

Chapter : Heights and Distances

LECTURER'S TASK

1. From the diagram :

$$\tan 30^\circ = \frac{h}{a}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a}$$

$$\Rightarrow h = \frac{a}{\sqrt{3}} \quad \dots\dots\dots (1)$$

Again $\tan 60^\circ = \frac{h}{b}$

$$\Rightarrow \sqrt{3} = \frac{h}{b}$$

Now $\Rightarrow h = b\sqrt{3} \quad \dots\dots\dots (2)$

New (1) x (2)

$$\Rightarrow h^2 = \frac{a}{\sqrt{3}} \times b\sqrt{3}$$

$$\Rightarrow h^2 = ab$$

$$\Rightarrow h = \sqrt{ab}$$

Ans: B

2. From the diagram

$$\tan \beta = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\tan \beta}$$

$$\Rightarrow x = h \cot \beta \quad \dots\dots\dots (i)$$

Now, $\tan \alpha = \frac{h}{d+x}$

$$\Rightarrow d+x = h \cot \alpha$$

$$\Rightarrow x = h \cot \alpha - d \quad \dots\dots\dots (ii)$$

from (i) and (ii)

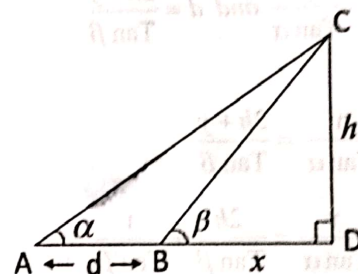
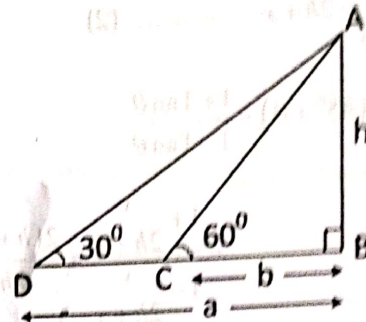
$$h \cot \beta = h \cot \alpha - d$$

$$\Rightarrow h \cot \alpha - h \cot \beta = d$$

$$\Rightarrow h(\cot \alpha - \cot \beta) = d$$

$$\Rightarrow h = \frac{d}{\cot \alpha - \cot \beta}$$

Ans: A



3. From diagram

$$\tan \theta = \frac{x}{d} \quad \dots\dots\dots (1)$$

$$\tan 45^\circ = \frac{2h+x}{d}$$

$$\Rightarrow d = 2h+x \quad \dots\dots\dots (2)$$

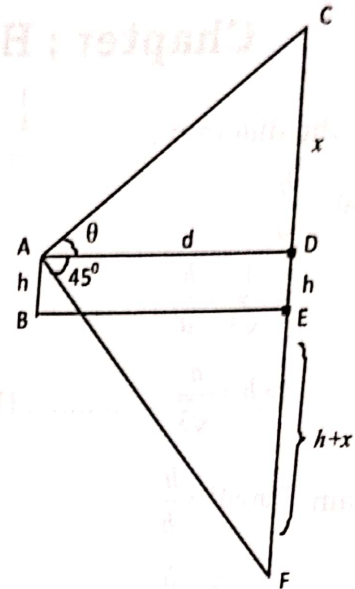
Now

$$\tan(45^\circ + \theta) = \frac{1 + \tan \theta}{1 - \tan \theta}$$

$$= \frac{1 + \frac{x}{2h+x}}{1 - \frac{x}{2h+x}} = \frac{2(h+x)}{2h} = \frac{h+x}{h}$$

$$\therefore h+x = h(\tan 45^\circ + \theta)$$

$\therefore$ The height of the cloud is $h(\tan 45^\circ + \theta)$



Ans: A

4. From diagram

$$\tan \alpha = \frac{y}{d}$$

$$\tan \beta = \frac{2h+y}{d}$$

$$\therefore d = \frac{y}{\tan \alpha} \text{ and } d = \frac{2h+y}{\tan \beta}$$

$$\therefore \frac{y}{\tan \alpha} = \frac{2h+y}{\tan \beta}$$

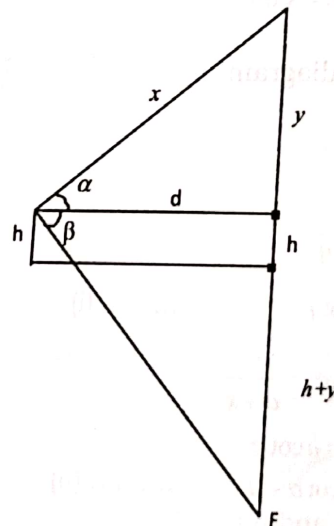
$$\Rightarrow \frac{y}{\tan \alpha} = \frac{2h}{\tan \beta} + \frac{y}{\tan \beta}$$

$$\Rightarrow \frac{y}{\tan \alpha} - \frac{y}{\tan \beta} = \frac{2h}{\tan \beta}$$

$$\Rightarrow y \left(\frac{\tan \beta - \tan \alpha}{\tan \alpha \cdot \tan \beta} \right) = \frac{2h}{\tan \beta}$$

$$\Rightarrow y \left(\frac{\tan \beta - \tan \alpha}{\tan \alpha} \right) = 2h$$

$$\Rightarrow \frac{y}{\tan \alpha} = \frac{2h}{\tan \beta - \tan \alpha}$$



$$\Rightarrow d = \frac{2h}{\tan \beta - \tan \alpha} \rightarrow (1)$$

$$\text{Now } \sec \alpha = \frac{x}{d}$$

$$\Rightarrow x = d \sec \alpha$$

$$\Rightarrow x = \frac{2h \sec \alpha}{\tan \beta - \tan \alpha} (\because \text{from (1)})$$

Hence, the distance between the cloud and the observer is $\frac{2h \sec \alpha}{\tan \beta - \tan \alpha}$

Ans: C

5. From the diagram

AB = Building

CE = Hill

A, B = Position of the observers

$$\text{Now, } \tan p = \frac{x}{d} \text{ and } \tan q = \frac{x+h}{d}$$

$$\Rightarrow d = x \cot p \quad \Rightarrow d = (x+h) \cot q$$

$$\therefore x \cot p = (x+h) \cot q$$

$$\Rightarrow x \cot p - x \cot q = h \cot q$$

$$\Rightarrow x(\cot p - \cot q) = h \cot q$$

$$\Rightarrow x = \frac{h \cot q}{\cot p - \cot q}$$

Now, height of the hill

$$x+h = \frac{h \cot q}{\cot p - \cot q} + h$$

$$= \frac{h \cot q + h \cot p - h \cot q}{\cot p - \cot q}$$

$$= \frac{h \cot p}{\cot p - \cot q}$$

Ans: C

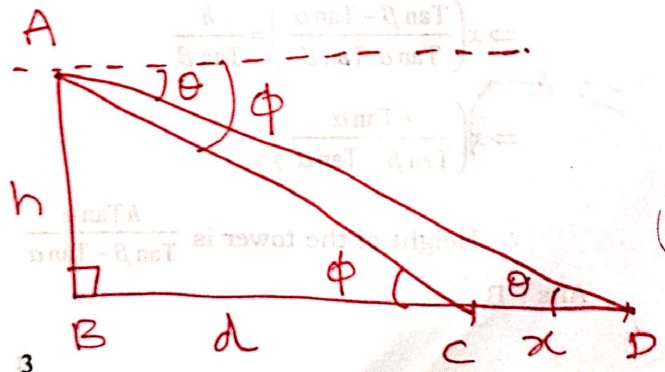
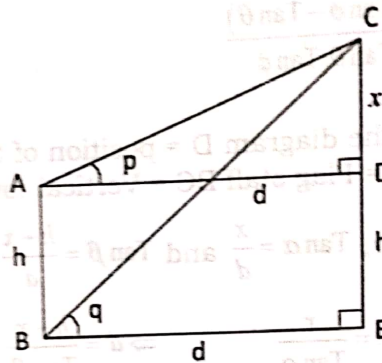
6. In the diagram

C, D = Position of the boats

A = Position of the observer

AB = Light hense house

$$\text{Now, } \tan \phi = \frac{h}{d} \text{ and } \tan \theta = \frac{h}{d+x}$$



$$\Rightarrow d = \frac{h}{\tan \phi} \quad \Rightarrow d + x = \frac{h}{\tan \theta}$$

$$\therefore \frac{h}{\tan \phi} + x = \frac{h}{\tan \theta}$$

$$\Rightarrow x = \frac{h}{\tan \theta} - \frac{h}{\tan \phi}$$

$$\therefore x = h \left(\frac{\tan \phi - \tan \theta}{\tan \theta \cdot \tan \phi} \right)$$

$\therefore$ The distance between the two boats is

$$\frac{h(\tan \phi - \tan \theta)}{\tan \theta \cdot \tan \phi}$$

Ans: B

7. In the diagram D = position of the observer
AB = Flag staff BC = Vertical tower

$$\text{Now, } \tan \alpha = \frac{x}{d} \text{ and } \tan \beta = \frac{h+x}{d}$$

$$\Rightarrow d = \frac{x}{\tan \alpha} \quad \Rightarrow d = \frac{h+x}{\tan \beta}$$

$$\therefore \frac{x}{\tan \alpha} = \frac{h+x}{\tan \beta}$$

$$\Rightarrow \frac{x}{\tan \alpha} = \frac{h}{\tan \beta} + \frac{x}{\tan \beta}$$

$$\Rightarrow \frac{x}{\tan \alpha} - \frac{x}{\tan \beta} = \frac{h}{\tan \beta}$$

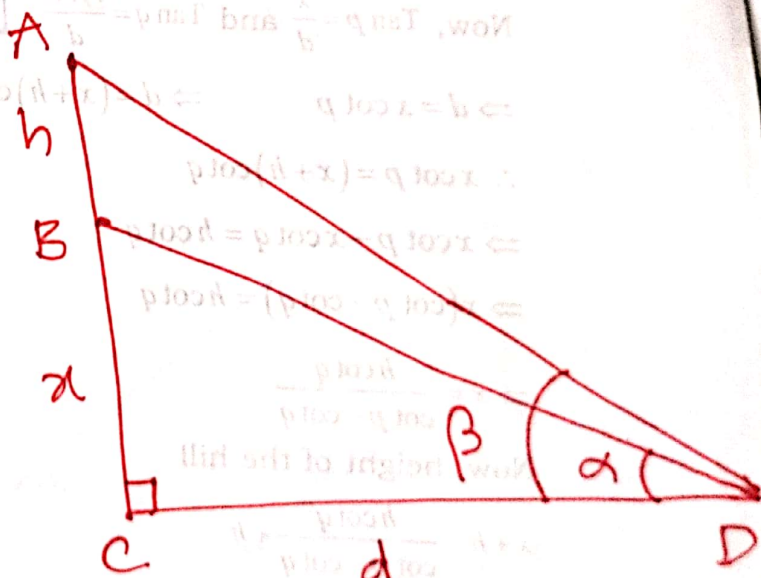
$$x \left(\frac{1}{\tan \alpha} - \frac{1}{\tan \beta} \right) = \frac{h}{\tan \beta}$$

$$\Rightarrow x \left(\frac{\tan \beta - \tan \alpha}{\tan \alpha \cdot \tan \beta} \right) = \frac{h}{\tan \beta}$$

$$\Rightarrow x \left(\frac{h \tan \alpha}{\tan \beta - \tan \alpha} \right)$$

$\therefore$ Height of the tower is $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$

Ans : B



8. In the diagram AB = Tower
P, Q = Position of the observer

Given $\alpha + \beta = 90^\circ$

$\Rightarrow \tan(\alpha + \beta) = \tan 90^\circ$

$\Rightarrow \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \text{Not defined}$

$\Rightarrow 1 - \tan \alpha \cdot \tan \beta = 0$

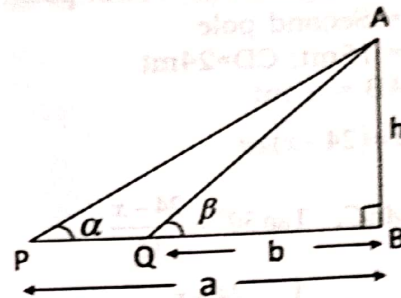
$\Rightarrow \tan \alpha \cdot \tan \beta = 1$

$\Rightarrow \frac{h}{a} \cdot \frac{h}{b} = 1$

$\Rightarrow h^2 = ab$

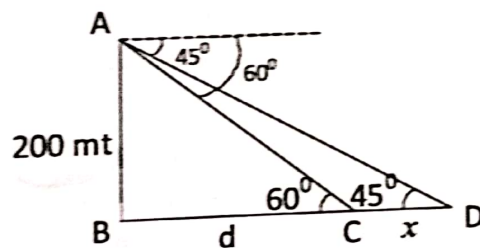
$\Rightarrow h = \sqrt{ab}$

$\therefore$ Height of the tower is $\sqrt{ab}$



Ans: A

9.



In the above diagram AB = Altitude of the aeroplane C, D = Opposite sides of the banks of a river

$CD = x$ = width of the river.

Now, $\tan 60^\circ = \frac{200}{d}$

$\Rightarrow \sqrt{3} = \frac{200}{d}$

$\Rightarrow d = \frac{200}{\sqrt{3}}$ and $\tan 45^\circ = \frac{200}{d+x} \rightarrow x$

$\Rightarrow d+x = 200 \rightarrow x$

$\therefore \frac{200}{\sqrt{3}} + x = 200$

$\Rightarrow x = 200 - \frac{200}{\sqrt{3}}$

$\Rightarrow x = 200 \left(\frac{\sqrt{3}-1}{\sqrt{3}} \right)$

Ans : B

10. In the diagram AB = First pole
 CD = Second pole
 BD = 15mt; CD=24mt
 Let AB = x mt
 $\therefore CE = (24-x)mt$

In $\triangle AEC$, $\tan 30^\circ = \frac{24-x}{15}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{24-x}{15}$$

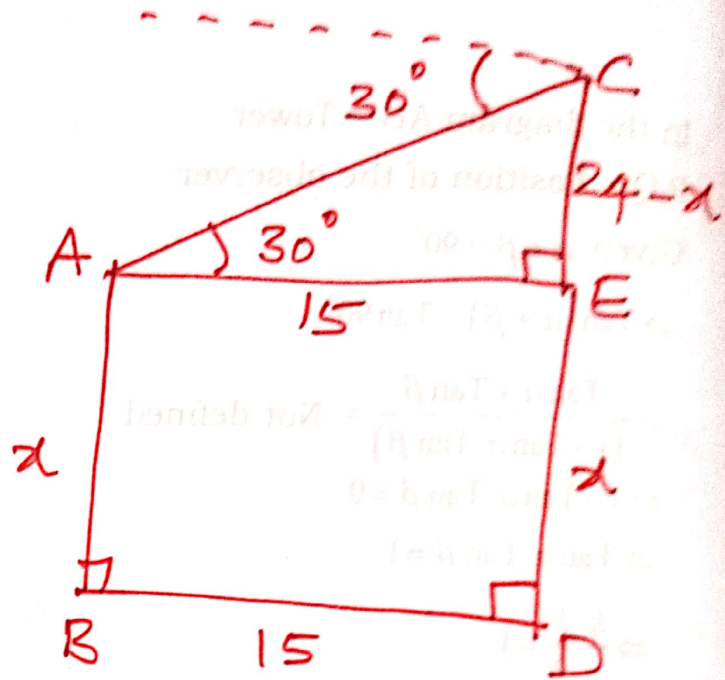
$$\Rightarrow 24-x = \frac{15}{\sqrt{3}}$$

$$\Rightarrow 24-x = 8.67$$

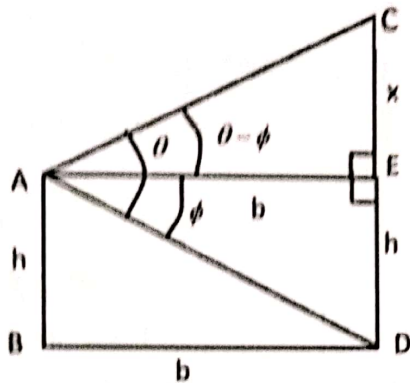
$$\Rightarrow x = 24 - 8.66$$

$$\Rightarrow x = 15.34 \text{ (approx)}$$

Ans : C



11. In the figure AB = Window, CD = Building, BD = Road, given AB = h



Now, $\tan \phi = \frac{h}{b} \rightarrow (i)$

$$\tan(\theta - \phi) = \frac{x}{b}$$

$$\Rightarrow \frac{\tan \theta - \tan \phi}{1 + \tan \theta \cdot \tan \phi} = \frac{x}{b}$$

$$\Rightarrow \frac{\frac{\sin \theta}{\cos \theta} - \frac{h}{b}}{1 + \frac{\sin \theta}{\cos \theta} \cdot \frac{h}{b}} = \frac{x}{b} \quad (\because \text{from (i)})$$

$$\Rightarrow \frac{b \sin \theta - h \cos \theta}{b \cos \theta + h \sin \theta} = \frac{x}{b}$$

$$\Rightarrow x = \frac{b(b \sin \theta - h \cos \theta)}{b \cos \theta + h \sin \theta}$$

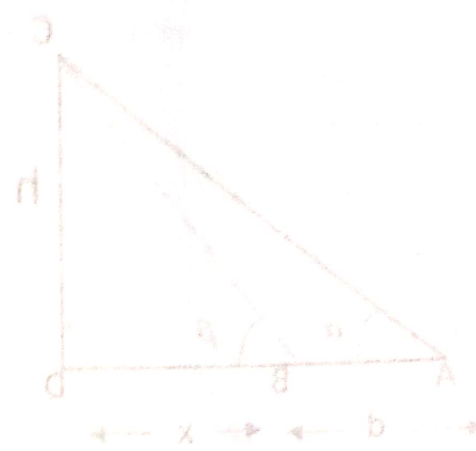
$\therefore$ Height of the building = $h + x$

$$\therefore h + x = \frac{b(b \sin \theta - h \cos \theta)}{b \cos \theta + h \sin \theta} + h$$

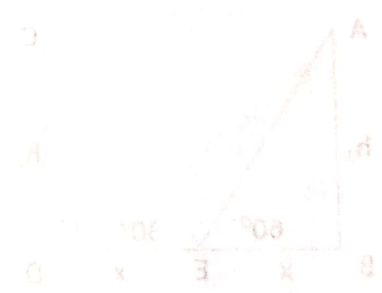
$$= \frac{b^2 \sin \theta - bh \cos \theta + bh \cos \theta + h^2 \sin \theta}{b \cos \theta + h \sin \theta}$$

$$= \frac{(b^2 + h^2) \sin \theta}{b \cos \theta + h \sin \theta}$$

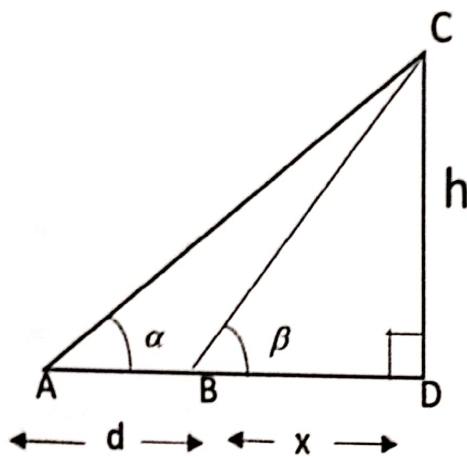
ANS: C



Let BD = x, CD = h
Now, $\tan \theta = \frac{h}{x}$ and $\tan \phi = \frac{h}{b}$
 $\Rightarrow x = h \cot \theta$ and $b = h \cot \phi$
 $\Rightarrow \frac{h}{x} = \cot \theta$ and $\frac{h}{b} = \cot \phi$
 $\Rightarrow \frac{h}{x} = \frac{\cos \theta}{\sin \theta}$ and $\frac{h}{b} = \frac{\cos \phi}{\sin \phi}$
 $\Rightarrow \frac{h}{x} = \frac{\cos \theta}{\sin \theta}$
 $\Rightarrow x = \frac{h \sin \theta}{\cos \theta}$
 $\therefore$ Height of the tower = $h + x$



12. In the figure A, B = positions of the observers, CD = Tower, AB = d



Let $BD = x$, $CD = h$

Now, $\tan \beta = \frac{h}{x}$ and $\tan \alpha = \frac{h}{d+x}$

$$\Rightarrow x = h \cot \beta \text{ and } d+x = h \cot \alpha$$

$$\therefore d + h \cot \beta = h \cot \alpha$$

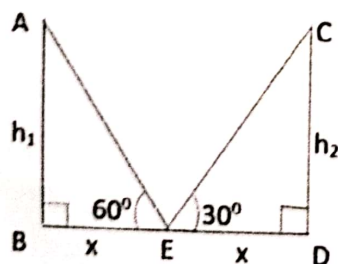
$$\Rightarrow h(\cot \alpha - \cot \beta) = d$$

$$\Rightarrow h = \frac{d}{\cot \alpha - \cot \beta}$$

$$\therefore \text{Height of the tower} = \frac{d}{\cot \alpha - \cot \beta}$$

ANS : B

13. Statement I



In the figure $\tan 60^\circ = \frac{h_1}{x}$

$$\tan 30^\circ = \frac{h_2}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h_1}{x}$$

$$\Rightarrow x = \frac{h_1}{\sqrt{3}} \rightarrow (i)$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h_2}{x}$$

$$\Rightarrow x = \sqrt{3}h_2 \rightarrow (ii)$$

From (i) and (ii)

$$\frac{h_1}{\sqrt{3}} = \sqrt{3}h_2$$

$$\Rightarrow h_1 = 3h_2$$

$$\Rightarrow h_1 : h_2 = 3 : 1$$

Hence, Statement I is correct

Statement II

In the figure

$$\tan \theta_1 = \frac{h_1}{x} \text{ and } \tan \theta_2 = \frac{h_2}{x}$$

$$\Rightarrow x = \frac{h_1}{\tan \theta_1} \text{ and } x = \frac{h_2}{\tan \theta_2}$$

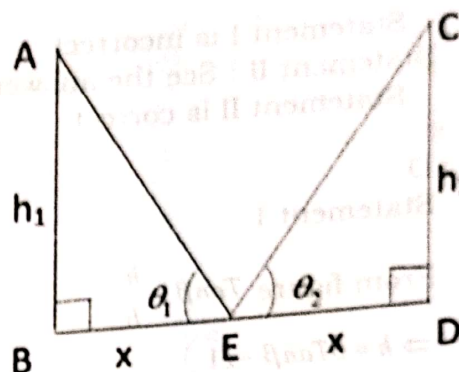
$$\Rightarrow \frac{h_1}{\tan \theta_1} = \frac{h_2}{\tan \theta_2}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

$$\Rightarrow h_1 : h_2 = \tan \theta_1 : \tan \theta_2$$

Hence, Statement II is correct.

Also, Statement II is the correct explanation of Statement I



ANS : A

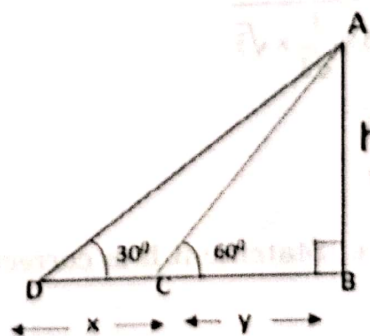
14. Statement I

$$\text{From the figure } \tan 30^\circ = \frac{h}{x+y}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y}$$

$$\Rightarrow x+y = \sqrt{3}h \rightarrow (1)$$

$$\text{Again } \tan 60^\circ = \frac{h}{y}$$



$$\Rightarrow \sqrt{3} = \frac{h}{y}$$

$$\Rightarrow y = \frac{h}{\sqrt{3}} \rightarrow (2)$$

$$\text{Now, from 1, } x + \frac{h}{\sqrt{3}} = \sqrt{3}h$$

It is not possible to find h value unless we know x value.

$\therefore$ Statement I is incorrect.

Statement II : See the answer given for question number 12.

$\therefore$ Statement II is correct.

ANS : D

15. Statement I

$$\text{From figure } \tan \beta = \frac{h}{b}$$

$$\Rightarrow h = b \tan \beta \rightarrow (1)$$

$$\text{Also, } \tan \alpha = \frac{h}{a}$$

$$\Rightarrow h = a \tan \alpha \rightarrow (2)$$

$$\text{Now, } (1) \times (2) \Rightarrow h^2 = ab \tan \alpha \cdot \tan \beta$$

$$\Rightarrow h = \sqrt{ab \tan \alpha \cdot \tan \beta}$$

Statement I is incorrect.

Statement II : Given $a = 9\text{mt}$, $b = 4\text{mt}$

$$\alpha = 30^\circ, \beta = 60^\circ$$

$$\text{Now, } h = \sqrt{ab \tan \alpha \cdot \tan \beta}$$

$$= \sqrt{9 \times 4 \times \tan 30^\circ \cdot \tan 60^\circ}$$

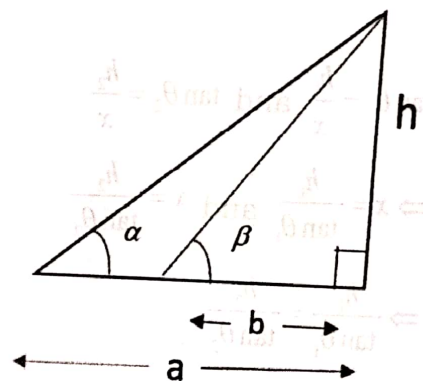
$$= \sqrt{36 \times \frac{1}{\sqrt{3}} \times \sqrt{3}}$$

$$= \sqrt{36}$$

$$= 6\text{mt}$$

Hence, Statement II is correct.

ANS : D



16. Statement I

From figure $\tan \alpha = \frac{h}{x}$ and $\tan \beta = \frac{h}{y}$

Also, $\alpha + \beta = 90^\circ$

$$\Rightarrow \tan(\alpha + \beta) = \tan 90^\circ$$

$$\Rightarrow \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \text{Not defined}$$

$$\Rightarrow 1 - \tan \alpha \cdot \tan \beta = 0$$

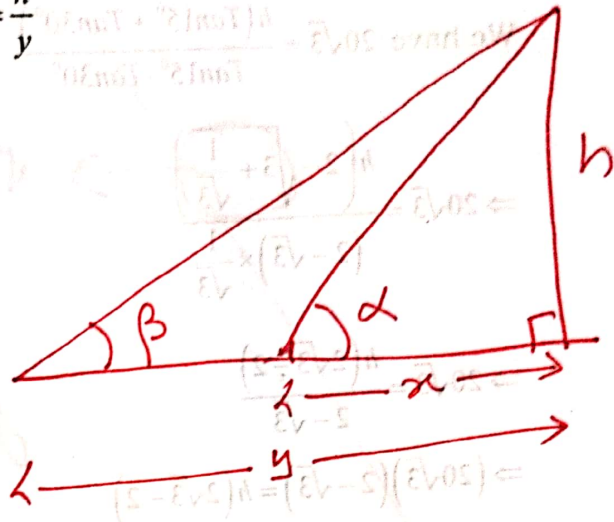
$$\Rightarrow \tan \alpha \cdot \tan \beta = 1$$

$$\Rightarrow \frac{h}{x} \cdot \frac{h}{y} = 1$$

$$\Rightarrow h^2 = xy$$

$$\Rightarrow h = \sqrt{xy}$$

Hence, Statement I is correct.



Statement II

Given $x = a^2 + b^2$, $y = 2ab$

We have, $h = \sqrt{xy}$

$$h = \sqrt{(a^2 + b^2)2ab} \neq a + b$$

Statement II is incorrect.

ANS: C

17. We have $d = \frac{h(\tan \alpha + \tan \beta)}{\tan \alpha \cdot \tan \beta}$

Given $h = 10 \text{ mts}$, $\alpha = 45^\circ$, $\beta = 30^\circ$

$$\therefore d = \frac{10(\tan 45^\circ + \tan 30^\circ)}{\tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{10\left(1 + \frac{1}{\sqrt{3}}\right)}{1 \cdot \frac{1}{\sqrt{3}}} = 10(\sqrt{3} + 1) \text{ mt}$$

ANS: A

18. Given $d = 20\sqrt{3}, \alpha = 15^\circ, \beta = 30^\circ$

We have $20\sqrt{3} = \frac{h(\tan 15^\circ + \tan 30^\circ)}{\tan 15^\circ \cdot \tan 30^\circ}$

$\Rightarrow 20\sqrt{3} = \frac{h\left(2 - \left(3 + \frac{1}{\sqrt{3}}\right)\right)}{(2 - \sqrt{3}) \times \frac{1}{\sqrt{3}}} \rightarrow \sqrt{3} + \frac{1}{\sqrt{3}}$

$\Rightarrow 20\sqrt{3} = \frac{h(2\sqrt{3} - 2)}{2 - \sqrt{3}}$

$\Rightarrow (20\sqrt{3})(2 - \sqrt{3}) = h(2\sqrt{3} - 2)$

$= \frac{10\left(1 + \frac{1}{\sqrt{3}}\right)}{1 + \frac{1}{\sqrt{3}}} = 10(\sqrt{3} + 1) \text{ m}$

ANS: A

8. Given $d = 20\sqrt{3}, \alpha = 15^\circ, \beta = 30^\circ$

We have $20\sqrt{3} = \frac{h(\tan 15^\circ + \tan 30^\circ)}{\tan 15^\circ \cdot \tan 30^\circ}$

$\Rightarrow 20\sqrt{3} = \frac{h\left(2 - \left(3 + \frac{1}{\sqrt{3}}\right)\right)}{(2 - \sqrt{3}) \times \frac{1}{\sqrt{3}}} \rightarrow \sqrt{3} + \frac{1}{\sqrt{3}}$

$\Rightarrow 20\sqrt{3} = \frac{h(2\sqrt{3} - 2)}{2 - \sqrt{3}}$

$\Rightarrow (20\sqrt{3})(2 - \sqrt{3}) = h(2\sqrt{3} - 2)$

$\Rightarrow 10\sqrt{3}(2 - \sqrt{3}) = h(\sqrt{3} - 1)$

$\Rightarrow h = \frac{10\sqrt{3}(2 - \sqrt{3})}{\sqrt{3} - 1}$

ANS: B

19. Given $\alpha = 45^\circ$, $d = 30\sqrt{2} \text{ m}$, $h = 2 \text{ m}$ then $\tan \beta$

$$\text{We have } 30\sqrt{2} = \frac{2(\tan 45^\circ + \tan \beta)}{\tan 45^\circ \cdot \tan \beta}$$

$$\Rightarrow 30\sqrt{2} = \frac{2(1 + \tan \beta)}{\tan \beta}$$

$$\Rightarrow 30\sqrt{2} \tan \beta = 2 + 2 \tan \beta$$

$$\Rightarrow (30\sqrt{2} - 2) \tan \beta = 2$$

$$\Rightarrow \tan \beta = \frac{2}{30\sqrt{2} - 2}$$

$$\Rightarrow \tan \beta = \frac{1}{15\sqrt{2} - 1}$$

ANS: B

20. We have $H = \frac{h(\tan \beta + \tan \alpha)}{\tan \beta - \tan \alpha}$

Given $\alpha = 30^\circ$, $\beta = 60^\circ$

$$\therefore H = \frac{h(\tan 60^\circ + \tan 30^\circ)}{\tan 60^\circ - \tan 30^\circ}$$

$$\Rightarrow H = \frac{h\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)}$$

$$\Rightarrow H = \frac{h(4)}{2}$$

$$\Rightarrow H = 2h$$

$$\Rightarrow h:H = 1:2$$

ANS: A

21. We have

$$\Rightarrow h = \frac{H(\tan \beta + \tan \alpha)}{\tan \beta - \tan \alpha}$$

$$\Rightarrow h = \frac{H\left(\frac{1}{\cot \beta} - \frac{1}{\cot \alpha}\right)}{\left(\frac{1}{\cot \beta} + \frac{1}{\cot \alpha}\right)}$$

$$H = \frac{h(\tan \beta + \tan \alpha)}{\tan \beta - \tan \alpha}$$

$$\Rightarrow h = \frac{H(\tan \beta - \tan \alpha)}{\tan \beta + \tan \alpha}$$

$$\Rightarrow h = \frac{H(\cot \alpha - \cot \beta)}{(\cot \alpha + \cot \beta)}$$

ANS : A

22. Given $\alpha = 0^\circ$

$$\text{Now } H = \frac{h(\tan \beta + \tan \alpha)}{(\tan \beta - \tan \alpha)}$$

$$\Rightarrow H = \frac{h(\tan \beta + \tan 0^\circ)}{(\tan \beta - \tan 0^\circ)}$$

$$\Rightarrow H = h$$

$$\text{Now, } h^2 + H^2 = 2h^2 \text{ or } 2H^2$$

$\therefore$ The possible answer is 2.

ANS: B

23.

In the above figure

AB and CD are ~~the~~ poles.

$\therefore AB = 16 \text{ mt}$, $CD = 10 \text{ mt}$

$\therefore AF = 6 \text{ mt}$

AC = Length of the wire.

$$\text{Now in } \triangle AFC, \sin 30^\circ = \frac{6}{l}$$

$$\Rightarrow \frac{1}{2} = \frac{6}{l}$$

$$\Rightarrow l = 12 \text{ cm}$$

24.

a) AB = Tower, BC = Shadow

$$\text{Now } \tan \theta = \frac{h}{\sqrt{3}h}$$

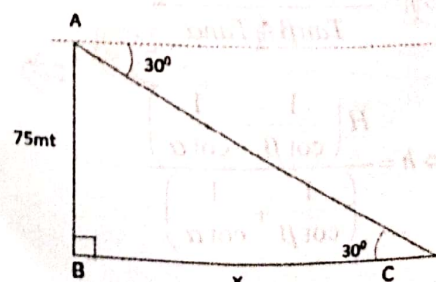
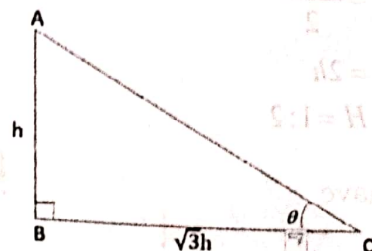
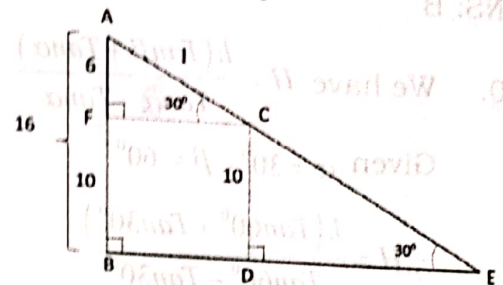
$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^\circ$$

b) A = Position of the observer
C = Position of the car

$$\text{Now } \tan 30^\circ = \frac{75}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x}$$

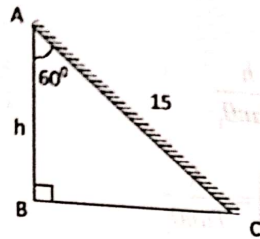


$$\Rightarrow x = 75\sqrt{3}$$

c) $\cos 60^\circ = \frac{h}{15}$

$$\Rightarrow \frac{1}{2} = \frac{h}{15}$$

$$\Rightarrow h = \frac{15}{2} \text{ mt}$$

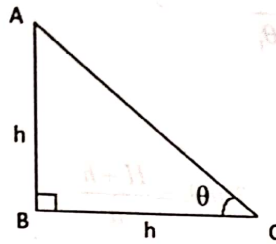


d) Here AB = Pole
Given AB = BC BC = Shadow of the pole

$$\therefore \tan \theta = \frac{AB}{BC}$$

$$\Rightarrow \tan \theta = 1$$

$$\Rightarrow \theta = 45^\circ$$



ANS: $a \rightarrow q, b \rightarrow t, c \rightarrow r, d \rightarrow s$

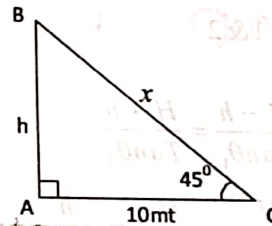
25. a) From figure, $\tan 45^\circ = \frac{h}{10}$ $\sin 45^\circ = \frac{h}{x}$

$$\Rightarrow 1 = \frac{h}{10}$$

$$\Rightarrow h = 10 \text{ mt}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{10}{x}$$

$$\Rightarrow x = 10\sqrt{2} \text{ mt}$$



The length of the entire tree = AB + BC

$$= h + x$$

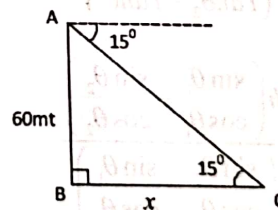
$$= 10 + 10\sqrt{2}$$

$$= 10(\sqrt{2} + 1) \text{ mt}$$

b) From figure $\tan 15^\circ = \frac{60}{x}$

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{60}{x}$$

$$\Rightarrow x = 60 \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \right)$$



c) Hence, AB = Building, CD = Hill A, B = Positions of the observer

Now, $\tan \theta_1 = \frac{H-h}{d}$

$$\tan \theta_2 = \frac{H}{d}$$

$$\Rightarrow d = \frac{H-h}{\tan \theta_1} \rightarrow 1$$

$$d = \frac{H}{\tan \theta_2} \rightarrow 2$$

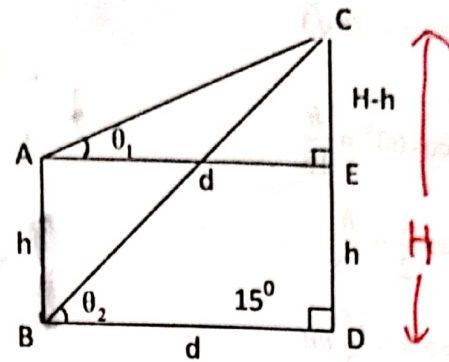
from 1 & 2

$$\frac{H-h}{\tan \theta_1} = \frac{H}{\tan \theta_2}$$

$$\Rightarrow \frac{H}{\tan \theta_1} - \frac{H}{\tan \theta_2} = \frac{h}{\tan \theta_1}$$

$$\Rightarrow H \left(\frac{\tan \theta_2 - \tan \theta_1}{\tan \theta_1 \tan \theta_2} \right) = \frac{h}{\tan \theta_1}$$

$$\Rightarrow H = \frac{h \tan \theta_2}{\tan \theta_2 - \tan \theta_1}$$



d) From the figure

$$\tan \theta_1 = \frac{H-h}{d}$$

$$\tan \theta_2 = \frac{H+h}{d}$$

$$\Rightarrow d = \frac{H-h}{\tan \theta_1} \rightarrow 1$$

$$\Rightarrow d = \frac{H+h}{\tan \theta_2} \rightarrow 2$$

from 1 & 2

$$\therefore \frac{H-h}{\tan \theta_1} = \frac{H+h}{\tan \theta_2}$$

$$\Rightarrow \frac{H}{\tan \theta_1} - \frac{H}{\tan \theta_2} = \frac{h}{\tan \theta_2} + \frac{h}{\tan \theta_1}$$

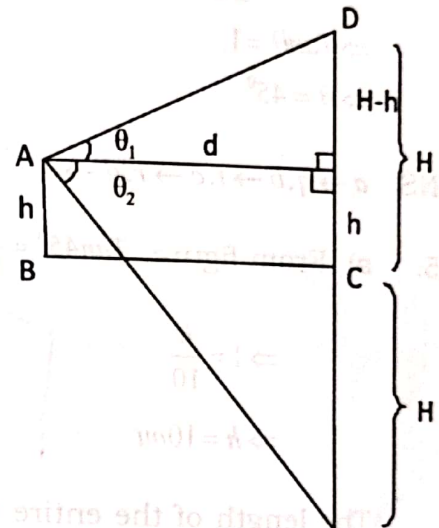
$$\Rightarrow H(\tan \theta_2 - \tan \theta_1) = h(\tan \theta_1 + \tan \theta_2)$$

$$\Rightarrow H = \frac{h(\tan \theta_1 + \tan \theta_2)}{(\tan \theta_2 - \tan \theta_1)}$$

$$\Rightarrow H = \frac{h \left(\frac{\sin \theta_1}{\cos \theta_1} + \frac{\sin \theta_2}{\cos \theta_2} \right)}{\left(\frac{\sin \theta_2}{\cos \theta_2} - \frac{\sin \theta_1}{\cos \theta_1} \right)}$$

$$\Rightarrow H = h \left(\frac{\sin \theta_1 \cdot \cos \theta_2 + \cos \theta_1 \cdot \sin \theta_2}{\sin \theta_2 \cdot \cos \theta_2 - \cos \theta_2 \cdot \sin \theta_1} \right)$$

$$\Rightarrow H = \frac{h \sin(\theta_1 + \theta_2)}{\sin(\theta_2 - \theta_1)}$$



ANS: $a \rightarrow t, b \rightarrow q, c \rightarrow r, d \rightarrow s$