9. PROPERTIES OF TRIANGLES-II

Class: IX. MATHEMATICS ①

FOUNDATION$^+$. SOLUTIONS

TEACHING TASK 2025/26

JEE MAINS

NOTE: In this chapter, Majority of the problems can be solved, by considering the given triangle is an equilateral triangle (Unless otherwise given). In the equilateral triangle, Assume that $a=b=c=1$ unit

We have,

① $a=b=c=1$

② $s=\frac{3}{2}$

③ $\Delta=\frac{\sqrt{3}}{4}$

④ $p_1=p_2=p_3=\frac{\sqrt{3}}{2}$

⑤ $r=\frac{\sqrt{3}}{6}$

⑥ $r_1=r_2=r_3=\frac{\sqrt{3}}{2}$

⑦ $R=\frac{1}{\sqrt{3}}$

01. $\frac{\Delta^2}{a^2+b^2+c^2}\left\{\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}+\frac{1}{r^2}\right\}$

(Consider this is an equilateral triangle)

$$=\frac{\left(\frac{\sqrt{3}}{4}\right)^2}{1^2+1^2+1^2}\left\{3\times\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}+\frac{1}{\left(\frac{\sqrt{3}}{6}\right)^2}\right\}$$

$$=\frac{1}{16}\left\{4+\frac{36}{3}\right\}=\frac{48}{48}=1$$

Ans. D

02 $\frac{s-a}{\Delta} = \frac{1}{8}, \quad \frac{s-b}{\Delta} = \frac{1}{12}, \quad \frac{s-c}{\Delta} = \frac{1}{24}$ ②

$$\frac{s-a+s-b+s-c}{\Delta} = \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$$

$$\Rightarrow \frac{3s-(a+b+c)}{\Delta} = \frac{1}{4}$$

$$\Rightarrow \frac{s}{\Delta} = \frac{1}{4}$$

$$\Rightarrow r = 4$$

$$\Delta = \sqrt{r \cdot r_1 r_2 r_3}$$

$$\Rightarrow \Delta = \sqrt{4 \cdot 8 \cdot 12 \cdot 24} = 96$$

$$\therefore s = 24 \quad \left(r = \frac{\Delta}{s}\right)$$

$$\frac{s-b}{\Delta} = \frac{1}{12} \Rightarrow \boxed{b = 16}$$

Ans: A

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03 let ΔABC be an equilateral triangle

$$= \frac{\sqrt{r r_1 r_2 r_3}}{2R \cdot r (\sin A + \sin B + \sin C)}$$

$$= \frac{\sqrt{\frac{\sqrt{3}}{6} \cdot \left(\frac{\sqrt{3}}{2}\right)^3}}{2 \cdot \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{6} \left(3 \cdot \sin 60^\circ\right)} = \frac{1}{2}$$

Ans. D

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04 $\left(\frac{1}{r} - \frac{1}{r_1}\right)\left(\frac{1}{r} - \frac{1}{r_2}\right)\left(\frac{1}{r} - \frac{1}{r_3}\right)$

$$= \left(\frac{6}{\sqrt{3}} - \frac{2}{\sqrt{3}}\right)^3 = \frac{64}{3\sqrt{3}}$$

opt: A: $\frac{abc}{\Delta^3} = \frac{1 \cdot 1 \cdot 1}{\left(\frac{\sqrt{3}}{4}\right)^3} = \frac{64}{3\sqrt{3}}$

Ans.. A

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05

$c = 2R \sin C$

$= 2R \sin 90^\circ$

$\boxed{c = 2R}$

$\tan \frac{C}{2} = \frac{r}{s-c}$

$\Rightarrow \tan 45^\circ = \frac{r}{s-c}$

$\Rightarrow s - c = r$

$\Rightarrow 2s - 2c = 2r$

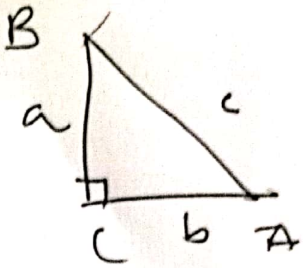
$\Rightarrow \boxed{a + b - c = 2r}$

Now

$= 2r + 2R$

$= a + b - c + c$

$= a + b$



Ans: A

06

$\frac{r_1 - r}{a} + \frac{r_2 - r}{b}$

$= \frac{\frac{\Delta}{s-a} - \frac{\Delta}{s}}{a} + \frac{\frac{\Delta}{s-b} - \frac{\Delta}{s}}{b}$

$\Delta\left(\frac{s-s+a}{as(s-a)} + \frac{s-s+b}{bs(s-b)}\right)$

$= \Delta\left(\frac{1}{s(s-a)} + \frac{1}{s(s-b)}\right)$

$= \frac{\Delta}{s}\left(\frac{s-b+s-a}{(s-a)(s-b)}\right)$

$= \frac{\Delta c}{s(s-a)(s-b)}$

$= \frac{\Delta \cdot c(s-c)}{s(s-a)(s-b)(s-c)}$

$= \frac{c(s-c)}{\Delta}$

$= \frac{c}{r_3}$

Ans: C

07

$\frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}}$

$= \frac{1}{\sqrt{\pi r_1^2}} + \frac{1}{\sqrt{\pi r_2^2}} + \frac{1}{\sqrt{\pi r_3^2}}$

$= \frac{1}{\sqrt{\pi}}\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right) = \frac{1}{\sqrt{\pi}} \cdot \frac{1}{r} = \frac{1}{\sqrt{\pi r^2}} = \frac{1}{\sqrt{A}}$

Ans. C

08

$\frac{\cos A}{p_1} + \frac{\cos B}{p_2} + \frac{\cos C}{p_3}$

$= 3 \cdot \frac{\cos A}{p_1}$

$= 3 \cdot \frac{\cos 60^\circ}{\left(\frac{\sqrt{3}}{2}\right)} = \sqrt{3}$

We have

$\therefore R = \frac{1}{\sqrt{3}}$

$\therefore \frac{1}{R} = \sqrt{3}$

Ans. B

09 $P_1 \cdot P_2 \cdot P_3 = (P_1)^3 = \left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{8}$

option D: $\boxed{\frac{2\Delta^2}{R}} = \frac{2\cdot\left(\frac{\sqrt{3}}{4}\right)^2}{\left(\frac{1}{\sqrt{3}}\right)} = \frac{3\sqrt{3}}{8}$

Ans. D

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10 $r_1 \cdot r_2 \cdot r_3 - r(r_1 r_2 + r_2 r_3 + r_3 r_1)$

$= (r_1)^3 - r\left[3(r_1 r_2)\right]$

$= \left(\frac{\sqrt{3}}{2}\right)^3 - \frac{\sqrt{3}}{6}\left[3\left(\frac{\sqrt{3}}{2}\right)^2\right] = 0$

Ans: A

JEE ADVANCED

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01 $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

$= \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$

$= \frac{1}{4}$

$\therefore \boxed{r=4}$

$\Delta^2 = r\, r_1 r_2 r_3$

$= 4 \cdot 8 \cdot 12 \cdot 24$

$\Delta = 96$

$\therefore r = \frac{\Delta}{s} \Rightarrow s = \frac{\Delta}{r}$

$= \frac{96}{4} = 24$

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$r_1 = \frac{\Delta}{s-a}$

$8 = \frac{96}{24-a}$

$\Rightarrow a = 12$

$r_2 = \frac{\Delta}{s-b}$

$12 = \frac{96}{24-b}$

$b = 16$

$r_3 = \frac{\Delta}{s-c}$

$24 = \frac{96}{24-c}$

$c = 20$

$\boxed{\text{Ans. } B, C, D}$

(5)

02 $(r_2-r_1)(r_3-r_1) = 2r_2r_3$

$= r_2r_3 - r_1r_2 - r_1r_3 + r_1^2 = 2r_2r_3$

$\Rightarrow \quad r_1^2 = r_1r_2 + r_2r_3 + r_3r_1$

$\Rightarrow \left(\frac{\Delta}{s-a}\right)^2 = \frac{\Delta^2}{(s-a)(s-b)} + \frac{\Delta^2}{(s-b)(s-c)} + \frac{\Delta^2}{(s-c)(s-a)}$

$\Rightarrow \frac{1}{(s-a)^2} = \frac{s-c+s-a+s-b}{(s-a)(s-b)(s-c)}$

$\Rightarrow \frac{1}{(s-a)} = \frac{3s-(a+b+c)}{(s-b)(s-c)} = \frac{s}{(s-b)(s-c)}$

$\Rightarrow$

$\Rightarrow \frac{s(s-a)}{(s-b)(s-c)} = 1$

$\Rightarrow \cot\left(\frac{A}{2}\right) = 1 \Rightarrow \frac{A}{2} = 45^\circ \Rightarrow A = 90^\circ$

$\therefore \angle B + \angle C = 90^\circ$ Ans: B, C

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03 Statement I: $a > b > c$

$\Rightarrow -a < -b < -c$

$s-a < s-b < s-c$

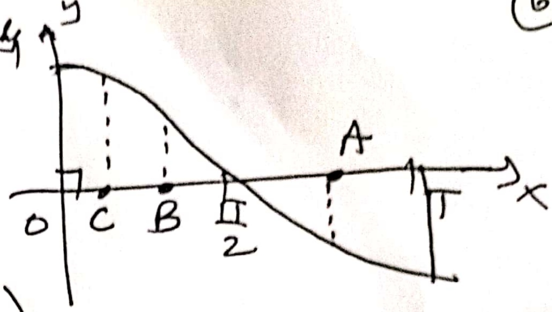
$\frac{s-a}{\Delta} < \frac{s-b}{\Delta} < \frac{s-c}{\Delta}$

$\frac{1}{r_1} < \frac{1}{r_2} < \frac{1}{r_3}$

$r_1 > r_2 > r_3$ (True)

Statement II: In $\triangle ABC$, we have $A+B+C=180°$ ⑥

We know, $\cos x$ is monotonically decreasing function in $(0, \pi)$



Given $a>b>c$

$\Rightarrow A>B>C$

$\Rightarrow \cos A < \cos B < \cos C$ (True)

Ans: A

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04 Statement I: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\Rightarrow \dfrac{3}{\sin 30°} = \dfrac{b}{\sin B}$

$\Rightarrow \sin B = \frac{3}{2}$, which is positive

$\sin B$ is positive in Q_1 and Q_2

$\Rightarrow$ B can take two values

∴ Hence, two triangles (True) ~~Ans~~:

Statement II: Let us consider it is an equilateral triangle

$$\frac{(a+b)\cos C+(b+c)\cos A+(c+a)\cos B}{\sin A+\sin B+\sin C}$$

$$= \frac{3(a+b)\cos C}{3\sin A} = \frac{(1+1)\cos 60°}{\sin 60} = \frac{2}{\sqrt{3}}$$

But $R = \frac{1}{\sqrt{2}}$ (False) Ans: C

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05 Statement I: $r_1 = r_2 = r_3$

$$\Rightarrow \frac{\Delta}{s-a} = \frac{\Delta}{s-b} = \frac{\Delta}{s-c}$$

$$\Rightarrow s-a = s-b = s-c$$

$$\Rightarrow a = b = c$$

ΔABC is an equilateral triangle (False)

Statement II: $r r_1 = r_2 r_3$

$$\Rightarrow \frac{\Delta}{s} \cdot \frac{\Delta}{s-a} = \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c}$$

$$\Rightarrow \frac{s(s-a)}{(s-b)(s-c)} = 1$$

$$\Rightarrow \cot\left(\frac{A}{2}\right) = \cot 45^\circ \quad \Rightarrow A = 90^\circ \text{ (False)}$$

Ans. B

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06 Assertion: Conceptual (True)

Reason: Conceptual (True) Ans: A

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07 Assertion: Conceptual (True) Ans. A

Reason: Conceptual (True)

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08 We know, A.M $\geq$ G.M.

$$\frac{\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}}{3} \geq \left(\frac{1}{P_1} \cdot \frac{1}{P_2} \cdot \frac{1}{P_3}\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{1}{6} \geq \left(\frac{1}{P_1 P_2 P_3}\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{1}{P_1 P_2 P_3} \leq \frac{1}{216}$$

$\Rightarrow P_1 P_2 P_3 \geq 216$

$\therefore$ least value $= 216$

Ans.- D

09 $\frac{b^2 p_1}{c} + \frac{c^2 p_2}{a} + \frac{a^2 p_3}{b}$

Since $\Delta = \frac{1}{2} a p_1$

$\Rightarrow p_1 = \frac{2\Delta}{a}$

$\Rightarrow \frac{b^2 \cdot 2\Delta}{ac} + \frac{c^2 2\Delta}{ab} + \frac{a^2 2\Delta}{ac}$

$= 2\Delta\left(\frac{b^2}{ac} + \frac{c^2}{ab} + \frac{a^2}{ac}\right)$

We know $A \cdot M \geq G \cdot M.$

$\Rightarrow \frac{\frac{b^2}{ac} + \frac{b^2}{ab} + \frac{c^2}{ac}}{3} \geq \left(\frac{b^2}{ac} * \frac{c^2}{ab} * \frac{a^2}{ac}\right)^{\frac{1}{3}}$

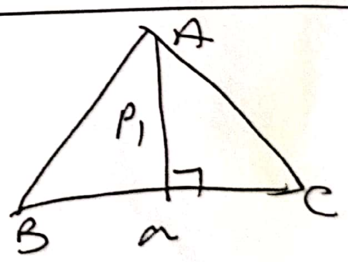
$\Rightarrow \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} \geq 3$

$\Rightarrow 2\Delta\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right) \geq 6\Delta$

$\therefore$ minimum value $= 6\Delta$

Ans. D

10 $p_1, p_2, p_3 \longrightarrow A \cdot P$



$2p_2 = p_1 + p_3$

$\Delta = \frac{1}{2} \cdot a \cdot p_1$

$p_1 = \frac{2\Delta}{a}$

$\Rightarrow 2 \cdot \frac{2\Delta}{b} = \frac{2\Delta}{a} + \frac{2\Delta}{c}$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \longrightarrow A \cdot P$

$\Rightarrow a, b, c \longrightarrow H \cdot P$

Ans: B

11 Let $\triangle ABC$ be an equilateral triangle

$$\frac{a}{\tan A} + \frac{b}{\tan B} + \frac{c}{\tan C}$$

$$= \frac{3a}{\tan A} = \frac{3 \cdot 1}{\tan 60^\circ} = \sqrt{3}$$

A) $2r = 2 \cdot \frac{\sqrt{3}}{6} = \frac{1}{\sqrt{3}}$

B) $r + 2R = \frac{\sqrt{3}}{6} + \frac{2}{\sqrt{3}} = \frac{5}{2\sqrt{3}}$

C) $2r + R = \frac{2\sqrt{3}}{6} + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

D) $2(r+R) = 2\left(\frac{\sqrt{3}}{6} + \frac{1}{\sqrt{3}}\right) = \sqrt{3}$ ✓

Ans: D

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12. Let $\triangle ABC$ be an equilateral triangle

$$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2}$$

$$= 3\cos^2\frac{A}{2} \qquad = 3 \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{9}{4}$$

$$= 3\cos^2 30^\circ$$

option: C $= 2 + \frac{r}{2R} = 2 + \frac{\left(\frac{\sqrt{3}}{6}\right)}{2\left(\frac{1}{\sqrt{3}}\right)} = \frac{9}{4}$

Ans. C

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13 Let $\triangle ABC$ be an equilateral triangle

$$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$$

$$= 3\sin^2\frac{A}{2} = 3\sin^2 30^\circ = \frac{3}{4}$$

option: D $= 1 - \frac{r}{2R} = 1 - \frac{\left(\frac{\sqrt{3}}{6}\right)}{2\left(\frac{1}{\sqrt{3}}\right)} = 1 - \frac{1}{4} = \frac{3}{4}$

Ans: D

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14. Consider $\triangle ABC$ is an equilateral triangle

$$\Sigma \frac{1}{\frac{3}{2}-1}\left(\tan\frac{60^\circ}{2} - \tan\frac{60^\circ}{2}\right) = 0$$

Ans. 0

15 Given r_1, r_2, r_3 are in H.P

$$\Rightarrow \frac{1}{r_1}, \frac{1}{r_2}, \frac{1}{r_3} \rightarrow A.P.$$

$$\Rightarrow \frac{2}{r_2} = \frac{1}{r_1} + \frac{1}{r_3}$$

$$\Rightarrow \frac{2}{r_2} = \frac{1}{r} - \frac{1}{r_2} \quad \text{since } \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$\Rightarrow \frac{3}{r_2} = \frac{1}{r}$$

$$\Rightarrow \frac{r_2}{r} = 3$$

Ans. 3

16 a) let $\triangle ABC$ be an equilateral triangle

$$= (r_1 - r)(r_2 - r)(r_3 - r)$$

$$= (r_1 - r)^3$$

$$= \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6}\right)^3 = \frac{\sqrt{3}}{9}$$

option: q : $4Rr^2 = 4 \cdot \frac{1}{\sqrt{3}} \cdot \left(\frac{\sqrt{3}}{6}\right)^2 = \frac{\sqrt{3}}{9}$

b) let $\triangle ABC$ be an equilateral triangle

$$: (r_1 + r_2)(r_2 + r_3)(r_3 + r_1)$$

$$= (r_1 + r_2)^3 = \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)^3 = 3\sqrt{3}$$

option r : $4Rs^2 = 4 \times \frac{1}{\sqrt{3}} \times \left(\frac{\sqrt{3}}{2}\right)^2 = 3\sqrt{3}$

c) $r_1 + r_2 + r_3 - r = 4R$ (Conceptual)

d) $r + r_3 + r_1 - r_2 = 4R\cos B$ (Conceptual)

Ans: q, r, p, t

17\. a) $r_1 = 3, \ r_2 = 10, \ r_3 = 15$

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{3} + \frac{1}{10} + \frac{1}{15} = \frac{1}{2}$$

$\therefore \boxed{r = 2}$

$\Delta^2 = r \cdot r_1 \cdot r_2 \cdot r_3 \Rightarrow \boxed{\Delta = 30}$

$r = \frac{\Delta}{s} \Rightarrow \boxed{s = 15}$

$r_1 = \frac{\Delta}{s-a} \Rightarrow 3 = \frac{30}{15-a} \Rightarrow \boxed{a = 5}$

Similarly $\boxed{b = 12}$, $\boxed{c = 13}$

We know $\Delta = \frac{abc}{4R}$

$$\Rightarrow R = \frac{abc}{4\Delta} = \frac{13}{2} \quad (s)$$

b) Consider ΔABC, is an equilateral triangle

$$\frac{\sqrt{r r_1 r_2 r_3}}{2Rr(\sin A + \sin B + \sin C)} = \frac{\sqrt{\Delta^2}}{2Rr(3\sin A)}$$

$$= \frac{\left(\frac{\sqrt{3}}{4}\right)}{2 \cdot \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{6} (3 \times \sin 60^\circ)} = \frac{1}{2} \quad (p)$$

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c) r_1, r_2, r_3 are in H.P $\Rightarrow \frac{r_2}{r} = 3$ (Repeated)

(12)

d) We have $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$

$= \frac{3}{2} - \frac{1}{2}(\cos A + \cos B + \cos C)$

Also, $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$

$= 1 + \frac{3}{7}$ since $7r = 3R$

$= \frac{10}{7}$ $\quad \frac{r}{R} = \frac{3}{7}$

$\therefore \sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$

$= \frac{3}{2} - \frac{1}{2}\left(\frac{10}{7}\right)$

$= \frac{3}{2} - \frac{5}{7} = \frac{11}{14}$ (q)

Ans: s, p, t, q

LEARNERS TASK (CUQ'S)

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01 $a = 13,\ b = 14,\ c = 15$

$s = \frac{13+14+15}{2} = \frac{42}{2} = 21.$

$\therefore r = \frac{\Delta}{s}$ $\quad \Delta = \sqrt{s(s-a)(s-b)(s-c)} = 84$

$\therefore r = \frac{84}{21} = 4$

Ans. A

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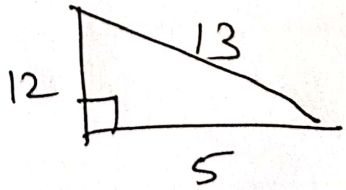
02 $a = 5,\ b = 12,\ c = 13$



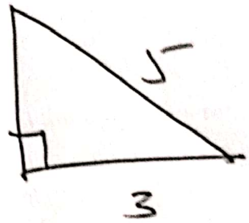
$\Delta = \frac{1}{2} \times 12 \times 5 = 30$

$s = \frac{5+12+13}{2} = \frac{30}{2} = 15$

$\therefore r_1 = \frac{\Delta}{s-a} = \frac{30}{15-5} = \frac{30}{10} = 3$ Ans. B

03 $r_3 = \frac{\Delta}{s-c} = \frac{48}{16-4} = \frac{48}{12} = 4$ Ans C

04 $a=4,\ b=5,\ c=3$



$\Delta = \frac{1}{2} \times 4 \times 3 = 6$

$s = \frac{4+5+3}{2} = 6$

$\therefore r = \frac{\Delta}{s} = \frac{6}{6} = 1$ Ans: A

05 $r_2 = \frac{\Delta}{s-b} = \frac{24}{15-7} = \frac{24}{8} = 3$ Ans. D

06 $\frac{1}{r} = \frac{1}{8} + \frac{1}{12} + \frac{1}{24} = \frac{1}{4} \Rightarrow \boxed{r=4}$

$\Delta^2 = 4 \cdot 8 \cdot 12 \cdot 24 \Rightarrow \boxed{\Delta = 96}$

$r = \frac{\Delta}{s} \Rightarrow s = \frac{\Delta}{r} = \frac{96}{4} = 24$ $\boxed{s=24}$

$\therefore r_1 = \frac{\Delta}{s-a}$

$\Rightarrow 8 = \frac{96}{24-a}$

$\Rightarrow \boxed{a=12}$ Ans B

07 $r \cdot r_1 \cdot r_2 \cdot r_3 = \Delta^2$

$r_1 r_2 r_3 = \frac{\Delta^2}{r} = \frac{(rs)^2}{r} = rs^2$ Ans: C

08 $\frac{1}{r_1} : \frac{1}{r_2} : \frac{1}{r_3}$

$= \frac{s-a}{\Delta} : \frac{s-b}{\Delta} : \frac{s-c}{\Delta}$

$s = \frac{30+24+18}{2}$

$\boxed{s = 36}$

$= s-a : s-b : s-c$

$= 36 - 30 : 36 - 24 : 36 - 18$

$= 6 : 12 : 18$

$= 1 : 2 : 3$

Ans.. A

Ans: D

09 $r = \frac{\Delta}{s} = \frac{8}{1.5} = \frac{80}{15} = \frac{16}{3}$

Ans: B

10 $\Delta^2 = 2 \cdot 4 \cdot 6 \cdot 12 \Rightarrow \Delta = 24$

JEE MAINS LEVEL

01. Conceptual $r = (s-b)\tan\left(\frac{B}{2}\right)$ Ans. B

02 Conceptual Ans: B

03 Let ΔABC be an equilateral triangle

$(r_1 - r)(r_2 - r)(r_3 - r)$

$= (r_1 - r)^3$

$= \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6}\right)^3 = \left(\frac{1}{\sqrt{3}}\right)^3 = \frac{1}{3\sqrt{3}}$

Opt: B. $4Rr^2 = 4 \cdot \frac{1}{\sqrt{3}} \cdot \left(\frac{\sqrt{3}}{6}\right)^2 = \frac{1}{3\sqrt{3}}$ Ans B

04. $\sqrt{\frac{r r_1}{r_2 r_3}} = \sqrt{\frac{\frac{\Delta}{s} \cdot \frac{\Delta}{s-a}}{\frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c}}} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \tan\frac{A}{2}$

Ans: B

05 We have

$r_1 - r = 4R\sin^2\left(\frac{A}{2}\right)$

$7r - r = 4\cdot 6r\cdot\sin^2\left(\frac{A}{2}\right)$

$\frac{1}{4} = \sin^2\left(\frac{A}{2}\right)$

$\sin\frac{A}{2} = \frac{1}{2}$

$\Rightarrow \frac{A}{2} = 30^\circ$

$\Rightarrow A = 60^\circ = \frac{\pi}{3}$ Ans. D

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06 $r\left(\cot\frac{B}{2} + \cot\frac{C}{2}\right)$

$= r\left[\sqrt{\frac{s(s-b)}{(s-a)(s-c)}} + \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}\right]$

$= r\left[\frac{\sqrt{s}(s-b) + \sqrt{s}(s-c)}{\sqrt{\blacksquare(s-a)(s-b)(s-c)}}\right]$

$= \frac{r}{\Delta}\left[s(2s-b-c)\right] = \cancel{\frac{r}{\Delta}}\cancel{s}\cdot a = a$

Ans. A

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07 Consider as ABC, is an equilateral triangle

$r^2\cot\frac{A}{2}\cdot\cot\frac{B}{2}\cdot\cot\frac{C}{2}$

$= r^2\left(\cot\frac{A}{2}\right)^3 = \left(\frac{\sqrt{3}}{6}\right)^2(\cot 30)^3 =$

$= \frac{3}{36}\times 3\sqrt{3} = \frac{\sqrt{3}}{4} = \Delta$ $\therefore$ Ans. C

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08 We have $r_2 + r_3 = 4R\cos^2\left(\frac{A}{2}\right)$

$= 4R\cos^2 45^\circ$ Since $\angle A = 90^\circ$

$= 2R$

$\therefore \cos^{-1}\left(\frac{R}{2R}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$ Ans: B

09

09 $r_1 + r = r_2 + r_3$

$$\frac{\Delta}{s-a} + \frac{\Delta}{s} = \frac{\Delta}{s-b} + \frac{\Delta}{s-c}$$

$$= \frac{s+s-a}{s(s-a)} = \frac{s-c+s-b}{(s-b)(s-c)}$$

$$= \frac{b+c}{s(s-a)} = \frac{a}{(s-b)(s-c)}$$

$$\Rightarrow \frac{b+c}{a} = \frac{s(s-a)}{(s-b)(s-c)} = \cot^2\left(\frac{A}{2}\right)$$

$$= \cot^2\left(\frac{60}{2}\right) = 3$$

$\therefore b+c = 3a$

$\Rightarrow a+b+c = 4a$

$\Rightarrow 2s = 4a$

$\Rightarrow s = 2a$

$\therefore \frac{s}{a} = 2$

Ans. B

10 $r_1 = 2r_2 = 3r_3$

$$\frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c}$$

let $\frac{1}{s-a} = \frac{2}{s-b} = \frac{3}{s-c} = k$

$s-a = \frac{1}{k}$; $s-b = \frac{2}{k}$, $s-c = \frac{3}{k}$

$(s-a) + (s-b) + (s-c) = \frac{6}{k}$

$$\boxed{s = \frac{6}{k}}$$

$s-a = \frac{1}{k}$

$a = s - \frac{1}{k} = \frac{6}{k} - \frac{1}{k}$

$= \frac{5}{k}$

Similarly

$b = \frac{4}{k}$

$c = \frac{3}{k}$

$\frac{a}{b} = \frac{\left(\frac{5}{K}\right)}{\left(\frac{4}{K}\right)} = \frac{5}{4}$

$\frac{b}{c} = \frac{4}{3}$; $\frac{c}{a} = \frac{3}{5}$

$\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$

$= \frac{5}{4} + \frac{4}{3} + \frac{3}{5}$

$= \frac{191}{60}$

Ans. D

JEE ADVANCED LEVEL

Ans.. A, B

Conceptual

Ans : A, C

Conceptual

Statement I: let $a = 2$, $b = 3$

$\Delta = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \cdot 2 \cdot 3 \cdot \sin C$

$= 3 \sin C$

We know $-1 \le \sin C \le +1$

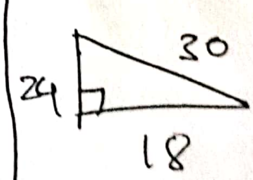
$-3 \le 3 \sin C \le 3$

$\therefore$ Area can not exceed $= 3$ (True)

Ans.. A

Statement II: Conceptual (True)

Statement I: $a = 18$, $b = 24$, $c = 30$



Now $s = 36$; $\Delta = \frac{1}{2} \times 18 \times 24 = 216$

$r_1 = \frac{\Delta}{s-a} = 12$, $r_2 = 18$, $r_3 = 36$

Now, $l = r_2 - r_1$
$= 18 - 12$
$= 6$

$m = r_3 - r_2$
$= 36 - 18$
$= 18$

$n = r_3 - r_1$
$= 36 - 12$
$= 14$

Ascending Order $= l, m, n$

Descending order $= n, m, l$ (False)

Statement II: $a=13,\ b=14,\ c=15$

$$s=\frac{13+14+15}{2}=21$$

$$\Delta=\sqrt{s(s-a)(s-b)(s-c)}=84$$

$$\therefore r_1=\frac{\Delta}{s-a}=\frac{84}{21-13}=\frac{21}{2}\ \text{(True)}$$

Ans: D

05 Assertion: Conceptual (True)

Reason: Conceptual (True)

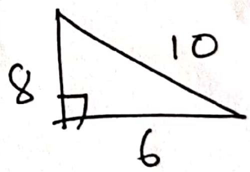
Ans.: A

06 Assertion: Conceptual (True)

Reason: In a right angled triangle, the centroid lies ~~its~~ interior of the triangle (False)

Ans.: C

07 $a=6,\ b=10,\ c=8$



$$s=\frac{6+10+8}{2}=12$$

$$\Delta=\frac{1}{2}\times 8\times 6=24$$

$$r=\frac{\Delta}{s}=\frac{24}{12}=2$$

Ans: B

08 $$r_1=\frac{\Delta}{s-a}=\frac{24}{12-6}=4$$

Ans: A

09 $$r_3=\frac{\Delta}{s-c}=\frac{24}{12-8}=6$$

Ans: C

10. Let us consider ΔABC is an equilateral tangle

$$r\left(\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}\right)=r\left(3.\cot\frac{A}{2}\right)$$

$= \frac{\sqrt{3}}{6}\left(3 \cdot \cot\frac{60^\circ}{2}\right) = \frac{\sqrt{3}}{2} \cdot \sqrt{3} = \frac{3}{2} = s$

Ans.. A

11. Let us Consider $\triangle ABC$, be equilateral $\triangle le$

$r_1 \cot\left(\frac{A}{2}\right) + r_2 \cot\left(\frac{B}{2}\right) + r_3 \cot\left(\frac{C}{2}\right)$

$= 3 r_1 \cot\left(\frac{A}{2}\right)$

$= 3 \cdot \frac{\sqrt{3}}{2} \cot 30^\circ = \frac{3\sqrt{3}}{2} \times \sqrt{3} = \frac{9}{2} = 3 \times \frac{3}{2} = 3s$

Ans. C

12 Repeated Q.no: 11 (above)

13 O, since in an equilateral triangle orthocentre and circumcentre coincides

Ans. O

14 $\frac{a}{r_1} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Ans: 3

15 a) $\Delta = 400\sqrt{3}$, $s = 20(2+\sqrt{3})$

$r = \frac{\Delta}{s} = \frac{400\sqrt{3}}{20(2+\sqrt{3})} = \frac{20\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 20(2\sqrt{3}-3)$

(q)

b) $\Delta = 400\sqrt{3}$, $s-a = 20\sqrt{3}$

$r_1 = \frac{\Delta}{s-a} = \frac{400\sqrt{3}}{20\sqrt{3}} = 20$ (s)

c) $a+b-c = 40(2-\sqrt{3})$

$2s-2c = 40(2-\sqrt{3})$

$\Rightarrow s-c = 20(2-\sqrt{3})$

$r_3 = \frac{\Delta}{s-c} = \frac{400\sqrt{3}}{20(2-\sqrt{3})}$

$= 20\sqrt{3}(2+\sqrt{3})$ (r)

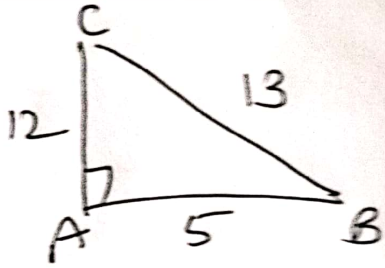
d) $\Delta = \dfrac{abc}{4R}$

$\Rightarrow 400\sqrt{3} = \dfrac{64000\sqrt{3}}{4R} \Rightarrow R = 40$ (p)

Ans: q, s, r, p

16 a) $\angle A = 90^\circ$

$\sin\frac{A}{2} = \sin 45^\circ = \frac{1}{\sqrt{2}}$ (r)



b) $a, b, c \longrightarrow AP$

$2b = a + c$

$\Rightarrow 3b = 2s$

$\tan\left(\frac{A}{2}\right) \cdot \tan\frac{C}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)} \cdot \dfrac{(s-a)(s-b)}{s(s-c)}}$

$= \dfrac{s-b}{s}$

$= \dfrac{2s-2b}{2s} = \dfrac{3b-2b}{3b} = \dfrac{1}{3}$ (p)

c) $3a = b + c$

$\Rightarrow 4a = a + b + c = 2s$

$\Rightarrow 2a = s$

$\cot\left(\frac{B}{2}\right) \cdot \cot\frac{C}{2} = \sqrt{\dfrac{s(s-b)}{(s-a)(s-c)} \cdot \dfrac{s(s-c)}{(s-a)(s-b)}} = \dfrac{s}{s-a}$

$= \cancel{\dfrac{2s}{2s-2a}} = \dfrac{2a}{2a-a} = 2$ (t)

d) $\dfrac{1}{r} = \dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}$

$= \dfrac{1}{8} + \dfrac{1}{12} + \dfrac{1}{24}$

$= \dfrac{3+2+1}{24} = \dfrac{6}{24} = \dfrac{1}{4}$

$\boxed{r = 4}$

$r_1 = \frac{\Delta}{s-a}$

$\Rightarrow 8 = \frac{96}{24-a}$

$\Rightarrow \boxed{a=12}$ (q)

$r = \frac{\Delta}{s} \Rightarrow 4 = \frac{96}{s}$

$\Rightarrow \boxed{s=24}$

Ans : r, p, t, q

ADDITIONAL PRACTICE QUESTIONS

01 Let ΔABC, be an equilateral triangle

$r_1^2 + r_2^2 + r_3^2 + r^2 = 3r_1^2 + r^2$

$= 3\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{6}\right)^2 = \frac{7}{3}$

opt: A: $16R^2 - (a^2+b^2+c^2)$

$= 16\left(\frac{1}{\sqrt{3}}\right)^2 - (1^2+1^2+1^2)$

$= \frac{16}{3} - 3 = \frac{7}{3}$

Ans.. A

02 Let ΔABC, be an equilateral triangle

$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2} = 3\sin^2\frac{A}{2} = 3\sin^2\frac{60°}{2}$

$= 3\left(\frac{1}{2}\right)^2 = \frac{3}{4}$

opt: C : $2 + \frac{r}{2R} = 2 + \frac{\left(\frac{\sqrt{3}}{6}\right)}{2\times\left(\frac{1}{\sqrt{3}}\right)} = 2 + \frac{1}{4} = \frac{9}{4}$ X

opt: D : $1 - \frac{r}{2R} = 1 - \frac{\left(\frac{\sqrt{3}}{6}\right)}{2\left(\frac{1}{\sqrt{3}}\right)} = 1 - \frac{1}{4} = \frac{3}{4}$ ✓

$\boxed{\text{Ans.. D}}$

03 | let $\triangle ABC$, be an equilateral tringle

$$\frac{1}{p_1}+\frac{1}{p_2}-\frac{1}{p_3}=\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}+\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}-\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}=\frac{2}{\sqrt{3}}$$

A). $$\frac{2ab\cos^2\frac{C}{2}}{\Delta(a+b+c)}=\frac{2.1.1.\cos^2 30^\circ}{\frac{\sqrt{3}}{4}(1+1+1)}=\frac{2}{\sqrt{3}}$$

Ans.. A

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04 | let $OABC$, be an equilateral tringle

$$r_1 r_2\sqrt{\frac{4R-r_1-r_2}{r_1+r_2}}$$

$$=\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\sqrt{\frac{4.\left(\frac{1}{\sqrt{3}}\right)-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}}=\frac{3}{4}\sqrt{\frac{1}{3}}=\frac{\sqrt{3}}{4}=\Delta$$

Ans: C

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05 | let $\triangle ABC$, be an epilateral tringle

$$\frac{r_1+r_2}{1+\cos C}=\frac{\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}{1+\cos 60^\circ}=\frac{\sqrt{3}}{1+\frac{1}{2}}=\frac{2}{\sqrt{3}}$$

Ans. C

option: C. $\frac{abc}{2\Delta}=\frac{1.1.1}{2.\frac{\sqrt{3}}{4}}=\frac{2}{\sqrt{3}}$

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06. | $x^3-2(r+R)x^2+s^2x-s^2r=0$

product of the roots $=-\frac{(-s^2r)}{1}=s^2r.$

$$=\left(\frac{3}{2}\right)^2.\frac{\sqrt{3}}{6}=\frac{3\sqrt{3}}{8}$$

opt: D: $r_1.r_2.r_3=\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{8}$

Ans.. D

⟹ THE END ⟸