B

JEE ADVANCED LEVEL

01 $\log_{10}(a+b)^2 = \log_{10}(a+b)$

$\Rightarrow \log_{10}(a+b)^2 - \log_{10}(a+b) = 0$

$\Rightarrow \log_{10} 2 \log_{10}(a+b) - \log_{10}(a+b) = 0$

$$\Rightarrow \log_{10}(a+b) = 0$$

$$\Rightarrow a+b = 10^0 = 1$$

$$\Rightarrow a+b = 1$$

Given $a=b \therefore a = \frac{1}{2} \text{ \& } b = \frac{1}{2}$

$$a^3 + b^3 = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

$$\text{Also } \sqrt[3]{\frac{1}{512}} = \sqrt[3]{\frac{1}{2^9}} = \frac{1}{2^3} = \frac{1}{8} \quad \times$$

$$\text{D) } \sqrt[3]{\frac{1}{64}} = \sqrt[3]{\frac{1}{4^3}} = \frac{1}{4} \quad \checkmark$$

Ans: B, D

Q2

$$x^3 + y^3 = 1$$

$$(x+y)(x^2 - xy + y^2) = 1$$

(Reciprocals)

Ans: A, C, D

Q3

$$x = 2^3$$

$$x = 8$$

$$y = 3^5$$

$$y = 243$$

$$\text{Now) A) } x = \frac{y-8}{15} = \frac{243-8}{15} = \frac{235}{15} \quad \times$$

$$\text{B) } x = \frac{y-3}{15} = \frac{243-3}{15} = \frac{240}{15} \quad \times$$

NOTE: Please discard this problem as it has insufficient data.

04 NOTE: Please, discard this problem as the problem needs some correction.

05 statement I:

$$a^{\frac{x}{5}} = y \Rightarrow \log a^{\frac{x}{5}} = \log y$$

$$\Rightarrow \frac{x}{5} \log a = \log y$$

$$\Rightarrow x \log a = 5 \log y \text{ (True)}$$

statement II: Conceptual (True)

Ans: A

06 Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans: A

07 Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans: A

$$08 \quad \log_{5^{1/4}} 5^{1/3} = \frac{(\frac{1}{3})}{(\frac{1}{4})} \log_5 5 = \frac{7}{3} \times 1 = \frac{7}{3}$$

Ans: B

$$09 \quad \log_{6^{1/2}} 36^{1/6} = 5 \Rightarrow \frac{(\frac{1}{6})}{(\frac{1}{2})} \log_6 6^2 = 5$$

$$\Rightarrow \frac{x}{6} \times 2 = 5$$

$$\Rightarrow x = 15$$

Ans: C

10

$$\log_{0.001} 0.01 = \frac{x}{y}$$

$$\Rightarrow \log_{(0.1)^3} (0.1)^2 = \frac{x}{y}$$

$$\Rightarrow \frac{2}{3} = \frac{x}{y}$$

$$3x = 2y$$

Ans. B

11

$$x = 10^p$$

$$10^{p+1} = 10^p \times 10^1 = 10x$$

Ans. B

12

$$10^{p+3} = 10^p \times 10^3 = 1000x$$

Ans. B

13

$$10^{p-2} = \frac{10^p}{10^2} = \frac{x}{100}$$

Ans: —

14

$$\log_{y^3} x^2 = 1 \Rightarrow \frac{2}{3} \log_y x = 1$$

$$\Rightarrow \log_y x = \frac{3}{2}$$

$$\Rightarrow x = y^{\frac{3}{2}}$$

$$\Rightarrow x^2 = y^3$$

$$\text{Now, } x^2 + y^3 = y^3 + y^3 = 2y^3$$

$$(or) x^2 + y^3 = x^2 + x^2 = 2x^2$$

Ans: —

15

$$a^2 = e^2 \Rightarrow a^2 = 4$$

$$\text{Now } \log_{a^2} b^2 = 1 \Rightarrow b^2 = a^2 = 4$$

$$\therefore b = \pm 2$$

Ans: ± 2

$$16 \quad a) \quad a^m = a^n \Rightarrow m = n \Rightarrow \log_a a^m = m \quad \left| \quad \log_a a^n = n \quad (12) \right. \\ \Rightarrow n = m \quad (x) \quad \left| \Rightarrow m = n \quad (p, x) \right.$$

$$b) \quad a^{\frac{1}{m}} = a^{\frac{1}{n}} \Rightarrow \frac{1}{m} = \frac{1}{n}$$

$$\therefore \log_a a^{\frac{1}{m}} = \frac{1}{m} \quad \left| \quad \log_a a^{\frac{1}{n}} = \frac{1}{n} \right. \\ \Rightarrow \frac{1}{m} = \frac{1}{n} \quad \left| \Rightarrow m = n \right. \\ \Rightarrow \frac{1}{n} = \frac{1}{m} \quad \left| \Rightarrow \frac{1}{m} = \frac{1}{n} \quad (p, x) \right.$$

$$c) \quad m^n = \left(\frac{1}{n}\right) \Rightarrow \log_m \left(\frac{1}{n}\right) = n \quad (q)$$

$$d) \quad \frac{1}{m} = \left(\frac{1}{n}\right)^n \Rightarrow \log_{\left(\frac{1}{n}\right)} \left(\frac{1}{m}\right) = n \quad (s)$$

Ans: (p, x), (p, r), q, s

$$17 \quad a) \quad \sqrt[3]{x} = x^a \Rightarrow x^{\frac{1}{3}} = x^a \Rightarrow a = \frac{1}{3}$$

$$\sqrt{a} = \sqrt{\frac{1}{3}} \quad (x)$$

$$b) \quad \sqrt[4]{a} = a^x \Rightarrow x = \frac{1}{4}$$

$$\Rightarrow \sqrt[3]{x} = \sqrt[3]{\frac{1}{4}} \quad (t)$$

$$c) \quad \sqrt[3]{a} = a^m \Rightarrow m = \frac{1}{3} \Rightarrow m^2 = \frac{1}{9} \quad (s)$$

$$d) \quad \sqrt[5]{x} = x^{2a} \Rightarrow 2a = \frac{1}{5} \Rightarrow a = \frac{1}{10} \\ \Rightarrow a^2 = \frac{1}{100} \quad (p)$$