

9. COMPOUND ANGLES ①

Class: VIII. MATHEMATICS

FOUNDATION⁺ SOLUTIONS

TEACHING TASK (2025/26) JEE MAINS LEVEL

01 $A+B+C=\pi$
 $\Rightarrow A+B=\pi-C$
 $\Rightarrow \tan(A+B) = \tan(\pi-C)$
 $\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = -\tan C$
 $\Rightarrow \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$
 $\Rightarrow \sum \tan A = \tan A \cdot \tan B \cdot \tan C$ Ans. A

02
$$\frac{\tan 225^\circ - \cot 81^\circ \cdot \cot 69^\circ}{\cot 261^\circ + \tan 21^\circ}$$

$$= \frac{1 - \cot 81^\circ \cdot \cot 69^\circ}{\cot 81^\circ + \cot 69^\circ}$$

$$= -\left(\frac{\cot 81^\circ \cdot \cot 69^\circ - 1}{\cot 81^\circ + \cot 69^\circ} \right)$$

$$= -\cot(81^\circ + 69^\circ) = -\cot 150^\circ = -\cot(180^\circ - 30^\circ)$$

$$= \cot 30^\circ = \sqrt{3}$$
 Ans. C

$\tan 225^\circ = \tan(180^\circ + 45^\circ)$
 $= \tan 45^\circ$
 $= 1$
 $\cot 261^\circ = \cot(180^\circ + 81^\circ)$
 $= \cot 81^\circ$

03
$$\frac{(1 + \tan 13^\circ)(1 + \tan 32^\circ)}{(1 + \tan 12^\circ)(1 + \tan 33^\circ)} =$$

$$= \frac{2}{2} = 1$$
 Ans. A

[NOTE]
 $A+B=45^\circ$
 $\Rightarrow (1 + \tan A)(1 + \tan B) = 2$

04 Given $A+B=300^\circ$

Let $A=B=150^\circ$

$$\cot 150^\circ = \cot (180^\circ - 30^\circ) = -\cot 30^\circ = -\sqrt{3}$$

$$\therefore (1+\sqrt{3} \cot A)(1+\sqrt{3} \cot B)$$

$$= (1+\sqrt{3}(-\sqrt{3}))(1+\sqrt{3}(-\sqrt{3}))$$

$$= (1-3)(1-3) = (-2)(-2) = 4$$

Ans. A

05 Given $A+B+C=360^\circ$

Let $A=B=C=120^\circ$

Now, $\frac{A}{4} = \frac{B}{4} = \frac{C}{4} = 30^\circ$

$$\cot \frac{A}{4} + \cot \frac{B}{4} + \cot \frac{C}{4} = K \cdot \cot \frac{A}{4} \cdot \cot \frac{B}{4} \cdot \cot \frac{C}{4}$$

$$\Rightarrow \sqrt{3} + \sqrt{3} + \sqrt{3} = K \cdot \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3}$$

$$\Rightarrow 3\sqrt{3} = K \cdot 3\sqrt{3} \Rightarrow K=1$$

Ans. D

06 $\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \cdot \tan 110^\circ} = \tan (160^\circ - 110^\circ) = \tan 50^\circ = \cot 40^\circ$

$$= \frac{1}{\tan 40^\circ} = \frac{1}{\tan 2 \cdot 20^\circ} = \frac{1}{\left(\frac{2 \tan 20^\circ}{1 - \tan^2 20^\circ} \right)} = \frac{1 - \rho^2}{2\rho}$$

Ans. B

07 $A+B=225^\circ \Rightarrow \cot (A+B) = \cot 225^\circ$

$$\Rightarrow \frac{\cot A \cot B - 1}{\cot A + \cot B} = 1$$

$$\Rightarrow \cot A \cdot \cot B - 1 = \cot A + \cot B \rightarrow \text{①}$$

Now, $\frac{\cot A}{1 + \cot A} \cdot \frac{\cot B}{1 + \cot B} = \frac{\cot A \cot B}{1 + \cot A + \cot B + \cot A \cdot \cot B}$

$$= \frac{\cot A \cdot \cot B}{\cot A \cdot \cot B + \cot A \cdot \cot B}$$

$$= \frac{\cot A \cdot \cot B}{2 \cot A \cdot \cot B} = \frac{1}{2}$$

(from ①) ③

Ans. B

08 $A+B+C=180^\circ$
 $\Rightarrow A+B=180^\circ-C$
 $\Rightarrow \sin(A+B) = \sin(180^\circ-C)$
 $\Rightarrow \sin A \cos B + \cos A \sin B = \sin C$
 $= \left(\frac{3}{5}\right) \cdot \frac{12}{13} + \left(-\frac{4}{5}\right) \cdot \frac{5}{13} = \sin C$
 $\Rightarrow \sin C = \frac{16}{65}$

Since
 A is obtuse
 $\Rightarrow \sin A = -\frac{4}{5}$
 Ans. C

09 In $\triangle ABC$, let $A=B=C=60^\circ$

$$\sum \frac{\sin(60^\circ+60^\circ) \sin(60^\circ-60^\circ)}{\sin^2 60^\circ} = 0$$

Ans. B

10. let $A=30^\circ$, $B=60^\circ$

$$\frac{\cos A}{\cos B} = n \Rightarrow \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = n \Rightarrow \boxed{n=\sqrt{3}}$$

$$\frac{\sin A}{\sin B} = m \Rightarrow \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = m \Rightarrow \boxed{m=\frac{1}{\sqrt{3}}}$$

Given $(m^2 - n^2) \sin^2 B$

$$= \left(\frac{1}{3} - 3\right) \cdot \frac{3}{4} = -\frac{8}{3} \times \frac{3}{4} = -2$$

option A: $1 - n^2 = 1 - 3 = -2$

Ans. A

$$11 \quad \angle R = \frac{\pi}{2} \Rightarrow P + Q = 90^\circ$$

$$\Rightarrow \frac{P}{2} + \frac{Q}{2} = 45^\circ$$

$$\Rightarrow \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = \tan 45^\circ$$

$$\Rightarrow \frac{\tan \frac{P}{2} + \tan \frac{Q}{2}}{1 - \tan \frac{P}{2} \cdot \tan \frac{Q}{2}} = 1$$

$$\frac{\left(\frac{b}{a}\right)}{1 + \frac{c}{a}} = 1 \quad (4)$$

$$\Rightarrow a + b = c$$

Ans: A

JEE ADVANCED LEVEL

01. $\sin 52\frac{1}{2}^\circ - \sin 22\frac{1}{2}^\circ$

$$= \sin\left(52\frac{1}{2}^\circ + 22\frac{1}{2}^\circ\right) \cdot \sin\left(52\frac{1}{2}^\circ - 22\frac{1}{2}^\circ\right)$$

$$= \sin 75^\circ \cdot \sin 30^\circ$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{4\sqrt{2}} \quad (\text{or}) \quad \frac{1}{2\sqrt{2}(\sqrt{3}+1)}$$

Ans: A, D

02. $\sin(\theta + \alpha) = \cos(\theta + \alpha)$

$$\Rightarrow \tan(\theta + \alpha) = 1$$

$$\Rightarrow \frac{\tan \theta + \tan \alpha}{1 - \tan \theta \cdot \tan \alpha} = 1$$

$$\Rightarrow \tan \alpha = \frac{1 - \tan \theta}{1 + \tan \theta}$$

Ans: B

03. Statement I:

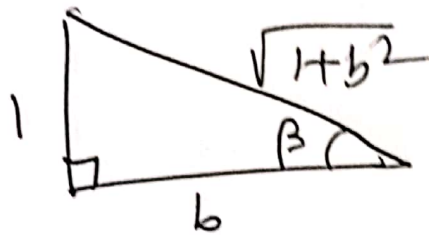
$$\sin \alpha = \frac{a}{\sqrt{1+a^2}}$$



$$\therefore \tan \alpha = a$$

$$\cos \beta = \frac{b}{\sqrt{1+b^2}}$$

(5)



$$\tan \beta = \frac{1}{b}$$

$$\tan(\alpha + \beta) = \frac{a + \frac{1}{b}}{1 - \frac{a}{b}} = \frac{ab+1}{b-a} \quad (\text{False})$$

Statement II:

$$\tan 2B = \tan[(A+B) - (A-B)] = \frac{m-n}{1+mn} \quad (\text{False})$$

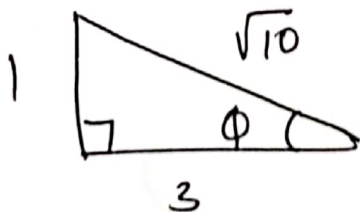
Ans.. B

04. Statement I:

$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta \\ &= \frac{12}{13} \cdot \left(-\frac{3}{5}\right) + \left(\frac{5}{13}\right) \left(-\frac{4}{5}\right) \\ &= \frac{-56}{65} \quad (\text{False}) \end{aligned}$$

Statement II:

$$\sin \phi = \frac{1}{\sqrt{10}}$$



$$\therefore \tan \phi = \frac{1}{3}$$

$$\tan 2\phi = \frac{2\left(\frac{1}{3}\right)}{1 - \left(\frac{1}{3}\right)^2} = \frac{3}{4}$$

$$\text{Now, } \tan(\phi + 2\phi) = \frac{\tan \phi + \tan 2\phi}{1 - \tan \phi \cdot \tan 2\phi}$$

$$= \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \cdot \frac{3}{4}} = \frac{25}{21} = 1 = \tan 45^\circ \quad (6)$$

hence, $\theta + 2\phi = 45^\circ$ (True)

Ans. D

Q5 Assertion: $\cot A + \cot B + \cot C = \sqrt{3}$

Let $A = B = C = 30^\circ$ so that $A + B + C = 90^\circ$

$$\cot 30^\circ + \cot 30^\circ + \cot 30^\circ = \sqrt{3} + \sqrt{3} + \sqrt{3} = 3\sqrt{3} \text{ (False)}$$

Reason: mathematically (True)

Ans. D

Q6 Assertion: Given $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = -\tan C$$

$$\Rightarrow \tan(A+B) = -\tan C = \tan(-C)$$

$$\Rightarrow A+B = n\pi + C$$

$$\Rightarrow A+B+C = n\pi \quad \forall n \in \mathbb{Z} \text{ (True)}$$

Reason: Conceptual (True)

Ans. A

Q7 $\cos^2\left(\frac{\pi}{4} + x\right) - \sin^2\left(\frac{\pi}{4} - x\right)$

$$= \cos\left(\frac{\pi}{4} + x + \frac{\pi}{4} - x\right) \cdot \cos\left(\frac{\pi}{4} + x - \frac{\pi}{4} + x\right)$$

$$= \cos\left(\frac{\pi}{2}\right) \cdot \cos 2x = 0$$

Ans. D

~~Q8 $\cos\left(\frac{\pi}{6}\right)$~~

08 $\cos^2\left(\frac{\pi}{6} + \theta\right) - \cos^2\left(\frac{\pi}{6} - \theta\right)$ (7)

$$= \sin\left(\frac{\pi}{6} + \theta + \frac{\pi}{6} - \theta\right) \cdot \sin\left(\frac{\pi}{6} - \theta - \frac{\pi}{6} - \theta\right)$$

$$= \sin\frac{\pi}{3} \cdot \sin(-2\theta) = -\frac{\sqrt{3}}{2} \sin 2\theta$$

Ans: B

09 $\cos(A+B) \cdot \cos(A-B) + \sin(A+B) \cdot \sin(A-B)$

$$= \cos^2 A - \sin^2 B + \sin^2 A - \sin^2 B$$

$$= 1 - 2\sin^2 B$$

$$= 1 - 2(1 - \cos^2 B)$$

$$= 2\cos^2 B - 1$$

Ans: C

10 $\tan 2A = \tan[(A+B) + (A-B)]$

$$= \frac{pq + 1}{p - q} = \frac{p+q}{1-pq}$$

Ans: C

11 $\cot 2B = \cot[(A+B) - (A-B)]$

$$= \frac{pq + 1}{p - q}$$

Ans: C

12 $\tan 2A \cdot \cot 2B + \tan 2B \cdot \cot 2A$

$$= \frac{p+q}{1-pq} \cdot \frac{pq+1}{p-q} + \frac{p-q}{pq+1} \cdot \frac{1-pq}{p+q}$$

Let $p=1, q=0$

$$= \frac{1}{1} \cdot \frac{1}{1} + \frac{1}{1} \cdot \frac{1}{1} = 2$$

Ans: D

13

$$\tan(A+B) = \frac{\frac{n}{n+1} + \frac{1}{2n+1}}{1 - \frac{n}{n+1} \cdot \frac{1}{2n+1}}$$

put $n=1$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = 1 \quad \text{hence } A+B=45^\circ$$

Ans: 45°

14

$$A+B+C=180^\circ$$

$$\Rightarrow A+B=180^\circ-C$$

$$\Rightarrow \sin(A+B) = \sin(180^\circ-C)$$

$$\Rightarrow \sin A \cos B + \cos A \sin B = \sin C$$

$$\Rightarrow \frac{3}{5} \cdot \frac{5}{13} + \frac{4}{5} \cdot \frac{12}{13} = \sin C$$

$$\Rightarrow 15 + 48 = 65 \sin C$$

$$\therefore 65 \sin C = 63$$

$$\sin A = \frac{3}{5}$$



$$\cos A = \frac{4}{5}$$



$$\cos B = \frac{5}{13}$$

Ans: 63

15

a) $A+B+C=180^\circ$

let $A=B=C=60^\circ$

$$\sum \frac{\cot A + \cot B}{\tan A + \tan B}$$

$$= \sum \frac{(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}})}{\sqrt{3} + \sqrt{3}}$$

$$= \sum \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

b) In $\triangle ABC$,
let $A=B=C=60^\circ$

$$\sum \frac{\cos(B-C)}{\sin B \cdot \sin C} = \sum \frac{1}{\left(\frac{\sqrt{3}}{2}\right) \cdot \left(\frac{\sqrt{3}}{2}\right)} = \sum \frac{4}{3}$$

$$= \frac{4}{3} + \frac{4}{3} + \frac{4}{3} = 4 \quad (9)$$

c) In $\triangle ABC$
let $A=B=C=60^\circ$

$$\sum \frac{\sin(A-B)}{\cos A \cdot \cos B} = \sum \frac{\sin(60^\circ - 60^\circ)}{\cos 60^\circ \cdot \cos 60^\circ} = 0 \quad (8)$$

d) $A+B+C=90^\circ$
let $A=B=C=30^\circ$

$$\frac{\cot A + \cot B + \cot C}{\cot A \cdot \cot B \cdot \cot C} = \frac{\cot 30^\circ + \cot 30^\circ + \cot 30^\circ}{\cot 30^\circ \cdot \cot 30^\circ \cdot \cot 30^\circ}$$

$$= \frac{3 \times \sqrt{3}}{(\sqrt{3})^3} = \frac{3\sqrt{3}}{3\sqrt{3}} = 1 \quad (1)$$

Ans: $(1), 9, 8, (1)$

16 a) $\frac{\sin \theta}{\sin\left(\frac{\pi}{8} + \frac{\theta}{2} + \frac{\pi}{8} - \frac{\theta}{2}\right) \cdot \sin\left(\frac{\pi}{8} + \frac{\theta}{2} - \frac{\pi}{8} + \frac{\theta}{2}\right)}$

$$= \frac{\sin \theta}{\sin \frac{\pi}{4} \cdot \sin \theta} = \sqrt{2} \quad (1)$$

b) $\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = a$

let $\theta = 0^\circ$

$\Rightarrow a = 2$

$$\text{Now, } \tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) \rightarrow \textcircled{7} \quad \textcircled{10}$$

$$\text{let } \theta = 0$$

$$\textcircled{1} \Rightarrow 2 \stackrel{\text{now}}{=} a^2 - 2 = (2)^2 - 2 = 2 = 2(\delta)$$

$$\text{c) } \cot A + 2 \tan(A-B)$$

$$= \cot A + 2 \left(\frac{\tan A - \tan B}{1 + \tan A \cdot \tan B} \right)$$

$$= \frac{\cot A + \tan B + 2 \tan A - 2 \tan B}{1 + \tan A \cdot \tan B}$$

$$= \frac{(2 \tan A + \cot A) - \tan B}{1 + \tan A \cdot \tan B} = \frac{\tan B - \tan B}{1 + \tan A \cdot \tan B} = 0 \quad \textcircled{9}$$

$$\text{d) } A + B + C = 90^\circ$$

~~let~~ We have

$$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1 \quad (\text{S})$$

Ans: t, r, q, s

LEARNER'S TASK (CUE'S)

$$\text{01. } \sin 40^\circ \cdot \cos 50^\circ + \cos 40^\circ \cdot \sin 50^\circ = \sin(40^\circ + 50^\circ) = \sin 90^\circ = 1$$

Ans: A

$$\text{02. } \cos 70^\circ \cdot \cos 20^\circ - \sin 70^\circ \cdot \sin 20^\circ$$

$$= \cos(70^\circ + 20^\circ) = \cos 90^\circ = 0$$

Ans: B

$$\text{03. } \frac{\tan 20^\circ + \tan 25^\circ}{1 - \tan 20^\circ \cdot \tan 25^\circ} = \tan(20^\circ + 25^\circ) = \tan 45^\circ = 1$$

Ans: A

04 $\tan(A+B) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \frac{5}{5} = 1 = \tan \frac{\pi}{4}$ (11) Ans. C

05 $\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ Ans. B

06 $\tan 90^\circ =$ Not defined Ans. D

07 $\cos^2 A - \sin^2 B$ (or) $\cos^2 B - \sin^2 A$ Ans. A

08 $\cos^2 45^\circ - \sin^2 15^\circ$
 $= \cos(45^\circ + 15^\circ) \cdot \cos(45^\circ - 15^\circ)$
 $= \cos 60^\circ \cdot \cos 30^\circ$
 $= \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$ Ans. C

09 $\cos(120^\circ + A) + \cos(120^\circ - A)$
 $= 2\cos 120^\circ \cdot \cos A = 2\left(-\frac{1}{2}\right) \cdot \cos A = -\cos A$ Ans. B

10 $\sin^2 C - \sin^2 B$ Ans. B

JEE MAINS LEVEL

01 $\sin 75^\circ \cdot \cos 75^\circ = \frac{1}{2} (2\sin 75^\circ \cos 75^\circ) = \frac{1}{2} \sin 2 \times 75^\circ$
 $= \frac{1}{2} \sin 150^\circ = \frac{1}{2} \sin(180^\circ - 30^\circ) = \frac{1}{2} \cdot \sin 30^\circ = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ Ans. C

02 $\sin^2 75^\circ + \cot^2 75^\circ = (2 + \sqrt{3})^2 + (2 - \sqrt{3})^2$
 $= 2[(2)^2 + (\sqrt{3})^2] = 14$ Ans. A

03

$$\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} = \frac{63}{16}$$

(12)
Ans. D

04

$$A + B = 45^\circ$$

$$\Rightarrow \tan(A + B) = \tan 45^\circ$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1$$

$$\Rightarrow \tan A + \tan B = 1 - \tan A \cdot \tan B$$

$$\Rightarrow \tan A + \tan B + \tan A \cdot \tan B = 1$$

$$= (1 + \tan A)(1 + \tan B) = 2$$

Ans. B

05

$$20^\circ = 55^\circ - 35^\circ$$

$$\Rightarrow \tan 20^\circ = \frac{\tan 55^\circ - \tan 35^\circ}{1 + \tan 55^\circ \cdot \tan 35^\circ}$$

$$\Rightarrow \tan 20^\circ = \frac{\tan 55^\circ - \tan 35^\circ}{1 + 1}$$

$$\Rightarrow 2 \tan 20^\circ = \tan 55^\circ - \tan 35^\circ$$

$$\Rightarrow 2 \tan 20^\circ + \tan 35^\circ = \tan 55^\circ$$

$$\therefore x = 55^\circ$$

Since

$$\tan 35^\circ = \cot 55^\circ$$

Ans. C

06

$$\cos^2 48^\circ - \sin^2 12^\circ$$

$$= \cos(48^\circ + 12^\circ) \cdot \cos(48^\circ - 12^\circ)$$

$$= \cos 60^\circ \cdot \cos 36^\circ$$

$$= \frac{1}{2} \times \frac{1 + \sqrt{5}}{4} = \frac{1 + \sqrt{5}}{8}$$

Ans. A

07.

$$\begin{aligned}
 x &= \frac{\sin A}{\cos A} - \frac{\sin B}{\cos B} \\
 &= \frac{\sin A \cos B - \cos A \sin B}{\cos A \cdot \cos B} \\
 &= \frac{\sin(A-B)}{\cos A \cdot \cos B}
 \end{aligned}$$

$$\begin{aligned}
 y &= \frac{\cos B}{\sin B} - \frac{\cos A}{\sin A} \quad (13) \\
 &= \frac{\sin A \cos B - \cos A \sin B}{\sin B \cdot \sin A} \\
 &= \frac{\sin(A-B)}{\sin A \cdot \sin B}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } \frac{1}{x} + \frac{1}{y} &= \frac{\cos A \cos B + \sin A \sin B}{\sin(A-B)} \\
 &= \frac{\cos(A-B)}{\sin(A-B)} = \cot(A-B) \quad \text{Ans: A}
 \end{aligned}$$

$$\begin{aligned}
 \text{or } \tan(A-B) &= \frac{1}{24} \\
 \Rightarrow \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B} &= \frac{1}{24} \\
 \Rightarrow \frac{\frac{4}{3} - \tan B}{1 + \frac{4}{3} \cdot \tan B} &= \frac{1}{24} \\
 \Rightarrow \tan B &= \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 \tan(A+B) &= \frac{\frac{4}{3} + \frac{3}{4}}{1 - \frac{4}{3} \cdot \frac{3}{4}} \\
 &= \text{Not defined} \\
 A+B &= \frac{\pi}{2}
 \end{aligned}$$

Ans: D

$$\begin{aligned}
 \text{09 } \tan(A+B+C) &= \frac{(1+2+3) - (1 \cdot 2 \cdot 3)}{1 - (1 \cdot 2 + 2 \cdot 3 + 3 \cdot 1)} = 0 \\
 \therefore A+B+C &= \pi \quad \text{Ans: B}
 \end{aligned}$$

$$\begin{aligned}
 \text{10 } (A+B+C=90^\circ \Rightarrow \tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A &= 1) \\
 \text{Now, } A+B+C &= 180^\circ \\
 \Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} &= 90^\circ
 \end{aligned}$$

$$\tan \frac{A}{2} \cdot \tan \frac{B}{2} + \tan \frac{B}{2} \cdot \tan \frac{C}{2} + \tan \frac{C}{2} \cdot \tan \frac{A}{2} = 1 \quad (14)$$

$$\Rightarrow \frac{5}{6} \cdot \frac{20}{37} + \frac{20}{37} \cdot \tan \frac{C}{2} + \tan \frac{C}{2} \cdot \frac{5}{6} = 1$$

$$\Rightarrow \tan \frac{C}{2} = \frac{122}{305}$$

Ans. A

JEE ADVANCED LEVEL

01 $A+B=\frac{\pi}{4} \Rightarrow (1+\tan A)(1+\tan B)=2$
 $(1+\cot A)(1+\cot B)=2 \cot A \cot B$
 Ans. A, C

02 $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ} = \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \tan 9^\circ}$
 $= \tan 54^\circ = \cot 36^\circ$ Ans. A, C

03 Statement I: $\tan A = x \tan B \Rightarrow x = \frac{\tan A}{\tan B}$

$$\frac{\sin(A-B)}{\sin(A+B)} = \frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B}$$

$$= \frac{\left(\frac{\tan A}{\tan B}\right) - 1}{\left(\frac{\tan A}{\tan B}\right) + 1} = \frac{x-1}{x+1} \quad (\text{True})$$

Statement II: Conceptual (True)

Ans. A

04 Statement I:

(15)

$$m \cot(\theta + \alpha) = n \cot(\theta - \alpha)$$

$$\Rightarrow \frac{m}{n} = \frac{\cos(\theta - \alpha)}{\cos(\theta + \alpha)}$$

$$\Rightarrow \frac{m+n}{m-n} = \frac{\cos(\theta - \alpha) + \cos(\theta + \alpha)}{\cos(\theta - \alpha) - \cos(\theta + \alpha)}$$

$$= \frac{2 \cos \theta \cdot \cos \alpha}{2 \sin \theta \cdot \sin \alpha} = \cot \theta \cdot \cot \alpha \text{ (False)}$$

Statement II:

$$\text{Given } \frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{\sin(\alpha + \beta) - \sin(\alpha - \beta)} = \frac{(a+b) + (a-b)}{(a+b) - (a-b)}$$

$$\Rightarrow \frac{2 \sin \alpha \cos \beta}{2 \cos \alpha \sin \beta} = \frac{2a}{2b}$$

$$\Rightarrow \tan \alpha \cdot \cot \beta = \frac{a}{b} \text{ (True)}$$

Ans. D

05 Assertion: LHS = $\tan 75^\circ - \cot 15^\circ$

$$= \tan(90^\circ - 15^\circ) - \cot 15^\circ$$

$$= \cot 15^\circ - \cot 15^\circ = 0 \text{ (True)}$$

Ans. A

Reason: Conceptual (True)

06 Assertion: mathematically (True)

Reason: Conceptual (True)

Ans. A

07

$$A + B = 45^\circ$$

$$\Rightarrow \cot(A+B) = \cot 45^\circ$$

$$\Rightarrow \frac{\cot A \cdot \cot B - 1}{\cot A + \cot B} = 1$$

$$\Rightarrow \cot A \cdot \cot B - \cot A - \cot B = 1$$

$$\Rightarrow (\cot A - 1)(\cot B - 1) = 2$$

Ans: C

08

$$A = 22\frac{1}{2}^\circ$$

$$\Rightarrow 2A = 45^\circ$$

$$\Rightarrow \tan 2A = \tan 45^\circ$$

$$\Rightarrow \frac{2 \tan A}{1 - \tan^2 A} = 1$$

$$\Rightarrow \frac{2x}{1-x^2} = 1$$

$$\Rightarrow x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{4+4}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}}{2}$$

$$= \sqrt{2} - 1$$

Ans: C

09

$$A = 67\frac{1}{2}^\circ$$

$$2A = 135^\circ$$

$$\Rightarrow \tan 2A = \tan 135^\circ$$

$$\Rightarrow \frac{2 \tan A}{1 - \tan^2 A} = -1$$

$$\Rightarrow \frac{2x}{1-x^2} = -1$$

$$\Rightarrow x^2 - 2x - 1 = 0$$

$$\Rightarrow x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$= \frac{2 + 2\sqrt{2}}{2} = \sqrt{2} + 1$$

Ans: B

10

A) Consider $(\sin 15^\circ + \cos 15^\circ)^2 = 1 + 2 \sin 15^\circ \cos 15^\circ$

$$= 1 + \sin 30^\circ$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2} \quad (\text{canceled})$$

$$\therefore \sin 15^\circ + \cos 15^\circ = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$$



B) $\tan 15^\circ - \cot 75^\circ$

$$= (2 - \sqrt{3}) - \frac{1}{(2 + \sqrt{3})}$$

$$= (2 - \sqrt{3}) - (2 - \sqrt{3}) = 0 \text{ (True)}$$

Ans: C

11.

$$x = \tan 15^\circ + \tan 75^\circ$$

$$= (2 - \sqrt{3}) + (2 + \sqrt{3}) = 4$$

$$\therefore x^2 = 16$$

Ans: D

12

$$80^\circ - 10^\circ = 70^\circ$$

$$\Rightarrow \tan(80^\circ - 10^\circ) = \tan 70^\circ$$

$$\Rightarrow \frac{\tan 80^\circ - \tan 10^\circ}{1 + \tan 80^\circ \cdot \tan 10^\circ} = \tan 70^\circ$$

$$\Rightarrow \frac{\tan 80^\circ - \tan 10^\circ}{\tan 70^\circ} = 2$$

Since
 $\tan 80^\circ - \tan 10^\circ =$

Ans. 2

13

In $\triangle ABC$, let $A = B = C = 60^\circ$

$$\therefore \tan \frac{A}{2} \cdot \tan \frac{B}{2} + \tan \frac{B}{2} \cdot \tan \frac{C}{2} + \tan \frac{C}{2} \cdot \tan \frac{A}{2}$$

$$= 3 \tan \frac{A}{2} \cdot \tan \frac{B}{2}$$

$$= 3 \tan 30^\circ \cdot \tan 30^\circ$$

$$= 3 \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = 1$$

Ans. 1

14 a) $\cos \theta + \cos(120^\circ - \theta) + \cos(120^\circ + \theta)$ (18)

$$\begin{aligned} \text{let } \theta &= 0^\circ \\ &= 1 + \cos 120^\circ + \cos 120^\circ \\ &= 1 - \frac{1}{2} - \frac{1}{2} = 0(t) \end{aligned}$$

b) $\cos \theta + \cos(240^\circ + \theta) + \cos(240^\circ - \theta)$

$$\begin{aligned} \text{let } \theta &= 0^\circ \\ &= 1 + \cos 240^\circ + \cos 240^\circ \\ &= 1 - \frac{1}{2} - \frac{1}{2} = 0(t) \end{aligned}$$

c) $\cos \theta - \cos(60^\circ + \theta) - \cos(60^\circ - \theta)$

$$\begin{aligned} \text{let } \theta &= 0^\circ \\ &= 1 - \cos 60^\circ - \cos 60^\circ \\ &= 1 - \frac{1}{2} - \frac{1}{2} = 0(t) \end{aligned}$$

d) $\sin \theta + \sin(120^\circ + \theta) + \sin(\theta - 60^\circ)$

$$\begin{aligned} \text{let } \theta &= 0^\circ \\ &= 0 + \sin 120^\circ - \sin 60^\circ \\ &= 0 + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0 \end{aligned}$$

Ans. t, t, t, t

15 a) $20^\circ + 40^\circ = 60^\circ$

$$\Rightarrow \tan(20^\circ + 40^\circ) = \tan 60^\circ$$

$$\Rightarrow \frac{\tan 20^\circ + \tan 40^\circ}{1 - \tan 20^\circ \cdot \tan 40^\circ} = \sqrt{3}$$

$$\Rightarrow \tan 20^\circ + \tan 40^\circ + \sqrt{3} \tan 20^\circ \cdot \tan 40^\circ = \sqrt{3} (s) \quad (15)$$

$$b) \quad 75^\circ - 30^\circ = 45^\circ$$

$$\tan(75^\circ - 30^\circ) = \tan 45^\circ$$

$$\Rightarrow \tan 75^\circ - \tan 30^\circ - \tan 75^\circ \cdot \tan 30^\circ = 1 (P)$$

$$c) \quad \frac{\cos 13^\circ - \sin 13^\circ}{\cos 13^\circ + \sin 13^\circ} + \frac{1}{\cot 44^\circ}$$

$$= \frac{1 - \tan 13^\circ}{1 + \tan 13^\circ} + \tan 44^\circ$$

$$= \frac{\tan 45^\circ - \tan 13^\circ}{1 + \tan 45^\circ \cdot \tan 13^\circ} + \tan(180^\circ - 32^\circ)$$

$$= \tan(45^\circ - 13^\circ) - \tan 32^\circ$$

$$= \tan 32^\circ - \tan 32^\circ = 0 (r)$$

$$d) \quad \frac{\tan \alpha + \tan \beta}{\tan(\alpha + \beta)} + \frac{\tan \alpha - \tan \beta}{\tan(\alpha - \beta)}$$

$$= \frac{\tan \alpha + \tan \beta}{\left(\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} \right)} + \left(\frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan \beta} \right)$$

$$= 1 - \tan \alpha \cdot \tan \beta + 1 + \tan \alpha \cdot \tan \beta$$

$$= 2 (t)$$

Ans: s, p, r, t

ADDITIONAL PRACTICE QUESTIONS

(20)

$$01 \quad \csc 15^\circ - \sec 15^\circ$$
$$= \frac{2\sqrt{2}}{\sqrt{3}-1} - \frac{2\sqrt{2}}{\sqrt{3}+1}$$

$$= \sqrt{2}(\sqrt{3}+1) - \sqrt{2}(\sqrt{3}-1)$$

$$= \sqrt{2}(\sqrt{3}+1 - \sqrt{3}+1) = 2\sqrt{2}$$

Ans. A

$$02 \quad 4x + 5x = 9x$$

$$\Rightarrow \tan(4x + 5x) = \tan 9x$$

$$\Rightarrow \frac{\tan 4x + \tan 5x}{1 - \tan 4x \cdot \tan 5x} = \tan 9x$$

$$= \tan 4x + \tan 5x - \tan 9x = -\tan 4x \cdot \tan 5x$$

Ans. B

$$\therefore k = -1$$

$$03 \quad \tan\left(\frac{\pi}{4} + \theta\right) \times \tan\left(\frac{3\pi}{4} + \theta\right)$$

$$\text{Let } \theta = 0^\circ$$

$$= \tan \frac{\pi}{4} \times \tan \frac{3\pi}{4} = \cancel{1} \times -1 = -1$$

Ans. B

$$04 \quad \cos \alpha + \cos \beta = 0 \Rightarrow \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = 0 \rightarrow (1)$$
$$\sin \alpha + \sin \beta = 0 \Rightarrow \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta = 0 \rightarrow (2)$$

$$(1) + (2) \Rightarrow 1 + 1 + 2 \cos(\alpha - \beta) = 0$$

$$\Rightarrow \cos(\alpha - \beta) = -1$$

Ans. C

$$05 \quad \frac{\cos A \cdot \cos B + \sin A \sin B}{\cos A \cos B - \sin A \sin B} + \frac{\cos C \cos D - \sin C \sin D}{\cos C \cos D + \sin C \sin D} = 0 \quad (21)$$

$$= \frac{1 + \tan A \cdot \tan B}{1 - \tan A \cdot \tan B} + \frac{1 - \tan C \cdot \tan D}{1 + \tan C \cdot \tan D} = 0$$

$$= 1 + \cancel{\tan C \tan D} + \cancel{\tan A \tan B} + \tan A \cdot \tan B \tan C \cdot \tan D$$

$$+ 1 - \cancel{\tan C \tan D} - \cancel{\tan A \tan B} + \tan A \cdot \tan B \tan C \cdot \tan D = 0$$

$$= \tan A \cdot \tan B \cdot \tan C \cdot \tan D = 1 \quad \text{Ans: B}$$

$\Rightarrow$ THE END $\Leftarrow$