3. QUADRATIC EXPRESSIONS-I ⓘ

Class: IX, FOUNDATION

SOLUTIONS 2025-26

TEACHING TASK

JEE MAINS LEVEL

01. $\frac{x^2-2x-3}{x+1}=0$

$\Rightarrow x^2-2x-3=0$ and $x+1 \neq 0 \Rightarrow x \neq -1$

$\Rightarrow (x-3)(x+1)=0$

$\Rightarrow x=3$ or -1

$\therefore$ solution set $=\{3\}$

Ans. C

02. $\sqrt{5x^2-6x+8}-\sqrt{5x^2-6x-7}=1$

let $5x^2-6x=a$

$\therefore \sqrt{a+8}-\sqrt{a-7}=1$

$\Rightarrow \sqrt{a+8}=1+\sqrt{a-7}$

S.O.B.S

$\Rightarrow a+8=1+a-7+2\sqrt{a-7}$

$\Rightarrow 14=2\sqrt{a-7}$

$\Rightarrow \sqrt{a-7}=7$

$\Rightarrow a-7=49$

$\Rightarrow a=56$

$\Rightarrow 5x^2-6x-56=0$

$\Rightarrow$

$(x-4)(5x+14)=0$

$\Rightarrow x=4$ or $x=-\frac{14}{5}$

Ans: B

03 a, b, c A.P. $\Rightarrow 2b = a + c$

$\Rightarrow ax^2 + bx + c = 0$

2 is a root

$\Rightarrow 4a + 2b + c = 0$

$\Rightarrow 4a + a + c + c = 0$

$\Rightarrow 5a + 2c = 0$

$\Rightarrow \frac{c}{a} = -\frac{5}{2}$ Ans: A

$\Rightarrow 2 \times \beta = -\frac{5}{2} \Rightarrow \beta = -\frac{5}{4}$

04 $30^\circ + 15^\circ = 45^\circ$

$\Rightarrow \tan(30^\circ + 15^\circ) = \tan 45^\circ$

$\Rightarrow \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \cdot \tan 15^\circ} = 1$

$\Rightarrow \frac{-p}{1-q} = 1 \quad \Rightarrow q - p = 1$ Ans. D

$\Rightarrow 2 + q - p = 3$

05 $f(x) = 2x^2 - 3x - 6 = 0$

$\alpha^2, \beta^2 \Rightarrow f(\sqrt{x}) = 2(\sqrt{x})^2 - 3\sqrt{x} - 6 = 0$

$\Rightarrow 4x^2 - 33x + 36 = 0$

$\alpha^2 + 2, \beta^2 + 2 \Rightarrow f(\sqrt{x - 2}) = 4(x-2)^2 - 33(x-2) + 36 = 0$ Ans. D

$\Rightarrow 4x^2 - 49x + 118 = 0$

06 $\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{(\alpha\beta)^3} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$

$= \frac{\left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)^3} = \frac{3abc - b^3}{c^3}$

Ans: C

07 $x^2-(5m-2)x+(4m^2+10m+25)=0$

perfect square $\Rightarrow \Delta=0$

$\Rightarrow b^2-4ac=0$

$[-(5m-2)]^2-4(1)(4m^2+10m+25)=0$

$\Rightarrow m=-\frac{4}{3}$ or 8

Ans: C

08 $(a^2+b^2)x^2+2(bc+ad)x+(c^2+d^2)=0$

Roots are real $\Rightarrow \Delta\geq 0$

$[2(bc+ad)]^2-4(a^2+b^2)(c^2+d^2)\geq 0$

$\Rightarrow ac=bd$

$\Rightarrow (bd)^2=a^2c^2$

$\Rightarrow a^2, bd, c^2$ are in G.P.

Ans. B

09 $x^{\sqrt{x}}=(\sqrt{x})^x$

$\Rightarrow x^{\sqrt{x}}=x^{\frac{x}{2}}$

$\Rightarrow \sqrt{x}=\frac{x}{2}$

$\Rightarrow x=\frac{x^2}{4}$

$x\left(1-\frac{x}{4}\right)=0$

$x=0$ or $\boxed{x=4}$

$x=0$ can not be possible

Also, by observation

$\boxed{x=1}$ is a root

$\therefore$ solution set $=\{1,4\}$

Ans. C

10. $(a+b+c)x^2-2(a+c)x+(a-b+c)=0$

Now, $\Delta=b^2-4ac_2$

$=[-2(a+c)]^2-4(a+b+c)(a-b+c)$

$= 4b^2 > 0$

Hence, the roots are rational

Ans. C

JEE ADVANCED

01 $\frac{\alpha+1}{\alpha} + \frac{\beta+1}{\beta}$

$= 1 + \frac{1}{\alpha} + 1 + \frac{1}{\beta} = 2 + \left(\frac{\alpha+\beta}{\alpha\beta}\right) = 2 + \frac{(-3)}{-2}$

$= \frac{7}{2}$ or $\frac{14}{4}$

Ans: A, C

02 $\frac{a+b}{2} = 9$ and $\sqrt{ab} = 4$

$a+b = 18$ and $ab = 16$

$\therefore$ $x^2 - 18x + 16 = 0$ (OR)

$\Rightarrow 2x^2 - 36x + 32 = 0$

Ans: B, C

03 statement I: $\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)^2$

$= \left(\frac{\alpha^2 - \beta^2}{\alpha\beta}\right)^2 = \frac{(\alpha+\beta)^2(\alpha-\beta)^2}{(\alpha\beta)^2}$

$= \frac{(\alpha+\beta)^2\left[(\alpha+\beta)^2 - 4\alpha\beta\right]}{(\alpha\beta)^2}$

$= \frac{\left(\frac{-b}{a}\right)^2\left[\left(\frac{-b}{a}\right)^2 - \frac{4c}{a}\right]}{\left(\frac{c}{a}\right)^2} = \frac{b^2(b^2-4ac)}{c^2a^2}$ (True)

statement II: Conceptual (True)

Ans: A

04 Statement I: $(b-c)x^2+2(c-a)x+(a-b)=0$

$\boxed{\Delta=b^2-4ac} \Rightarrow [2(c-a)]^2-4(b-c)(a-b)$

$=4[a^2+b^2+c^2-ab-bc-ca]$

$=\cancel{4}\,2[2a^2+2b^2+2c^2-2ab-2bc-2ca]$

$=2[(a-b)^2+(b-c)^2+(c-a)^2] \geq 0$

$\therefore$ Roots are real and distinct (True)

Statement II: Conceptual (True) Ans: B

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05 Assertion: $x^2+1=0$ (True)

$\Delta=0^2-4.1.1=-4<0$

Reason: Conceptual (True) Ans.. A

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06 Assertion: Conceptual (True)

Reason: Conceptual (True) $\boxed{\text{Ans. A}}$

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07 $x^2-15x+1=0$

α is a root $\Rightarrow \alpha^2-15\alpha+1=0$

$\Rightarrow 1-15\alpha=-\alpha^2$

$\Rightarrow 1-15\beta=-\beta^2$

Now, $\left(\frac{1}{\alpha}-15\right)^{-2}+\left(\frac{1}{\beta}-15\right)^{-2}$

$=\left(\frac{1-15\alpha}{\alpha}\right)^{-2}+\left(\frac{1-15\beta}{\beta}\right)^{-2}$

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$$= \left(\frac{-\alpha^2}{\alpha}\right)^{-2} + \left(\frac{-\beta^2}{\beta}\right)^{-2}$$

$$= \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{(15)^2 - 2\cdot 1}{(1)^2} = 225 - 2 = 223$$

~~Ans: A~~

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08 Given $\frac{1}{\alpha} + \frac{1}{\beta} = 4 \Rightarrow \frac{\alpha+\beta}{\alpha\beta} = 4$

$$\Rightarrow \frac{\left(-\frac{q}{p}\right)}{\left(\frac{r}{p}\right)} = 4 \Rightarrow \frac{-q}{r} = 4$$

$$\Rightarrow -2q = 8r$$

$$\Rightarrow -(p+r) = 8r$$

$$\Rightarrow -p = 9r$$

$$\Rightarrow \boxed{\frac{r}{p} = \frac{-1}{9}}$$

Also, $-q = 4r$

$$\Rightarrow -q = 4\left(\frac{-p}{9}\right) \Rightarrow \boxed{\frac{q}{p} = \frac{4}{9}}$$

Now $|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$$= \sqrt{\left(\frac{-q}{p}\right)^2 - 4\left(\frac{r}{p}\right)}$$

$$= \sqrt{\left(\frac{4}{9}\right)^2 - 4\left(\frac{-1}{9}\right)} = \frac{2\sqrt{13}}{9}$$

Ans: D

(7)

09 $\alpha = 2+\sqrt{3} \Rightarrow \beta = 2-\sqrt{3}$

$x^2 - 4x + (4-3) = 0$

$\Rightarrow x^2 - 4x + 1 = 0$

Ans: B

10 $x^2 - \left(-\frac{2}{3} + \frac{3}{7}\right)x + \left(-\frac{2}{3} \cdot \frac{3}{7}\right) = 0$

$\Rightarrow 21x^2 + 5x - 6 = 0$

Ans. D

11 $22^\circ + 23^\circ = 45^\circ$

$\Rightarrow \tan(22^\circ + 23^\circ) = \tan 45^\circ$

$\Rightarrow \dfrac{\tan 22^\circ + \tan 23^\circ}{1 - \tan 22^\circ \cdot \tan 23^\circ} = 1$

$\Rightarrow \dfrac{-a}{1-b} = 1 \Rightarrow a - b = -1$

Ans. -1

12 $|\alpha^2 - \beta^2| = (\alpha+\beta)(\alpha-\beta) = 24$

$\Rightarrow (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 24$

$\Rightarrow (6)\sqrt{(6)^2 - 4q} = 24$

$\Rightarrow q = 5$

Ans: 5

13 $\alpha, \beta \longrightarrow ax^2 + bx + c$

a) $cx^2 + bx + c = 0 \longrightarrow$ roots $\frac{1}{\alpha}, \frac{1}{\beta}$

b) $ax^2 - bx + c = 0 \longrightarrow -\alpha, -\beta$

c) $a\left(\frac{x}{2}\right)^2 + b\left(\frac{x}{2}\right) + c = 0 \longrightarrow 2\alpha, 2\beta$

d) $a(2x)^2 + b(2x) + c = 0 \longrightarrow \frac{\alpha}{2}, \frac{\beta}{2}$

Ans: t, s, r, q

8

14. a) $x^2-4x+4=0 \Rightarrow (x-2)^2=0 \Rightarrow x=2$

Roots are real and equal

b) $x^2+4x+3=0$

$\Delta = 16-4.1.3 = 4>0$

Roots are real and distinct and rational

c) $x^2+x-3=0$

$\Delta = 1-4.1.(-3)=0$ $\quad 1+12=13>0$

Real and Irrational

d) $x^2+x+1=0$

$\Delta = 1-4.1.1 = -3<0$

roots are imaginary

Ans. q, r, s, t

LEARNER'S TASK (CUQ'S)

01. Conceptual

Ans: D

02 $(x-2)(8-x)=0 \Rightarrow x^2-10x+16=0$

Linear term $=-10x$

Ans. B

03 $(x-a)(x-b)=b^2$

$\Rightarrow x^2-(a+b)x+ab-b^2=0$

$\Delta = [-(a+b)]^2-4\cdot 1.(ab-b^2)$

$= (a+b)^2-4ab+4b^2$

$= (a-b)^2+4b^2>0$

$\therefore$ Roots are real and distinct

Ans. A

04.	$x^2+x-1=0$ $\Delta=(1)^2-4(1)(-1)$ $=5>0$, not perfect square $\therefore$ Roots are Irrational Conjugate — Ans. D
05	$x^2-15-m(2x-8)=0$ $\Rightarrow x^2-2mx+8m-15=0$ $\Delta=0$ $\Rightarrow (-2m)^2-4(1)(8m-15)=0$ $\Rightarrow m^2-8m+15=0$ $\Rightarrow m=3$ or 5 — Ans. D
06	$f(x)=x^2+11x+13=0$ $f(x+4)=(x+4)^2+11(x+4)+13=0$ $\Rightarrow x^2+19x+73=0$ — Ans. B
07	$3-2i$ — Ans: A
08	α^2, β^2 — Ans: B
09	$\Delta=a^2-4bc$ — Ans: C
10	$ax^2+bx+c=0$ $\alpha+\beta=-\frac{b}{a}$ $\alpha+\alpha=-\frac{b}{a}$ $2\alpha=-\frac{b}{a}$ $\alpha=-\frac{b}{2a}$ $\alpha.\beta=\frac{c}{a}$ $\alpha.\alpha=\frac{c}{a}$ $\alpha=\sqrt{\frac{c}{a}}$ — Ans. D

JEE MAINS LEVEL

01 $x^2 - S_1 x + S_2 = 0$ Ans.. A

02 $a + b + c = 0$

$(b+c-a)x^2 + (c+a-b)x + (a+b-c) = 0$

$(-2a)x^2 + (-2b)x + (-2c) = 0$

$\Rightarrow ax^2 + bx + c = 0$

$\Delta = b^2 - 4ac$

$= (-a-c)^2 - 4ac$

$= (a+c)^2 - 4ac = (a-c)^2 > 0$ Since $a \neq c$

$\therefore$ Roots are real and distinct Ans. B

03 $\alpha = 3 - 2i \Rightarrow \beta = 3 + 2i$

$x^2 - 6x + (3+2i)(3-2i) = 0$ since $i^2 = -1$

$\Rightarrow x^2 - 6x + 9 + 4 = 0$ Ans: C

$\Rightarrow x^2 - 6x + 13 = 0$

04 $\sin\theta + \cos\theta = -\frac{b}{a} \rightarrow (1)$

$\sin\theta \cdot \cos\theta = \frac{c}{a} \rightarrow (2)$

$(1)^2 \Rightarrow 1 + \frac{2c}{a} = \frac{b^2}{a^2}$ Ans: A

$\Rightarrow a^2 + 2ac = b^2$

05 $\frac{-b}{a} = -1 \Rightarrow -\frac{(2a+3)}{a+1} = -1 \Rightarrow a = -2$

product $= \frac{3a+4}{a+1} = \frac{3(-2)+4}{(-2)+1} = 2$ Ans. B

06 $a(x-1)^2+b(x-1)+c=0$

$\Rightarrow a(x^2-2x+1)+b(x-1)+c=0$

$\Rightarrow ax^2+(b-2a)x+a-b+c=0$

$2x^2+8x+2=0$

$\boxed{a=2}$	$b-2a=8$ $\boxed{b=12}$	$a-b+c=2$ $2-12+c=2$ $\boxed{c=12}$

Ans. C

$\therefore\ b=c$

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07 $x^2+px+12=0$

$\Rightarrow 4^2+p(4)+12=0$

$\Rightarrow p=-7$

$x^2-7x+q=0$

Equal roots $\Delta=0$

$49-4q=0$

$\Rightarrow q=\frac{49}{4}$

Ans. D

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08 $x^{\frac{2}{3}}+x^{\frac{1}{3}}-2=0$

Let $x^{\frac{1}{3}}=a$

$\Rightarrow a^2+a-2=0$

$\Rightarrow (a+2)(a-1)=0$

$\Rightarrow a=1$ or -2

$x^{\frac{1}{3}}=1$ $\Rightarrow x=1$	$x^{\frac{1}{3}}=-2$ $x=-8$

Two roots

Ans. B

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09. $f(x)=4x^2+7x+2=0$

$f(\sqrt{x})=4(\sqrt{x})^2+7(\sqrt{x})+2=0$

$\Rightarrow 16x^2-33x+4=0$

Ans. D

10 $x^2+4x+3=0$

$\Rightarrow (x+1)(x+3)=0$

$\Rightarrow x=-1,-3$ Ans. B

JEE ADVANCED LEVEL

01) $x^2+3x-1=0$

$\Rightarrow x=\frac{-3\pm\sqrt{9+4}}{2}=\frac{-3\pm\sqrt{13}}{2}$ Ans. A, B

02 $x^2+x-3=0$

$\Delta=(1)^2-4(1)(-3)$

$=13>0$, Not a perfect square Ans. C, D

03 Statement I: $x^2-4x+4=0 \Rightarrow (x-2)^2=0$

$\Rightarrow x=2,2$ (True)

Statement II: Conceptual (True) Ans. A

04 Statement I: $f(x)=ax^2+bx+c$

$f(-x)=ax^2-bx+c$ (True)

Statement II: $x^2-3x+4=0$

$\Rightarrow$ Sum $=3$ (False) Ans: C

05 Assertion: Conceptual (False)

Reason: Conceptual (True) Ans: C

06 Assertion: $2x^2+3x+5=0$

$\Delta = 3^2 - 4.2.5$

$= -31 < 0$

$\therefore$ The equation does not have real roots (False)

Reason: True

Ans: D

07 $f(x) = x^2+x+1=0$

$f(x-1) = (x-1)^2+(x-1)+1=0$

Ans. C

$\Rightarrow x^2 - x + 1 = 0$

08 $f(x) = x^2+2x+5=0$

$f(x+1) = (x+1)^2+2(x+1)+5=0$

Ans: B

$\Rightarrow x^2+4x+8=0$

09 $x^2+x+3=0$

$\Delta = 1 - 4.1.3 = -11 < 0$

Ans: C

Roots are Imaginary

Ans. D

10 All options $\Delta = 0$

11 $(a^2-5a+3)x^2+(3a-1)x+2=0$

Root be $\alpha, 2\alpha$

$\alpha + 2\alpha = \frac{1-3a}{a^2-5a+3} \Rightarrow \alpha = \frac{1-3a}{3(a^2-5a+3)}$

$\alpha \cdot 2\alpha = \frac{2}{a^2-5a+3} \Rightarrow \alpha^2 = \frac{1}{a^2-5a+3}$

$$\Rightarrow \left[\frac{1-3a}{3(a^2-5a+3)}\right]^2 = \frac{1}{a^2-5a+3}$$

$$\Rightarrow \quad 3a = 2$$

Ans: 2

12 (a) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (5)^2 - 2.6 = 13$

Ans: 13

13 a) $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= \left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)$

$= \frac{3abc - b^3}{a^3}$

b) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right) = \frac{b^2-2ac}{a^2}$

c) $|\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{\left(-\frac{b}{a}\right)^2 - \frac{4c}{a}} = \frac{\sqrt{b^2-4ac}}{|a|}$

d) $\alpha+\beta = -\frac{b}{a}$

Ans: t, p, s, q

14 a) $K\alpha, K\beta \Rightarrow f\left(\frac{x}{k}\right) = 0$

b) $\frac{\alpha}{K}, \frac{\beta}{K} \Rightarrow f(Kx) = 0$

c) $\alpha^2, \beta^2 \Rightarrow f(\sqrt{x}) = 0$

d) $\alpha+K, \beta+K \Rightarrow f(x-k) = 0$

Ans: s, p, q, r

$\Longrightarrow$ THE END $\Longleftarrow$