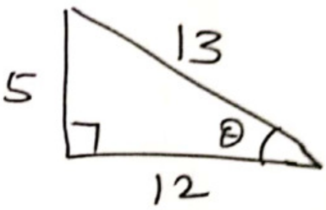
10. MULTIPLE AND SUB-MULTIPLE ANGLES

Class: VIII, MATHEMATICS FOUNDATION: SOLUTIONS

TEACHING TASK (2025/26)

JEE MAINS LEVEL

01. $90^\circ \le \theta \le 180^\circ \Rightarrow \theta \in Q_2$



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)$

$= \frac{-120}{169}$

Ans.. B

02. $\sqrt{2}\cdot \operatorname{cosec} 20^\circ \cdot \sec 20^\circ$

$= \sqrt{2}\cdot \frac{1}{\sin 20^\circ}\cdot \frac{1}{\cos 20^\circ}$

$= \frac{2\sqrt{2}}{2\sin 20^\circ \cdot \cos 20^\circ}$

$= \frac{4}{\sqrt{2} \times \sin 2 \times 20^\circ}$

$= 4 \cdot \sin 45^\circ \cdot \operatorname{cosec} 40^\circ$

Ans. D

03. $\tan 2\alpha - \tan\alpha(1 + \sec 2\alpha)$

Let $\alpha = 30^\circ$

$= \tan 60^\circ - \tan 30^\circ(1 + \sec 60^\circ)$

$= \sqrt{3} - \frac{1}{\sqrt{3}}(1 + 2)$

$= \sqrt{3} - \sqrt{3}$

$= 0$

Ans: C

04 $\cos\frac{\pi}{11}\cdot\cos\frac{2\pi}{11}\cdot\cos\frac{3\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{5\pi}{11}$

$$=\frac{1}{2\sin\frac{\pi}{11}}\left(2\sin\frac{\pi}{11}\cdot\cos\frac{\pi}{11}\right)\cos\frac{2\pi}{11}\cdot\cos\frac{3\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{5\pi}{11}$$

$$=\frac{1}{4\sin\frac{\pi}{11}}\left(2\sin\frac{2\pi}{11}\cdot\cos\frac{2\pi}{11}\right)\cos\frac{4\pi}{11}\cdot\cos\frac{3\pi}{11}\cdot\cos\frac{5\pi}{11}$$

$$=\frac{1}{8\sin\frac{\pi}{11}}\cdot\left(2\sin\frac{4\pi}{11}\cdot\cos\frac{4\pi}{11}\right)\cos\frac{3\pi}{11}\cdot\cos\frac{5\pi}{11}$$

$$=\frac{1}{8\sin\frac{\pi}{11}}\cdot\sin\frac{8\pi}{11}\cdot\cos\frac{3\pi}{11}\cdot\cos\frac{5\pi}{11}$$

$$=\frac{1}{8\sin\frac{\pi}{11}}\cdot\sin\left(\pi-\frac{3\pi}{11}\right)\cdot\cos\frac{3\pi}{11}\cdot\cos\frac{5\pi}{11}$$

$$=\frac{1}{16\sin\frac{\pi}{11}}\cdot\left(2\sin\frac{3\pi}{11}\cos\frac{3\pi}{11}\right)\cos\frac{5\pi}{11}$$

$$=\frac{1}{16\sin\frac{\pi}{11}}\sin\frac{6\pi}{11}\cdot\cos\frac{5\pi}{11}$$

$$=\frac{1}{16\sin\frac{\pi}{11}}\cdot\sin\left(\pi-\frac{5\pi}{11}\right)\cdot\cos\frac{5\pi}{11})$$

$$=\frac{1}{32\sin\frac{\pi}{11}}\cdot\left(2\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}\right)$$

$$=\frac{1}{32\sin\frac{\pi}{11}}\cdot\sin\frac{10\pi}{11}$$

$$=\frac{1}{32\sin\frac{\pi}{11}}\cdot\sin\left(\pi-\frac{\pi}{11}\right)$$

$$=\frac{1}{32\sin\frac{\pi}{11}}\cdot\sin\frac{\pi}{11}$$

$$=\frac{1}{32}$$

Ans: D

05 $\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left[\frac{1}{2}\left(a+\frac{1}{a}\right)\right]^2 - 1$

$= \frac{1}{2}\left(a+\frac{1}{a}\right)^2 - 1$

$= \frac{1}{2}\left(a^2+\frac{1}{a^2}+2\right) - 1$

$= \frac{1}{2}\left(a^2+\frac{1}{a^2}\right)$

Ans: A

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06 $\frac{\cos A+\sin A}{\cos A-\sin A} - \frac{\cos A-\sin A}{\cos A+\sin A}$

$= \frac{(\cos A+\sin A)^2-(\cos A-\sin A)^2}{(\cos A-\sin A)(\cos A+\sin A)}$

$= \frac{4\cos A\sin A}{\cos^2 A-\sin^2 A} = \frac{2\sin 2A}{\cos 2A} = 2\tan 2A$

Ans: A

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07 $\sin 18^\circ = \frac{\sqrt{5}-1}{4} \Rightarrow \sin^2 18^\circ = \frac{3-\sqrt{5}}{8} \rightarrow \alpha$

$\cos 36^\circ = \frac{\sqrt{5}+1}{4} \rightarrow \cos^2 36^\circ = \frac{3+\sqrt{5}}{8} \rightarrow \beta$

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - \left(\frac{3}{4}\right)x + \frac{1}{16} = 0 \Rightarrow 16x^2 - 12x + 1 = 0$

Ans: A

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08 $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = K\cos\theta$

let $\theta = 0^\circ < \frac{\pi}{16}$

$\sqrt{2+\sqrt{2+\sqrt{2+2}}} = K$

$K = 2$

Ans: A

~~Statement I:~~

09. Given $\frac{\tan 3A}{\tan A} = a$

Let $A = 45^\circ$

$\Rightarrow \frac{\tan 135^\circ}{\tan 45^\circ} = a \Rightarrow \boxed{a = -1}$

Now, $\frac{\sin 3A}{\sin A} = \frac{\sin 135^\circ}{\sin 45^\circ} = \frac{\sin(180^\circ - 45^\circ)}{\sin 45^\circ} = \frac{\sin 45^\circ}{\sin 45^\circ}$

$= 1.$

Now, option: B : $\frac{2a}{a-1} = \frac{2(-1)}{-1-1} = 1$ Ans: B

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10 Given $x = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}} = \tan\frac{\theta}{2}$

Now, $\frac{2x}{1-x^2} = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \tan\theta$ Ans. C

JEE ADVANCED LEVEL

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01. Given $\tan\theta = \frac{p^2-q^2}{2pq}$

We know, $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

$\Rightarrow \frac{p^2-q^2}{2pq} = \frac{2x}{1-x^2}$ ($\because$ let $\tan\frac{\theta}{2} = x$)

$\Rightarrow (p^2-q^2)x^2 + 4pqx - (p^2-q^2) = 0$

$\Rightarrow x = \frac{-4pq \pm \sqrt{16p^2q^2 + 4(p^2-q^2)^2}}{2(p^2-q^2)}$

$$= \frac{-4pq \pm 2\sqrt{(p^2-q^2)^2 + 4p^2q^2}}{2(p^2-q^2)}$$

$$= \frac{-4pq \pm 2\sqrt{(p^2+q^2)^2}}{2(p^2-q^2)}$$

$$= \frac{-\cancel{2}pq \pm (p^2+q^2)}{(p^2-q^2)}$$

$$x = \frac{-\cancel{2}pq + p^2+q^2}{(p^2-q^2)} = \frac{(p-q)^2}{p^2-q^2} = \frac{(p-q)(p-q)}{(p+q)(p-q)}$$

$$= \frac{p-q}{p+q}$$

$$\therefore \tan\frac{\theta}{2} = \frac{p-q}{p+q}$$

Ans. A, D

$$\therefore \cot\frac{\theta}{2} = \frac{p+q}{p-q}$$

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02 Given $xy + yz + zx = 1$

let $x = \tan A$, $y = \tan B$, $z = \tan C$

Now, $\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A = 1$

$\Rightarrow \tan A \cdot \tan B + \tan B \cdot \tan C = 1 - \tan C \cdot \tan A$

$\Rightarrow \tan B(\tan A + \tan C) = 1 - \tan C \cdot \tan A$

$$\Rightarrow \tan B = \frac{1 - \tan C \cdot \tan A}{\tan A + \tan C}$$

$$\Rightarrow \tan B = \frac{1}{\tan(A+C)}$$

$$\Rightarrow \tan B = \cot(A+C)$$

$$\Rightarrow \cot\left(\frac{\pi}{2}-B\right) = \cot(A+C)$$

$$\Rightarrow \frac{\pi}{2} - B = A + C$$

$$\Rightarrow A+B+C = \frac{\pi}{2}$$

$$\Rightarrow 2A+2B+2C = \pi$$

$$\Rightarrow 2A+2B = \pi - 2C$$

$$\Rightarrow \tan(2A+2B) = \tan(\pi - 2C)$$

$$\Rightarrow \frac{\tan 2A + \tan 2B}{1 - \tan 2A \cdot \tan 2B} = -\tan 2C$$

$$\Rightarrow \tan 2A + \tan 2B + \tan C = \tan 2A \cdot \tan 2B \cdot \tan 2C$$

$$\Rightarrow \sum \tan 2A = \prod \tan 2A$$

$$= \sum \frac{2\tan A}{1-\tan^2 A} = \prod\left(\frac{2\tan A}{1-\tan^2 A}\right)$$

$$= \sum\left(\frac{2x}{1-x^2}\right) = \prod\left(\frac{2x}{1-x^2}\right)$$

Similarly, we can prove

$$\sum\left(\frac{2x}{1-x^2}\right) = \prod\left(\frac{2y}{1-y^2}\right) = \prod\left(\frac{2z}{1-z^2}\right)$$

Ans: B, C, D

Q3 Statement I:

We know, $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$ and $\cos 72^\circ = \frac{\sqrt{5}-1}{4}$

Now, $LHS = \dfrac{\cos 36^\circ - \cos 72^\circ}{\cos 36^\circ + \cos 72^\circ}$

$$= \frac{\left(\frac{\sqrt{5}+1}{4}\right) - \left(\frac{\sqrt{5}-1}{4}\right)}{\left(\frac{\sqrt{5}+1}{4}\right) + \left(\frac{\sqrt{5}-1}{4}\right)} = \frac{1}{\sqrt{5}} \neq 2 \text{ (False)}$$

Statement II:

$LHS = \cos 15^\circ$

$= \cos(45^\circ - 30^\circ)$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}+\sqrt{2}}{4} = RHS$ (True) Ans. D

04 Statement I:

$LHS = 6\sin 20^\circ - 8\sin^3 20^\circ$

$= 2(3\sin 20^\circ - 4\sin^3 20^\circ)$

$= 2 \times \sin 3 \times 20^\circ$

$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3} \neq \frac{3\sqrt{3}}{2}$ (False)

Statement II: Conceptual (True) Ans. D

05 Assertion: Conceptual (True)

Reason: Conceptual (True) Ans. A

06 Assertion: $\cos 2A = \frac{1}{2}$

$\Rightarrow \cos 2A = \cos \frac{\pi}{6}$

$\Rightarrow 2A = 2n\pi \pm \frac{\pi}{6}$

$A = n\pi \pm \frac{\pi}{12}, n \in Z$

$\therefore$ A has infinite no. of solutions, (False)

Reason: Conceptual (~~Fool~~ Fails)

Ans: —

07. Let $A = 11\frac{1}{4}^\circ$

$\Rightarrow 2A = 22\frac{1}{2}^\circ$

$\Rightarrow \tan 2A = \tan 22\frac{1}{2}^\circ$

$\Rightarrow \frac{2\tan A}{1-\tan^2 A} = \sqrt{2}-1$

$\Rightarrow \frac{2x}{1-x^2} = \sqrt{2}-1$ (Let $x = \tan A$)

$\Rightarrow (\sqrt{2}-1)x^2 + 2x - (\sqrt{2}-1) = 0$

$\therefore x = \frac{-2 \pm \sqrt{(2)^2 + 4(\sqrt{2}-1)^2}}{2(\sqrt{2}-1)}$

$= \frac{-2 \pm \sqrt{16-8\sqrt{2}}}{2(\sqrt{2}-1)}$

$= \frac{-2 \pm 2\sqrt{4-2\sqrt{2}}}{2(\sqrt{2}-1)}$

$= \left[\frac{-1 \pm \sqrt{4-2\sqrt{2}}}{\sqrt{2}-1}\right] \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \frac{-(\sqrt{2}+1) \pm \sqrt{(4-2\sqrt{2})(\sqrt{2}+1)^2}}{2-1}$

$= -(\sqrt{2}+1) \pm \sqrt{(4-2\sqrt{2})(3+2\sqrt{2})}$

$= -(\sqrt{2}+1) \pm \sqrt{12+8\sqrt{2}-6\sqrt{2}-8}$

$x = -(\sqrt{2}+1) \pm \sqrt{4+2\sqrt{2}}$

$x = -(\sqrt{2}+1) + \sqrt{4+2\sqrt{2}}$ since, $11\frac{1}{4} \in Q_1$

$x = \tan 11\frac{1}{4} > 0$

$\therefore x = \sqrt{4+2\sqrt{2}} - (\sqrt{2}+1)$

Ans: C

$\boxed{08}$ $\cos^2 22\frac{1}{2}^\circ + \sin^2 22\frac{1}{2}^\circ = 1$ $\boxed{\text{Ans: A}}$

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09 $\cos^3\theta + \cos^3(120^\circ+\theta) + \cos^3(120^\circ-\theta)$

let $\theta = 0^\circ$

$\cos^3 0^\circ + \cos^3 120^\circ + \cos^3 120^\circ$

$= (1)^3 + 2[\cos(180^\circ - 60^\circ)]^3$

$= 1+2[-\cos 60^\circ)^3 = 1+2\left(\frac{-1}{8}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

Opt A: $\frac{3}{4}\cos 3\theta = \frac{3}{4}\cos 3\cdot 0^\circ = \frac{3}{4} \times 1 = \frac{3}{4}$ Ans: A

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10. $\cos^3\theta + \cos^3\left(\frac{2\pi}{3}+\theta\right) + \cos^3\left(\frac{4\pi}{3}+\theta\right) = a\cos 3\theta$

let $\theta = 0^\circ$

$\cos^3 0^\circ + \cos^3\frac{2\pi}{3} + \cos^3\frac{4\pi}{3} = a\cdot\cos 3\cdot 0^\circ$

$= 1 + \left(\frac{-1}{2}\right)^3 + \left(\frac{-1}{2}\right)^3 = a$

$= 1 - \frac{1}{8} - \frac{1}{8} = a$

$\Rightarrow a = \frac{3}{4}$

Ans.. A

11. Given $\cos^3 20° + \cos^3 100° + \cos^3 140°$ (10)

We have $\cos^3\theta + \cos^3(120°+\theta) + \cos^3(120°-\theta) = \frac{3}{4}\cos 3\theta$

put $\theta = 20°$

$\Rightarrow \cos^3 20° + \cos^3(120°+20°) + \cos^3(120°-20°) = \frac{3}{4}\cos 3\times 20°$

$\Rightarrow \cos^3 20° + \cos^3 100° + \cos^3 140° = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

Ans. B

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12. $a\cos\theta + b\sin\theta = c$

$\Rightarrow a\cos\theta = c - b\sin\theta$

$\Rightarrow a^2\cos^2\theta = c^2 + b^2\sin^2\theta - 2bc\sin\theta$

$\Rightarrow a^2(1-\sin^2\theta) = c^2 + b^2\sin^2\theta - 2b\sin\theta$

$\Rightarrow (a^2+b^2)\sin^2\theta - 2bc\sin\theta + c^2 - a^2 = 0$

This is a Q.E. in $\sin\theta$

Ans. A

$\therefore \sin\alpha \cdot \sin\beta = \frac{c^2-a^2}{a^2+b^2}$

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13. Given $a\cos\theta + b\sin\theta = c$

$\Rightarrow b\sin\theta = c - a\cos\theta$

Squaring and simplifying, we get

$(a^2+b^2)\cos^2\theta - 2ac\cos\theta + c^2 - b^2 = 0$

$\cos\alpha \cdot \cos\beta = \frac{c^2-b^2}{a^2+b^2}$

Ans. A

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14. Given $\cos(\alpha+\beta)$

$= \cos\alpha \cos\beta - \sin\alpha \cdot \sin\beta$

$= \left(\frac{c^2-b^2}{a^2+b^2}\right) - \left(\frac{c^2-a^2}{a^2+b^2}\right)$

$= \frac{a^2-b^2}{a^2+b^2}$ Ans: D

15 Given $(\sin 3A + \sin A)\sin A + (\cos 3A - \cos A)\cos A$

let $A = 0°$

$= (\sin 0° + \sin 0)\cos 0 + (\cos 0 - \cos 0)\cos 0$

$= 0$ Ans: D

16 $\cos 2\theta + \sin^2\alpha$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta} + \sin^2\alpha$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta} + 1 + \sin^2\alpha - 1$

$= \frac{2}{1+\tan^2\theta} + \sin^2\alpha - 1$

$= \frac{2}{1+2\tan^2\alpha+1} + \sin^2\alpha - 1$

$= \frac{1}{1+\tan^2\alpha} + \sin^2\alpha - 1$

$= \cos^2\alpha + \sin^2\alpha - 1 = 0$ Ans: D

17 a) $\frac{\sin 2\theta}{1+\cos 2\theta} = \frac{2 \overset{\sin}{\cancel{\tan}}\theta \cos\theta}{2\cos^2\theta} = \tan\theta$ (q)

b) $\frac{\sin 2\theta}{1-\cos 2\theta} = \frac{2\sin\theta\cos\theta}{2\sin^2\theta} = \cot\theta$ (p)

c) $\frac{1+\sin 2\theta+\cos 2\theta}{1+\sin 2\theta-\cos 2\theta}$

let $\theta = 30°$

$$= \frac{1+\sin 60^\circ + \cos 60^\circ}{1+\sin 60^\circ - \cos 60^\circ} = \sqrt{3} = \cot 30^\circ = \cot\theta \ (P)$$

d) $\dfrac{1+\sin\theta - \cos\theta}{1+\sin\theta+\cos\theta}$

Let $\theta = 60^\circ$

$$= \frac{1+\sin 60^\circ - \cos 60^\circ}{1+\sin 60^\circ + \cos 60^\circ} = \frac{1}{\sqrt{3}} = \tan 30^\circ = \tan\left(\frac{60^\circ}{2}\right)$$

$$= \tan\frac{\theta}{2} \ (r)$$

Ans: q, P, P, r

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18 a) $\sin^2\frac{3\pi}{5} + \sin^2\frac{4\pi}{5}$

$= \sin^2 108^\circ + \sin^2 144^\circ$

$= \sin^2(90^\circ + 18^\circ) + \sin^2(180^\circ - 36^\circ)$

$= \cos^2 18^\circ + \sin^2 36^\circ$

$= \left(\dfrac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 + \left(\dfrac{\sqrt{10-2\sqrt{5}}}{4}\right)^2 = \dfrac{5}{4} \ (r)$

b) $\cos^2\frac{3\pi}{5} + \cos^2\frac{4\pi}{5}$

$= \cos^2 108^\circ + \cos^2 144^\circ$

$= \sin^2 18^\circ + \cos^2 36^\circ$

$= \left(\dfrac{\sqrt{5}-1}{4}\right)^2 + \left(\dfrac{\sqrt{5}+1}{4}\right)^2 = \dfrac{3}{4} \ (P)$

c) $\cos^2\frac{\pi}{5} - \cos^2\frac{2\pi}{5}$

$= \cos^2 36^\circ - \cos^2 72^\circ$

$= \cos^2 36^\circ - \sin^2 18^\circ$

$= \left(\frac{\sqrt{5}+1}{4}\right)^2 - \left(\frac{\sqrt{5}-1}{4}\right)^2 = \frac{\sqrt{5}}{4}$ (q)

d) $\cos^2\frac{\pi}{12} + \cos^2\frac{3\pi}{12} + \cos^2\frac{5\pi}{12}$

$= \cos^2 15^\circ + \cos^2 45^\circ + \cos^2 75^\circ$

$= \cos^2 15^\circ + \cos^2 45^\circ + \sin^2 15^\circ$

$= 1 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{3}{2}$ (S)

Ans: r, p, q, s

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19 a) $\cos 290^\circ = \cos(270^\circ + 20^\circ) = \sin 20^\circ$

Also, $\sin 250^\circ = \sin(270^\circ - 20^\circ) = -\cos 20^\circ$

$\therefore \frac{1}{\sin 20^\circ} + \frac{1}{\sqrt{3}(-\cos 20^\circ)}$

$= \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sqrt{3}\sin 20^\circ \cos 20^\circ}$

$= \frac{\left(\frac{\sqrt{3}}{2}\right)\cos 20^\circ - \frac{1}{2}\sin 20^\circ}{\frac{\sqrt{3}}{4}\cdot 2\sin 20^\circ \cdot \cos 20^\circ} = \frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\frac{\sqrt{3}}{4}\cdot \sin 40^\circ}$

$= \frac{4}{\sqrt{3}} \frac{\sin(60^\circ - 20^\circ)}{\sin 40^\circ} = \frac{4}{\sqrt{3}}$ (s)

b) Given $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$

$= \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cdot \cos 10^\circ} = \frac{\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ}{\frac{1}{4}\cdot 2\sin 10^\circ \cdot \cos 10^\circ}$

$= \frac{4(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ)}{\sin 20^\circ}$

$= 4\,\frac{\sin(30^\circ - 10^\circ)}{\sin 20^\circ} = 4$ (P)

c) $\sqrt{3}\operatorname{cosec} 20^\circ - \sec 20^\circ$

$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$

$= \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cdot \cos 20^\circ} = \frac{\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ}{\frac{1}{4}\cdot 2\sin 20^\circ \cdot \cos 20^\circ}$

$= 4$ (P)

d) $6\sin 20^\circ - 8\sin^3 20^\circ = 2(3\sin 20^\circ - 4\sin^3 20^\circ)$

$= 2\times \sin 3\times 20^\circ = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$ (Q)

Ans: S, P, P, Q

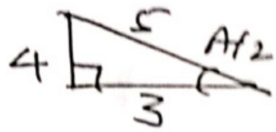
LEARNERS TASK (CUQ'S).

01 $\sin\theta = 1 \Rightarrow \cancel{\theta = 90^\circ}$

$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2(1)^2 = -1$ Ans. B

02 $\cos\theta = x \Rightarrow \sin\theta = \sqrt{1-x^2}$

$\sin 2\theta = 2\sin\theta\cos\theta = 2x\cdot\sqrt{1-x^2}$ Ans: B

03	$\sin 18^\circ \cdot \cos 36^\circ$ $= \frac{\sqrt{5}-1}{4} \cdot \frac{\sqrt{5}+1}{4} = \frac{1}{4}$	Ans. A
04	$\cos 2A = 2\cos^2 A - 1 = 2\left(\frac{3}{5}\right)^2 - 1 = \frac{-7}{25}$	Ans: C
05	$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2(2)}{1-(2)^2} = \frac{-4}{3}$	Ans. D
06	$\cos 3A = 4\cos^3 A - 3\cos A$ $= 4\theta^3 - 3\theta$	Ans: C
07	$\cos \frac{A}{2} = \frac{4}{5}$  $\sin \cancel{\cos} \frac{A}{2} = \frac{3}{5}$ $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$	Ans. B
08	$\tan 22\frac{1}{2}^\circ + \cot 22\frac{1}{2}^\circ$ $= (\sqrt{2}-1) + (\sqrt{2}+1) = 2\sqrt{2}$	Ans. B
09	$\sin 72^\circ \cdot \cos 72^\circ$ $= \frac{\sqrt{10+2\sqrt{5}}}{4} \cdot \left(\frac{\sqrt{5}-1}{4}\right)$	Ans. B
10	$\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A} = \frac{3x-x^3}{1-3x^2}$	Ans. D
	JEE MAINS LEVEL	

01. $\dfrac{3\cos\theta + \cos 3\theta}{3\sin\theta - \sin 3\theta}$

let $\theta = 30^\circ$

$= \dfrac{3\cos 30^\circ + \cos 90^\circ}{3\sin 30^\circ - \sin 90}$

$= \dfrac{3\cdot\frac{\sqrt{3}}{2} + 0}{3(\frac{1}{2}) - 1} = 3\sqrt{3}$

$\therefore$ opt: C $= \cot^3 A$

$= \cot^3 30^\circ$

$= 3\sqrt{3}$

Ans: C

02 $\dfrac{\sin 3A}{1+2\cos 2A}$

let $A = 30^\circ$

$= \dfrac{\sin 90^\circ}{1+2\cos 60}$

$= \dfrac{1}{1+2\cdot\frac{1}{2}} = \frac{1}{2}$

opt: A $\therefore \sin A = \sin 30^\circ = \frac{1}{2}$

Ans: A

03 $\cos\theta + \sin\theta = 1$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = a^2$

$\Rightarrow \sin 2\theta = a^2 - 1$

Ans: B

04 $\sin^2 2\alpha + \cos^2 2\alpha + \tan^2 2\alpha$

$= 1 + \tan^2 2\alpha$

$= \sec^2 2\alpha$

Ans: B

05 $x + \frac{1}{x} = 2\cos\theta$

C.O.B.S

$x^3 + \frac{1}{x^3} + 3\cdot x\cdot\frac{1}{x}\cdot\left(x + \frac{1}{x}\right) = 8\cos^3\theta$

$x^3 + \frac{1}{x^3} + 3(2\cos\theta) = 8\cos^3\theta$

$\Rightarrow x^3 + \frac{1}{x^3}$

$$\Rightarrow x^3 + \frac{1}{x^3} = 2(4\cos^3\theta - 3\cos\theta)$$
$$= 2\cdot\cos 3\theta$$

Ans.-B

06 $\frac{\sin\alpha}{a} = \frac{\cos\alpha}{b} \Rightarrow \tan\alpha = \frac{a}{b}$

Now, $a\sin2\alpha + b\cos2\alpha$

$$= a\left(\frac{2\tan\alpha}{1+\tan^2\alpha}\right) + b\left(\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\right)$$

$$= a\left(\frac{2\left(\frac{a}{b}\right)}{1+\left(\frac{a}{b}\right)^2}\right) + b\left(\frac{1-\left(\frac{a}{b}\right)^2}{1+\left(\frac{a}{b}\right)^2}\right) = b$$

Ans.-B

07 Given $\frac{\cos A - \cos 3A}{\cos A} + \frac{\sin A + \sin 3A}{\sin A}$

let $A = 30^\circ$

$$= \frac{\frac{\sqrt{3}}{2} - 0}{\frac{\sqrt{3}}{2}} + \frac{\frac{1}{2}+1}{\frac{1}{2}} = 4$$

Ans.-D

08 Given $\frac{\sin^2 3A}{\sin^2 A} - \frac{\cos^2 3A}{\cos^2 A}$

let $A = 30^\circ$

$$= \frac{\sin^2 90^\circ}{\sin^2 30^\circ} - \frac{\cos^2 90^\circ}{\cos^2 30^\circ}$$

$$= \frac{(1)^2}{\left(\frac{1}{2}\right)^2} - 0$$

$$= 4$$

opt B $\rightarrow 8\cos 2A$

$$= 8\cos 60^\circ$$
$$= 4$$

Ans.B

09 $\dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ}$

$= \dfrac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cdot \cos 10^\circ}$

$= \dfrac{\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ}{\frac{1}{4} \cdot 2\sin 10^\circ \cdot \cos 10^\circ} = 4$ Ans. A

10 $\tan(45^\circ + A) + \tan(45^\circ - A)$

= put $A = 0^\circ$

$\tan 45^\circ + \tan 45^\circ = 1 + 1 = 2$

Opt: B $\Rightarrow 2\sec 2A$

$= 2\sec 2 \times 0^\circ = 2\sec 0^\circ = 2$ Ans. B

11 $\tan 2A - \sec A \cdot \sin A$

Let $A = 30^\circ$

$= \sqrt{3} - \dfrac{2}{\sqrt{3}} \cdot \dfrac{1}{2} = \dfrac{2}{\sqrt{3}}$

Opt: B. $\tan A \cdot \sec 2A$

$= \tan 30^\circ \cdot \sec 60^\circ$ Ans. B

$= \dfrac{1}{\sqrt{3}} \cdot 2 = \dfrac{2}{\sqrt{3}}$

JEE ADVANCED LEVEL

01. $\tan 7\frac{1}{2}^\circ = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$

$= \sqrt{6}-\sqrt{3}-2+\sqrt{2}$

Also $(\sqrt{3}-\sqrt{2})(\sqrt{2}-1) = (\sqrt{2}-\sqrt{3})(1-\sqrt{2})$

Ans.. A, C, D

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02 $\cos 4A$

let $A = 30^\circ$

$\cos 120^\circ = \cos(180^\circ - 60^\circ) = -\cos 60^\circ = \frac{-1}{2}$

opt: A $\rightarrow \cos^2 2A - \sin^2 2A$

$= \cos^2 60^\circ - \sin^2 60$

$= \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$ ✓

opt: B : $1 - 2\sin^2 2A$

$= 1 - 2\sin^2 60^\circ$

$= 1 - 2\left(\frac{\sqrt{3}}{2}\right)^2$

$= 1 - \frac{3}{2} = \frac{-1}{2}$ ✓

op: D $= \frac{1-\tan^2 2A}{1+\tan^2 2A}$

$= \frac{1-\tan^2 60^\circ}{1+\tan^2 60^\circ}$

$= \frac{1-3}{1+3} = \frac{-1}{2}$ ✓

Ans. A, B, D

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03. Statement I: $\frac{x}{\cos\theta} = \frac{y}{\cos(\theta - \frac{2\pi}{3})} = \frac{z}{\cos(\theta + \frac{2\pi}{3})}$

let $\theta = 0^\circ$

$\therefore \frac{x}{1} = \frac{y}{-\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

$\therefore$ let $\frac{x}{1} = \frac{y}{(-\frac{1}{2})} = \frac{z}{-1/2} = K$ when $K \neq 0$

$\therefore x = K,\ y = \frac{-K}{2},\ z = \frac{-K}{2}$

Now, $x + y + z = K - \frac{K}{2} - \frac{K}{2} = 0$ (False)

Statement II:

$\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = K \cdot \sec 2\theta$

Let $\theta = 0^\circ$

$\Rightarrow 1 + 1 = K \cdot 1 \Rightarrow K = 2$ (True)

Ans: D

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04. Statement I:

$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$

$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{63}{65}$ (True)

Ans - A

Statement II: Conceptual (True)

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05 Assertion: ~~Fam~~ True since at $\sin 180^\circ = 0$ (True)

Ans - A

Reason: Conceptual (True)

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06 Assertion: $\cos 2A = 1 - 2\sin^2 2A$

True since $-1 \leq \cos 2A \leq 1$

Reason: Conceptual (True)

Ans - A

07 Let $A = 142\frac{1}{2}^\circ$

$\Rightarrow 2A = 285^\circ$

$\Rightarrow \tan 2A = \tan 285^\circ = \tan(270^\circ + 15^\circ) = -\cot 15^\circ$

$\Rightarrow \tan 2A = -\cot 15^\circ$

$\Rightarrow \dfrac{2\tan A}{1 - \tan^2 A} = -(2+\sqrt{3})$ (let $\tan A = x$)

$\Rightarrow \dfrac{2x}{1-x^2} = -(2+\sqrt{3})$

$\Rightarrow (2+\sqrt{3})x^2 - 2x - (2+\sqrt{3}) = 0$

$\Rightarrow x = \dfrac{2 \pm \sqrt{4 + 4(2+\sqrt{3})^2}}{2(2+\sqrt{3})}$

$= \dfrac{2 \pm 2\sqrt{8+4\sqrt{3}}}{2(2+\sqrt{3})}$

$= \dfrac{1 \pm \sqrt{8+4\sqrt{3}}}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= (2-\sqrt{3}) \pm \sqrt{\cancel{8}\,4(2+\sqrt{3})(2-\sqrt{3})^2}$

$= (2-\sqrt{3}) \pm \sqrt{4(2-\sqrt{3})}$

$= (2-\sqrt{3}) \pm \sqrt{8-4\sqrt{3}}$

$= (2-\sqrt{3}) \pm \sqrt{(\sqrt{6}-\sqrt{2})^2}$

$= (2-\sqrt{3}) \pm (\sqrt{6}-\sqrt{2})$

$= (2-\sqrt{3}) - (\sqrt{6}-\sqrt{2})$

$= 2+\sqrt{2}-\sqrt{3}-\sqrt{6}$ This value should be negative since $142\frac{1}{2}^\circ \in Q_2$

hence, $\tan 142\frac{1}{2}^\circ < 0$ Ans.- D

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08 $\cot 7\frac{1}{2}^\circ = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$ Ans. A

(see solved Examples)

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09 We know, $\cos\theta \cdot \cos(60^\circ+\theta)\cdot\cos(60^\circ-\theta) = \frac{1}{4}\cdot\cos 3\theta$

Let $\theta = 10^\circ$

$\cos 10^\circ \cdot \cos 70^\circ \cdot \cos 50^\circ = \frac{1}{4}\cos 30^\circ$

$\cos 10^\circ \cdot \cos 30^\circ \cdot \cos 70^\circ \cdot \cos 50^\circ = \frac{1}{4}\cos^2 30^\circ$

$= \cos 10^\circ \cdot \cos 30^\circ \cdot \cos 70^\circ \cdot \cos 50^\circ = \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{16}$ Ans.- D

---

10 Given $\cos\theta \cdot \cos(60^\circ+\theta)\cdot\cos(60^\circ-\theta) = \frac{1}{4}\cdot\cos 3\theta$

Let $\theta = 20^\circ$

$\therefore \cos 20^\circ \cdot \cos 80^\circ \cdot \cos 40^\circ = \frac{1}{4}\cos 60^\circ$

$\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{4}\cos^2 60^\circ$

$= \frac{1}{4}\left(\frac{1}{2}\right)^2 = \frac{1}{16}$

Ans.- A

11 We know, $\cos\theta \cdot \cos(60^\circ+\theta) \cdot \cos(60^\circ-\theta) = \frac{1}{4} \cdot \cos 3\theta$ (23)

Let $\theta = 25^\circ$

$\Rightarrow \cos 25^\circ \cdot \cos 85^\circ \cdot \cos 35^\circ = \frac{1}{4} \cos 75^\circ$

$= \frac{1}{4}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

$= \frac{\sqrt{3}-1}{8\sqrt{2}}$ Ans. C

---

12 $\cos 2\theta + b \sin 2\theta$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta} + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \frac{1-b^2}{1+b^2} + \frac{2b^2}{1+b^2} = \frac{1+b^2}{1+b^2} = 1$ Ans. 1

---

13 $\cos^2 A(3-4\cos^2 A)^2 + \sin^2 A(3-4\sin^2 A)^2$

$= (3\cos A - 4\cos^3 A)^2 + (3\sin A - 4\sin^3 A)^2$

$= (-\cos 3A)^2 + (\sin 3A)^2 = \cos^2 3A + \sin^2 3A = 1$

Ans: 1

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14 a) $\cos 2A = \sin 3A \Rightarrow 2A + 3A = 90^\circ$

$\Rightarrow A = 18^\circ$ (s)

b) $\cos 3A = \sin 7A \Rightarrow 3A + 7A = 90^\circ$

$\Rightarrow A = 9^\circ$ (r)

c) $\tan A = \cot 3A \Rightarrow A + 3A = 90^\circ$

$\Rightarrow A = 22\frac{1}{2}^\circ$ (p)

(24)

d) $\cot A + \tan 2A \Rightarrow A + 2A = 90^\circ$

$A = 30^\circ$ (q)

Ans: S, r, p, q

15 a) $\dfrac{1+\sin\theta - \cos\theta}{1+\sin\theta + \cos\theta}$

Let $\theta = 30^\circ$

$\dfrac{1+\frac{1}{2}-\frac{\sqrt{3}}{2}}{1+\frac{1}{2}+\frac{\sqrt{3}}{2}}$

$= \dfrac{3-\sqrt{3}}{3+\sqrt{3}}$

$= \dfrac{\sqrt{3}(\sqrt{3}-1)}{\sqrt{3}(\sqrt{3}+1)} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \tan 15^\circ$

$= \tan\dfrac{30^\circ}{2}$

$= \tan\dfrac{\theta}{2}$ (r)

b) $\dfrac{\sin\theta + \sin 2\theta}{1+\cos\theta + \cos 2\theta}$

Let $\theta = 30^\circ$

$= \dfrac{\frac{1}{2}+\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}+\frac{1}{2}}$

$= \dfrac{1+\sqrt{3}}{3+\sqrt{3}} = \dfrac{1+\sqrt{3}}{\sqrt{3}(\sqrt{3}+1)}$

$= \dfrac{1}{\sqrt{3}}$

$= \tan 30^\circ$

$= \tan\theta$ (p)

$+ \left[\cos 180^\circ \cos 35^\circ + \sin 180^\circ \sin 35^\circ\right]^2$

$= 2\left[\cos^2 60^\circ \cdot \cos^2 35^\circ + \sin^2 60^\circ \cdot \sin^2 35^\circ\right] + \cos^2 35^\circ$

$= 2\left[\frac{1}{4}\cos^2 35^\circ + \frac{3}{4}\sin^2 35^\circ\right] + \cos^2 35^\circ$

$= \left(\frac{1}{2}\cos^2 35^\circ + \cos^2 35^\circ\right) + \frac{3}{2}\sin^2 35^\circ$

$= \frac{3}{2}\left(\cos^2 35^\circ + \sin^2 35^\circ\right) = \frac{3}{2}$ Ans. B

$\Rightarrow$ THE END

Q c)

$$\frac{\boxed{3\cos\theta}+\cos 3\theta}{3\sin\theta-\sin 3\theta}$$

$$=\frac{3\cos\theta+(4\cos^3\theta-3\cos\theta)}{3\sin\theta-(3\sin\theta-4\sin^3\theta)}$$

$$=\cot^3\theta$$

d)

$$\frac{\cos 3\theta}{2\cos 2\theta-1}$$

$$=\frac{4\cos^3\theta-3\cos\theta}{2(2\cos^2\theta-1)-1}=\frac{\cos\theta(4\cos^2\theta-3)}{4\cos^3\theta-3}$$

$$=\cos\theta \quad (s)$$

Ans: r, $\boxed{p, \text{—}}$, s)

ADDITIONAL PRACTICE QUESTIONS

01. $\cot A + \tan A$

let $A = 30^\circ$

$= \cot 30^\circ + \tan 30^\circ$

$= \sqrt{3} + \frac{1}{\sqrt{3}}$

$= \frac{4}{\sqrt{3}}$

opt: C $\rightarrow 2\operatorname{cosec} 2\theta$

$= 2\operatorname{cosec} 30^\circ$

$= 2\left(\frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}}$

Ans: C

02 $\cos^6\theta + \sin^6\theta + K\sin^2 2\theta = 1$

$\Rightarrow$ let $\theta = 45^\circ$

$\left(\frac{1}{\sqrt{2}}\right)^6 + \left(\frac{1}{\sqrt{2}}\right)^6 + K(1)^2 = 1$

$\Rightarrow \frac{1}{8} + \frac{1}{8} + K = 1 \Rightarrow K = \frac{3}{4}$ Ans A

03 $\cos^2 25^\circ + \cos^2 95^\circ + \cos^2 145^\circ$

$= \cos^2(60^\circ - 35^\circ) + \cos^2(60^\circ + 35^\circ) + \cos^2(180^\circ - 35^\circ)$

$= [\cos 60^\circ \cdot \cos 35^\circ + \sin 60^\circ \cdot \sin 35^\circ]^2 + [\cos 60 \cos 35^\circ - \sin 60 \sin 35^\circ]^2$

$+ [\cos 180^\circ \cos 35^\circ + \sin 180^\circ \sin 35^\circ]^2$

$= 2[\cos^2 60^\circ \cdot \cos^2 35^\circ + \sin^2 60^\circ \cdot \sin^2 35^\circ] + \cos^2 35^\circ$

$= 2[\frac{1}{4}\cos^2 35^\circ + \frac{3}{4}\sin^2 35^\circ] + \cos^2 35^\circ$

$= \left(\frac{1}{2}\cos^2 35^\circ + \cos^2 35^\circ\right) + \frac{3}{2}\sin^2 35^\circ$

$= \frac{3}{2}(\cos^2 35^\circ + \sin^2 35^\circ) = \frac{3}{2}$ Ans: B

$\rightarrow$ THE END