FOOT OF THE PERPENDICULAR, IMAGE OF A POINT w.r.t. A LINE, DISTANCE OF A POINT FROM A LINE, ANGLE BETWEEN TWO LINES

Class: IX, Mathematics

F+

SOLUTIONS

TEACHING TASK

01. $A(\underset{x_1}{a\cos\theta}, \underset{y_1}{a\sin\theta})$, $B(\underset{x_2}{a\cos\phi}, \underset{y_2}{a\sin\phi})$

Eqn of the line

$$y - a\sin\theta = \left(\frac{a\sin\phi - a\sin\theta}{a\cos\phi - a\cos\theta}\right)(x - a\cos\theta)$$

$$\Rightarrow y - a\sin\theta = \frac{2\cos\left(\frac{\theta+\phi}{2}\right)\cdot\sin\left(\cancel{\frac{\phi-\theta}{2}}\right)}{-2\sin\left(\frac{\phi+\theta}{2}\right)\sin\left(\cancel{\frac{\phi-\theta}{2}}\right)}(x - a\cos\theta)$$

$$\Rightarrow (y - a\sin\theta)\sin\left(\frac{\phi+\theta}{2}\right) = -(x - a\cos\theta)\cos\left(\frac{\theta+\phi}{2}\right)$$

$$\Rightarrow \cos\left(\frac{\theta+\phi}{2}\right)x + \sin\left(\frac{\theta+\phi}{2}\right)y - a\sin\theta\sin\left(\frac{\theta+\phi}{2}\right) - a\cos\theta\cdot\cos\left(\frac{\theta+\phi}{2}\right) = 0$$

$$= \cos\left(\frac{\theta+\phi}{2}\right)x + \sin\left(\frac{\theta+\phi}{2}\right)y - a\left[\cos\left(\theta - \frac{\theta+\phi}{2}\right)\right] = 0$$

$$= \cos\left(\frac{\theta+\phi}{2}\right)x + \sin\left(\frac{\theta+\phi}{2}\right)y - a\cos\left(\frac{\theta-\phi}{2}\right) = 0 \rightarrow (1)$$

Now, let $P(h, k)$ be the foot of the $\perp$ from $(0,0)$ on the line (1)

We have

$$\frac{h-0}{\cos\left(\frac{\theta+\phi}{2}\right)} = \frac{k-0}{\sin\left(\frac{\theta+\phi}{2}\right)} = \frac{-\left(-a\cos\left(\frac{\theta-\phi}{2}\right)\right)}{1}$$

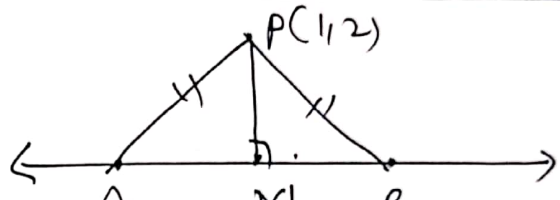
$$h = a\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right), \quad k = a\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$h = \frac{a}{2}(\cos\theta+\cos\phi), \quad k = \frac{a}{2}(\sin\theta+\sin\phi)$$

Ans: C

~~∴ (h,k~~

02 M is nothing but foot of the ⊥r.



$$\frac{h-1}{2} = \frac{k-2}{-1} = \frac{-(2(1)-2+3)}{4+1}$$

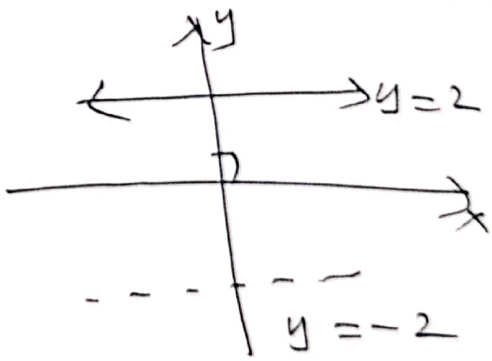
$$\therefore (h,k) = \left(\frac{-1}{5}, \frac{13}{5}\right)$$

Ans. A

03 A(7,2), B(−3,2)

Eqn of the line $y=2$

Image of the line $\Rightarrow y=-2$



Ans. D

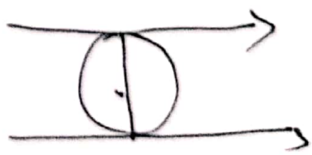
**04 $3x+4y-14=0 \rightarrow (1)$

$(1)\times 2 \Rightarrow 6x+8y-28=0$

$(2) \Rightarrow 6x+8y-7=0$

$$\therefore \text{Diameter} = \frac{|-28+7|}{\sqrt{36+84}} = \frac{21}{10}$$

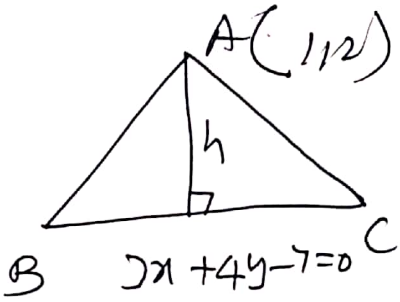
$\therefore$ radius $= \frac{21}{20}$



Ans: —

05.

$$h = \frac{|3\cdot 1 + 4\cdot 2 - 7|}{\sqrt{9+16}}$$



$$= \frac{4}{5}$$

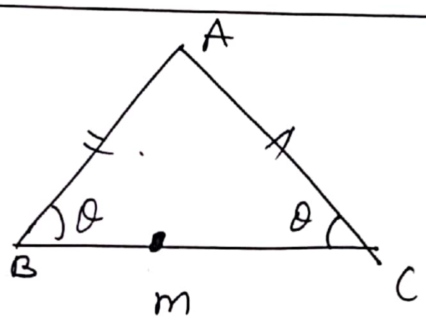
$$\therefore \frac{\sqrt{3}}{2} a = \frac{4}{5} \Rightarrow a = \frac{8}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{15}$$

Ans: C

06.

$$7x - y + 3 = 0 \qquad \Rightarrow m_1 = 7$$

$$x + y - 3 = 0 \qquad \Rightarrow m_2 = -1$$



$$\left|\frac{m-7}{1+7m}\right| = \left|\frac{m+1}{1-m}\right|$$

~~S:O:B:S~~

$$\Rightarrow \cancel{\frac{m^2 - 14m + 49}{1 + 49m^2 + 14m}} \quad \frac{m-7}{1+7m} = \pm \frac{m+1}{1-m}$$

Case (i) $\dfrac{m-7}{1+7m} = \dfrac{m+1}{1-m}$

$$\Rightarrow (m-7)(1-m) = (m+1)(1+7m)$$

$$\Rightarrow \cancel{m} - m^2 - 7 + \cancel{7m} = \cancel{m} + 7m^2 + 1 + \cancel{7m}$$

$$\Rightarrow 8m^2 + 7 = 0.$$

Case (ii) $\dfrac{m-7}{1+7m} = -\dfrac{m+1}{1-m}$

$$\Rightarrow (m-7)(1-m) = -(m+1)(1+7m)$$

$$\Rightarrow m - m^2 - 7 + 7m = -m - 7m^2 - 1 - 7m$$

$$\Rightarrow 6m^2 + 16m - 6 = 0$$

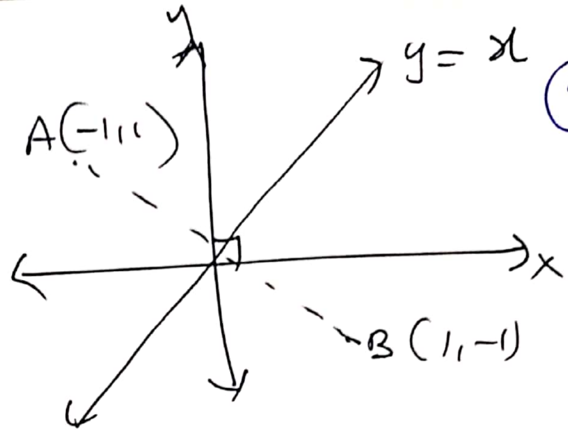
$$\Rightarrow 3m^2 + 8m - 3 = 0$$

Ans: C

4

07. $A(-1, 1)$, $B(1,-1)$

mid-point $(0,0)$, lies on $y = x$



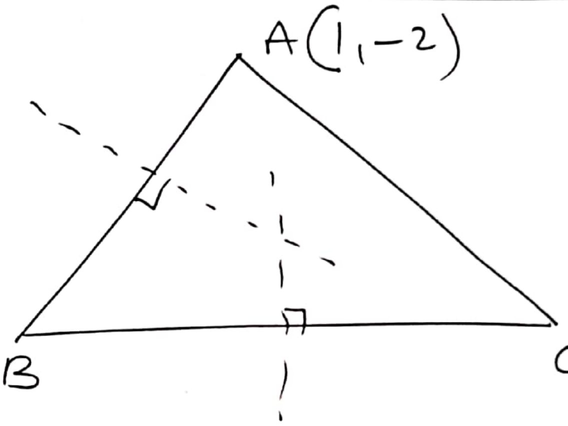
Ans: B

08. $A(1,-2)$, let $B(h,k)$

$x - y - 5 = 0$

$$\frac{h-1}{1} = \frac{k+2}{-1} = \frac{-2(1+2-5)}{1+1}$$

$= 1$



$B(h,k) = (3,-4)$

let $B(3,-4)$, let $C(h,k)$, $x + 2y = 0$

$$\frac{h-3}{1} = \frac{k+4}{2} = -2\frac{(3-8)}{1+4} = 2$$

$\therefore (h,k) = (5,0)$

Ans: C

09. $x\sec\alpha - y\,\mathrm{cosec}\,\alpha - a = p$

$$p = \frac{|-a|}{\sqrt{\sec^2\alpha + \mathrm{cosec}^2\alpha}} = a\sin\alpha\cos\alpha = \frac{1}{2}a\sin 2\alpha$$

$\therefore 2p = a\cos 2\alpha \rightarrow (1)$

Also, $x\cos\alpha + y\sin\alpha - a\cos 2\alpha = 0$

$$q = \frac{|-a\cos 2\alpha|}{\sqrt{\cos^2\alpha + \sin^2\alpha}} = a\cos 2\alpha \rightarrow (2)$$

$(1)^2 + (2)^2 \Rightarrow 4p^2 + q^2 = a^2$

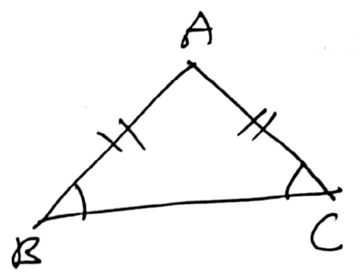
Ans: A

(5)

10. $(a+b)x + (a-b)y - 2ab = 0 \rightarrow (1)$

$(a-b)x + (a+b)y - 2ab = 0 \rightarrow (2)$

$x + y = 0 \rightarrow (3)$



slope of (1) $\Rightarrow m_1 = -\left(\frac{a+b}{a-b}\right)$

$= \frac{a+b}{b-a}$

slope of (2) $\Rightarrow m_2 = -\left(\frac{a-b}{a+b}\right) = \frac{b-a}{a+b}.$

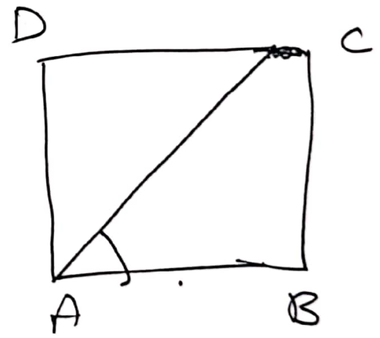
$$\tan A = \left| \frac{\frac{a+b}{b-a} - \frac{b-a}{a+b}}{1 + \left(\frac{a+b}{b-a}\right)\left(\frac{b-a}{a+b}\right)} \right|$$

$$\tan A = \left| \frac{(a+b)^2 - (b-a)^2}{2(a^2-b^2)} \right| = \left| \frac{2ab}{a^2-b^2} \right|$$

Ans : B

11. Eqn. of AC $\Rightarrow 8x - 15y = 0$

let B(1, 2).



slope of AC $= m_1 = \frac{8}{15}$

Slope of AB $= m$

$$\tan 45^\circ = \left| \frac{m - \frac{8}{15}}{1 + \frac{8m}{15}} \right|$$

$$\left|1 + \frac{8m}{15}\right| = \left|m - \frac{8}{15}\right|$$

$$1 + \frac{8m}{15} = \pm\left(m - \frac{8}{15}\right)$$

case (i) $1 + \frac{8m}{15} = m - \frac{8}{15}$

$\Rightarrow m - \frac{8m}{15} = 1 + \frac{8}{15}$

$\Rightarrow 7m = 23$

$\Rightarrow m = \frac{23}{7}$

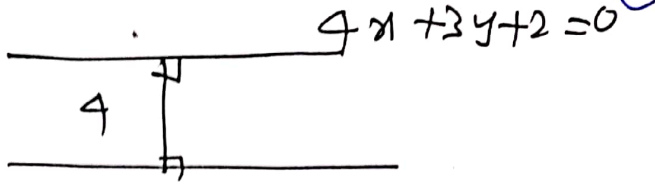
case (ii) $m = \frac{-7}{23}$

Ans: A, B

12.

$4x+3y+2=0$

$4x+3y+c=0$



$$4=\frac{|2-c|}{\sqrt{16+9}}$$

$\Rightarrow |2-c|=20$

$\Rightarrow 2-c=\pm 20$

$2-c=20$ | $2-c=-20$

$c=-18$ | $c=22$

$\therefore$ Required eq^ns^ are $4x+3y-18=0$ and

$4x+3y+22=0$

Ans: A, C

13 Statement I:

$x\cos\alpha+y\sin\alpha-p=0$

let $x\cos\alpha+y\sin\alpha+k=0$

Now, distance $p'-p=\dfrac{|k+p|}{\sqrt{\cos^2\alpha+\sin^2\alpha}}$

$\Rightarrow |k+p|=p'-p$

$\Rightarrow k+p=\pm(p'-p)$

$\Rightarrow k+p=p'-p$ (or) $k+p=p-p'$

$\Rightarrow k=p'-2p$ (or) $k=-p'$

$\therefore$ Required equation are

$x\cos\alpha+y\sin\alpha+p'-2p=0$ (True)

Statement II: Conceptual (True)

Ans: A

14. Statement I: $2x+3y+4=0$

$\lambda x + ky + 2 = 0$

$$\frac{2}{\lambda} = \frac{3}{k} = \frac{4}{2} = 2$$

$\therefore \lambda = 1 \quad k = \frac{3}{2}$

$\therefore 3\lambda - 2k = 3(1) - 2\left(\frac{3}{2}\right) = 0$ (True)

Statement II: Conceptual (True)

Ans: A

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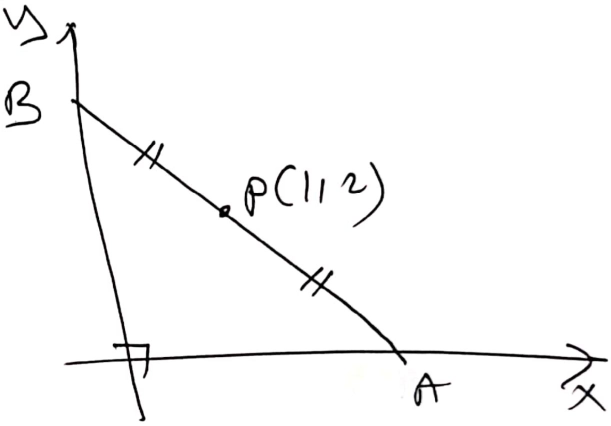
15. $A(a,0)$, $B(0,b)$



$\left(\frac{a}{2}, \frac{b}{2}\right) = (1,2)$

$a=2, \quad b=4$

$\therefore \frac{x}{2} + \frac{y}{4} = 1$

$\Rightarrow 2x + y - 4 = 0.$

$\perp$ distance from the origin $= \frac{|-4|}{\sqrt{4+1}} = \frac{4}{\sqrt{5}}$

Ans: C

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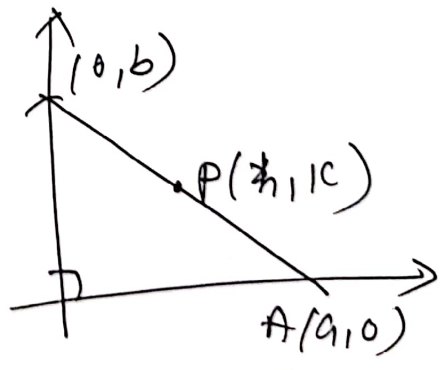
16. $\left(\frac{a}{2}, \frac{b}{2}\right) = (h,k)$



$a = 2h, \quad b = 2k$

$\therefore \frac{x}{2h} + \frac{y}{2k} = 1$

$\Rightarrow kx + hy - 2hk = 0$

$p = \frac{2hk}{\sqrt{k^2+h^2}}$ | $p^2 = \frac{4h^2k^2}{h^2+k^2}$

Locus $\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

Ans: C

17. $(q-r)x+(r-p)y+(p-q)=0$

$(q^3-r^3)x+(r^3-p^3)y+(p^3-q^3)=0$

represents same line

$$\frac{q^3-r^3}{q-r}=\frac{r^3-p^3}{r-p}=\frac{p^3-q^3}{p-q}$$

$\Rightarrow q^2+qr+\cancel{r^2}=\cancel{r^2}+rp+p^2$

$\Rightarrow q^2-p^2=rp-qr$

$\Rightarrow (q+p)(q-p)=r(p-q)$

$\Rightarrow (q+p)(q-p)+r(q-p)=0$

$\Rightarrow (p+q+r)(q-p)=0$

$\Rightarrow p+q+r=0$

Ans : A

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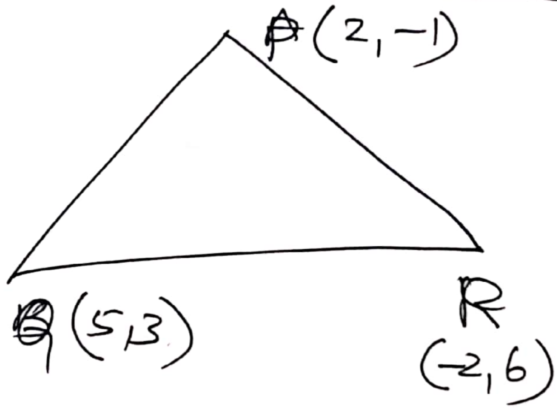
18. slope of $PQ=\frac{3+1}{5-2}=\frac{4}{3}$



slope of $QR=\frac{6-3}{-2-5}$

$=-\frac{3}{7}$

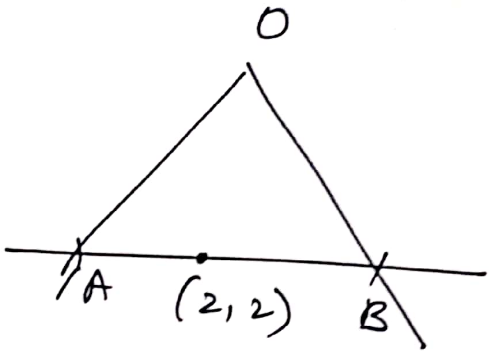
$\therefore \tan\theta=\left|\frac{\frac{4}{3}+\frac{3}{7}}{1-\frac{4}{3}\cdot\frac{3}{7}}\right|=\left|\frac{37}{9}\right|$

Ans: D

19. $\sqrt{3}x + y = 0$

slope $= -\sqrt{3} \Rightarrow m_1$

Let Slope of AB $= m_2$



$$\tan 60^\circ = \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right|$$

$$\sqrt{3} = \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right|$$

$$\Rightarrow \pm\sqrt{3} = \frac{m+\sqrt{3}}{1-\sqrt{3}m}$$

Case (i) $\sqrt{3} = \dfrac{m+\sqrt{3}}{1-\sqrt{3}m}$

$\Rightarrow \cancel{\sqrt{3}} - 3m = m + \cancel{\sqrt{3}}$

$\Rightarrow m = 0.$

$\therefore$ Eq$\underline{n}$ of the line $(2, 2)$, $m = 0$

$y - 2 = 0$

$\Rightarrow y = 2 \Rightarrow y = K$

Ans: 2

20. $4x - 3y - 8 = 0$ | $2x + 5y - 4 = 0$

$m_1 = \frac{4}{3}$ | $m_2 = -\frac{2}{5}$

$$\tan\theta = \left|\frac{\frac{4}{3}+\frac{2}{5}}{1-\frac{4}{3}\cdot\frac{2}{5}}\right| = \left|\frac{26}{7}\right|$$

$$= 3.7$$

Intergral value $= 3$

Ans: 3

21 (i) $2x - 3y + 7 = 0$ | $7x + 4y - 9 = 0$

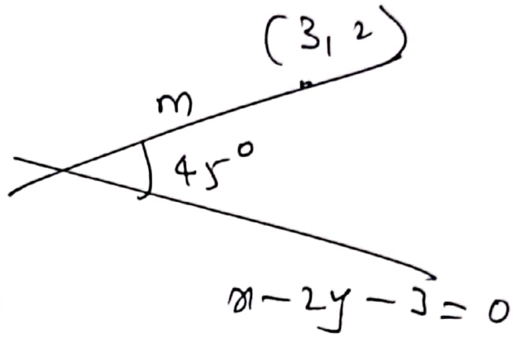
$m_1 = \frac{2}{3}$ | $m_2 = -\frac{7}{4}$

$$\tan\theta = \left| \frac{\frac{2}{3} + \frac{7}{4}}{1 - \frac{2}{3}\cdot\frac{7}{4}} \right| = \left| \frac{29}{-2} \right|$$

$$\therefore \theta = \tan^{-1}\left(\pm \frac{29}{2}\right)$$

(ii) $x - 2y - 3 = 0$

slope $= m = \frac{1}{2}$



$$\tan 45^\circ = \left| \frac{m - \frac{1}{2}}{1 + \frac{m}{2}} \right|$$

$$1 = \left| \frac{2m-1}{2+m} \right|$$

$$\frac{2m-1}{2+m} = \pm 1$$

$$\Rightarrow \frac{2m-1}{2+m} = 1$$

$$\Rightarrow 2m - 1 = 2 + m$$

$$\Rightarrow m = 3.$$

$\therefore$ eqn $(3, 2)$, $m = 3$

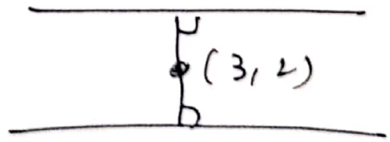
$$y - 2 = 3(x - 3)$$

$$\Rightarrow y - 2 = 3x - 9$$

$$\Rightarrow 3x - y - 7 = 0$$

(iii) $x + 2y - 5 = 0$

$$x + 2y + c = 0$$



$$\frac{|3 + 2(2) - 5|}{\sqrt{1+4}} = \frac{|3 + 2(2) + c|}{\sqrt{1+4}}$$

$$|2| = |7 + c|$$

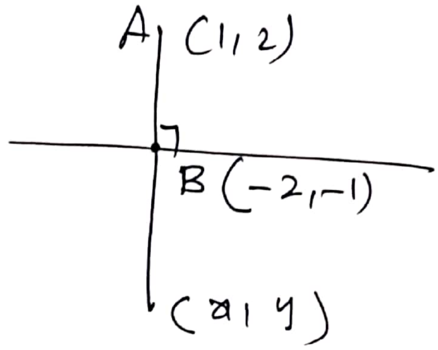
$$\pm 2 = 7 + c$$

$$c = -5 \text{ or } -9$$

$$x + 2y - 9 = 0$$

(iv) $\left(\frac{x+1}{2}, \frac{y+2}{2}\right) = (-2,-1)$

$(x,y) = (-5,-4)$



Ans: t, p, q, r

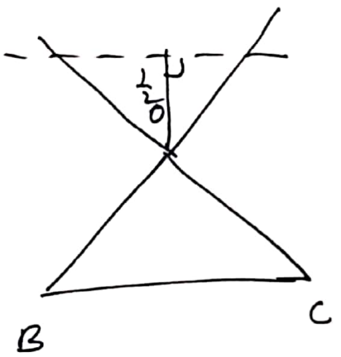
22 (i)

$(-2,-7) \rightarrow (x-2y,\ -3x+y)$
x, y

$\rightarrow (-2-2(-7),\ -3(-2)-7)$

$\rightarrow (12,\ -1)$

(ii) $B(-3,-1)$, $C(-1,-3)$



slope of $BC = m = \frac{-3+1}{-1+3}$

$= \frac{-2}{2} = -1$

Eqn of BC $\Rightarrow y+1 = -1(x+3)$

$\Rightarrow x+3+y+1 = 0$

$\Rightarrow x+y+4 = 0.$

Eqn of Required line $\quad x+y+k=0$

$\frac{1}{2} = \frac{|k|}{\sqrt{1+1}} \Rightarrow k = \pm\frac{\sqrt{2}}{2} \Rightarrow k = \frac{1}{\sqrt{2}}$

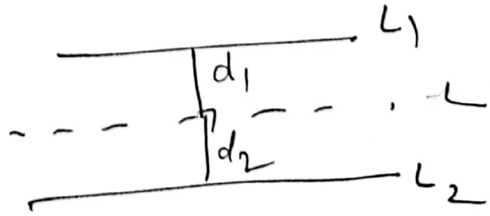
Required line is $\quad x+y+\frac{1}{\sqrt{2}} = 0$

$\Rightarrow \sqrt{2}x + \sqrt{2}y + 1 = 0$

(iii) $L_1 = 3x+4y+2=0$

$L = 3x+4y+5=0$

$L_2 = 3x+4y-5=0$



$d_1 = \frac{|2-5|}{\sqrt{3^2+4^2}} = \frac{3}{5}$

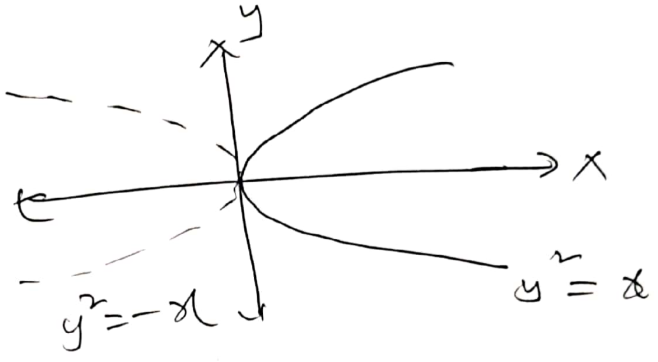
$d_2 = \frac{|2+5|}{\sqrt{3^2+4^2}} = \frac{7}{5}$

$\therefore d_1 : d_2 = 3:7$

(iv) $y = \sqrt{x}$

$\Rightarrow y^2 = x$

$\therefore y = \sqrt{-x}$



Ans: r, —, p, t

STUDENTS TASK

CUQ'S

01.	Conceptual	Ans: B
02	Conceptual	Ans: C
03.	Conceptual	Ans: A
04.	Conceptual	Ans: C
05	Conceptual	Ans: D
06.	Conceptual	Ans: D
07.	Conceptual	Ans: C
08	Conceptual	Ans B
09	Conceptual	Ans: A
10	Conceptual	Ans: C

JEE MAINS LEVEL

01. Image of $(2,3)$ wrt y-axis $= (-2,3)$

$A(-2,3)$, $B(-1,-1)$, $C(-4,\lambda)$

Collinear $\Rightarrow$ slope of AB = slope of BC

$$\Rightarrow \frac{-1-3}{-1+2} = \frac{\lambda+1}{-4+1}$$

$$\Rightarrow \lambda = 11$$

Ans: C

02
$$d = \frac{|-1|}{\sqrt{\frac{1}{p^2\sec^2\alpha} + \frac{1}{p^2\operatorname{cosec}^2\alpha}}}$$

$$= \frac{|p|}{\sqrt{\cos^2\alpha + \sin^2\alpha}} = |p|$$

Ans: D

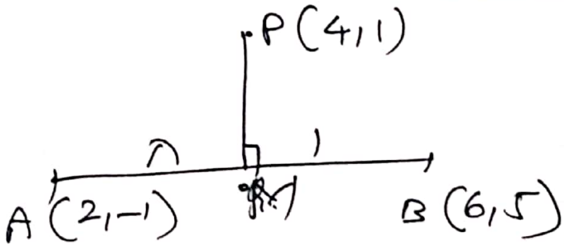
03
$$-L_{11} : L_{22} = 4:9$$

$$-(3\cdot 1 + 4\cdot 2 - 7) : (3\cdot k + 4\cdot 1 - 7) = 4:9$$

$$\Rightarrow k = -2$$

Ans: B

04. Let ratio be $\lambda : 1$



$$M = \left(\frac{6\lambda+2}{\lambda+1}, \frac{5\lambda-1}{\lambda+1}\right)$$

$$S_{PM} \times S_{AB} = -1$$

$$\frac{\frac{5\lambda-1}{\lambda+1} - 1}{\frac{6\lambda+2}{\lambda+1} - 4} \times \frac{5+1}{6-2} = -1$$

upon solving

$$\lambda = \frac{5}{8}$$

$$\therefore \lambda : 1 = 5:8$$

Ans: B

05 $x\cos\alpha + y\sin\alpha = P_1 \rightarrow (1)$

$x\cos\beta + y\sin\beta = P_2 \rightarrow (2)$

slope of (1) $\Rightarrow m_1 = -\dfrac{\cos\alpha}{\sin\alpha}$

Slope of (2) $\Rightarrow m_2 = -\dfrac{\cos\beta}{\sin\beta}$

$$\therefore \tan\theta = \left| \frac{-\frac{\cos\alpha}{\sin\alpha} + \frac{\cos\beta}{\sin\beta}}{1 + \frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}} \right|$$

$$= \left| \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta} \right|$$

$$= \left| \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} \right| = \tan(\alpha-\beta)$$

$\therefore \theta = \alpha - \beta$

Ans: B

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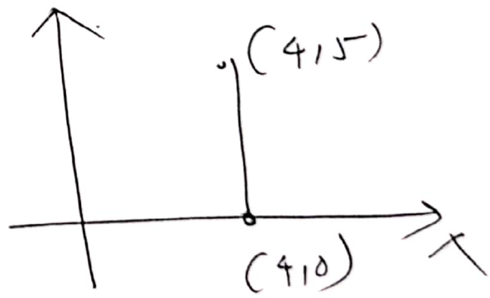
06 Foot of the ⊥ from $(4,5)$ on X-axis $= (4,0)$.

Image of $(4,5)$ w.r.t y-axis $= (-4,5)$



Required Equ $(4,0), (-4,5) \rightarrow 5x + 8y - 20 = 0$

Ans : B

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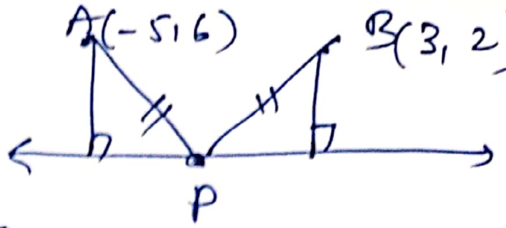
07. $\left(-\frac{7}{5}, -\frac{6}{5}\right), (1,2)$

mid-point $= \left(\dfrac{-\frac{7}{5}+1}{2}, \dfrac{-\frac{6}{5}+2}{2}\right) = \left(-\frac{1}{5}, \frac{2}{5}\right)$

$3x + 4y = 1$ satisfies above pt : Ans: B

08 Let $P(h,k)$ be the required point.



$P(h,k)$ lies on $4x-y-2=0$

$4h-k-2=0 \longrightarrow (1)$

Given $PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (h+5)^2 + (k-6)^2 = (h-3)^2 + (k-2)^2$

$\Rightarrow h^2+10h+25+k^2-12k+36 = h^2-6h+9+k^2-4k+4$

$\Rightarrow 16h-8k+48=0$

$\Rightarrow 2h-k+6=0 \longrightarrow (2)$

Solving (1) & (2) $(h,k) = (4,14)$

Ans: B

---

09. Option verification method. Opt: C

$$d = \frac{|2(-2)-1+5|}{\sqrt{4+1}} = 0.$$

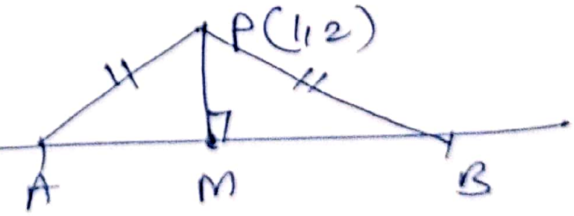
$\therefore$ C$(-2,1)$ lies on the line.

Hence it is nearest point

Ans: C

---

10. M is foot of the ⊥r from $P(1,2)$ upon the line $2x-y+3=0$.



$$\frac{h-1}{2} = \frac{k-2}{-1} = -\frac{(2.1-2+3)}{4+1}$$

$\therefore (h,k) = \left(\frac{-1}{5}, \frac{13}{5}\right)$

Ans: A

11. $4x - y - 7 = 0$ | $kx - 5y - 9 = 0$

$m_1 = 4$ | $m_2 = \frac{k}{5}$

$$\tan 45^\circ = \left| \frac{4 - \frac{k}{5}}{1 + \frac{4k}{5}} \right|$$

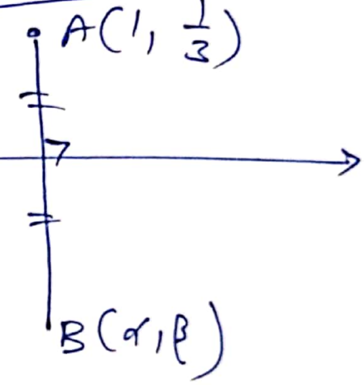
$$1 = \left| \frac{20 - k}{5 + 4k} \right|$$

$$\therefore \frac{20-k}{5+4k} = \pm 5 \Rightarrow k = \frac{-25}{3}, 3$$

Ans: A, B

---

12. $B(\alpha, \beta)$ is the image of $(1, \frac{1}{3})$ w.r.t. $2x + 3y - 5 = 0$.



$$\frac{\alpha - 1}{2} = \frac{\beta - \frac{1}{3}}{3} = \frac{-2\left(2.1 + 3\left(\frac{1}{3}\right) - 5\right)}{2^2 + 3^2}$$

$$= \frac{4}{13}$$

$\therefore \alpha = \frac{8}{13} + 1, \quad \beta = \frac{12}{13} + \frac{1}{3}$ $\qquad \frac{36}{13}$

$\therefore \alpha = \frac{21}{13}, \quad \beta = \frac{49}{39}$

Ans: A, B

---

13. Statement I: $y - 3kx + 4 = 0$

$3kx - y - 4 = 0$

Slope $= m_1 = 3k$

$(2k-1)x - (8k-1)y - 6 = 0$

Slope $= m_2 = \frac{2k-1}{8k-1}$

For $\perp \Rightarrow m_1 \times m_2 = -1$

$$(3k) \times \left(\frac{2k-1}{8k-1}\right) = -1$$

$$\Rightarrow k = \frac{1}{6}$$

(True)

Statement II: Conceptual (True) Ans: A

14. Statement I:

$$7 = \frac{|3\cdot 1 + 4\cdot 1 + c|}{\sqrt{9+16}}$$

$\Rightarrow |7+c| = 35$

$\Rightarrow 7+c = \pm 35$

$c = 28$ or -42 (True)

17

Statement II: Conceptual (True) Ans: B

15. $\frac{h-4}{5} = \frac{k+13}{1} = \frac{-2(5\cdot 4 - 13 + 6)}{25+1}$

$\therefore (h,k) = (-1,-14)$ Ans: A

16. $\frac{h-3}{1} = \frac{k-5}{-1} = \frac{-2(3-5+1)}{1+1} = 1$

$\frac{h-3}{1} = 1$ | $\frac{k-5}{-1} = 1$

$\Rightarrow h = \cancel{3}4$ | $k = 4$

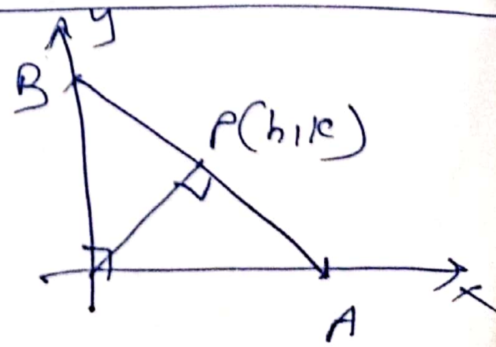
$\therefore (h,k) = (\cancel{3}4, 4)$

$\therefore$ opt B: $(x-2)^2 + (y-4)^2 = 4$. satisfies Ans: B

17. $3x + y - \lambda = 0$ O(0,0)

$\frac{h-0}{3} = \frac{k-0}{1} = -\frac{(3\cdot 0 + 0 - \lambda)}{9+1}$

$= \frac{\lambda}{10}$



$(h,k) = \left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$.

This pt lies on $\cancel{3x+y-\lambda=0}$

$\cancel{3\left(\frac{3\lambda}{10}\right)+\left(\frac{\lambda}{10}\right)}$

Now $3x+y-\lambda=0$

$\Rightarrow 3x+y=\lambda$

$\Rightarrow \frac{x}{\left(\frac{\lambda}{3}\right)}+\frac{y}{\lambda}=1$

$\therefore A\left(\frac{\lambda}{3},0\right), B(0,\lambda)$

Required ratio: $x_1-x : x-x_2$

AP : PB $= \frac{\lambda}{3}-\frac{3\lambda}{10} : \frac{3\lambda}{10}-0$

$\Rightarrow \frac{1}{3}-\frac{3}{10} : \frac{3}{10}$

$\Rightarrow \frac{1}{30} : \frac{3}{10}$

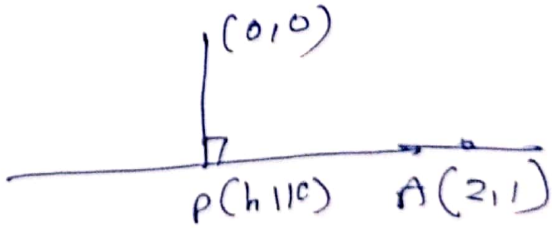
$\Rightarrow 1:9$

$\therefore$ BP : PA $= 9:1$

Ans: A

---

18. Slope of OP $= m_1 = \frac{k}{h}$



Slope of AB $m_2 = \frac{k-1}{h-2}$

$\therefore m_1 \times m_2 = -1$

$\frac{k}{h} \times \frac{k-1}{h-2} = -1$

$k^2-k = -h^2+2h$

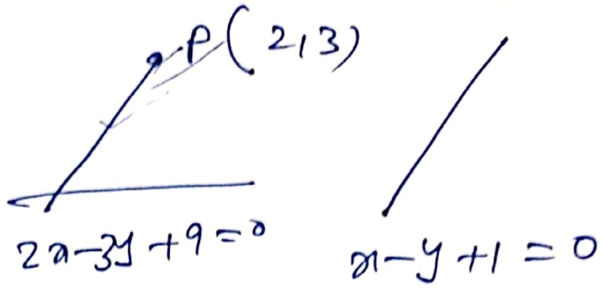
$h^2+k^2-2h-k=0$

Locus $\rightarrow x^2+y^2-2x-y=0$

Ans. B

19. slope of $x-y+1=0$

$m = \frac{-1}{-1} = 1$



$\therefore \tan\alpha = 1$

$\alpha = 45^\circ$

$$d = \left|\frac{ax_1 + by_1 + c}{a\cos\alpha + b\sin\alpha}\right| = \left|\frac{2\cdot 2 - 3\cdot 3 + 9}{2\cdot\frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}}}\right|$$

$\frac{\frac{y}{9}}{13}$

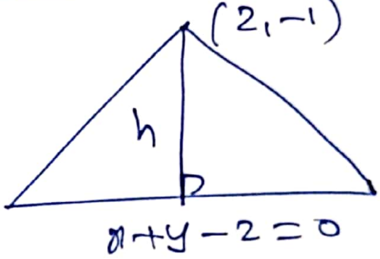
$= \sqrt{2}\times 4$

$\therefore d^2 = 32$

Ans: 32

---

20. $h = \frac{|2-1-2|}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$



$\therefore \frac{\sqrt{3}}{2} d = \frac{1}{\sqrt{2}}$

$\Rightarrow d = \frac{2}{\sqrt{6}}$

$\Rightarrow d^2 = \frac{4}{6} = 0.6$

Integral value $= 0$

Ans. 0

---

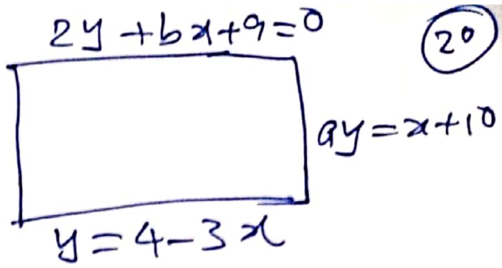
21 (i) Reflection of $x=3$ in y-axis is $x=-3$

(ii) $3x+4y+3=0 \rightarrow (1)$

$(1)\times 2 \Rightarrow 6x+8y+6=0$

$6x+8y+11=0$

$$d = \frac{|11-6|}{\sqrt{36+64}} = \frac{5}{10} = \frac{1}{2}$$



(iii) $3x + y - 4 = 0$

$bx + 2y + 9 = 0$

Lines are parallel

$\Rightarrow -\frac{b}{2} = \frac{-3}{1} \Rightarrow \boxed{b = 6}$

$x - ay + 10 = 0$

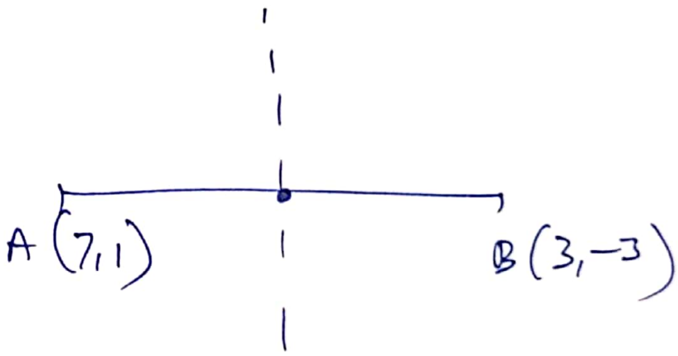
$3x + y - 4 = 0$

Lines are $\perp$r

$\Rightarrow \frac{1}{a} \times -3 = -1$

$\Rightarrow \boxed{a = 3}$

$\therefore ab = 18$



(iv) mid-point

$= \left(\frac{7+3}{2}, \frac{1-3}{2}\right)$

$= (5, -1)$

Ans : S, P, Q, R

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22 (i) $\tan 45^\circ = \left|\frac{m-1}{1+m}\right|$

$1 = \left|\frac{m-1}{1+m}\right|$

$\frac{m-1}{1+m} = \pm 1$

$\Rightarrow \frac{m-1}{1+m} = 1$ or $\frac{m-1}{1+m} = -1$

$\Rightarrow m - 1 = 1 + m$ $\quad$ $m - 1 = -1 - m$

X

22) (i) Point $(3,2)$ Given line $x-2y=3$

$$\text{Slope} = m_1 = \frac{1}{2}$$

$$\therefore \tan 45^\circ = \left|\frac{m-\frac{1}{2}}{1+\frac{m}{2}}\right|$$

$$\Rightarrow 1 = \left|\frac{2m-1}{2+m}\right|$$

$$\frac{2m-1}{2+m} = \pm 1$$

$$\Rightarrow 2m-1 = 2+m$$

$$\Rightarrow m = 3$$

$$\frac{2m-1}{2+m} = -1$$

$$2m-1 = -2-m$$

$$3m = -1$$

$$m = -1/3$$

(ii) $y = 2x+1$

$$\text{Slope} = m = 2$$

$$\tan 45^\circ = \left|\frac{m-2}{1+2m}\right|$$

$$\frac{m-2}{1+2m} = \pm 1$$

$$m-2 = 1+2m$$

$$\Rightarrow m = -3$$

eqn of the line $(1,2)$, $m=-3$

$$3x+y=5$$

(iii) $(1,1)$, $(x+y+5=0$

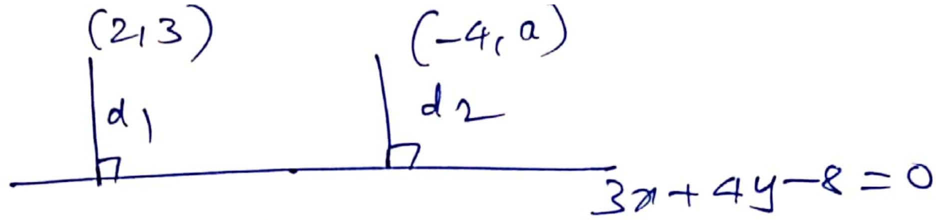
$$\frac{h-1}{1} = \frac{k-1}{1} = -2\frac{(1+1+5)}{1+1}$$

$$\therefore (h,k) = (-6,-6)$$

$$\text{Distance} = \sqrt{36+36}$$

$$= 6\sqrt{2}$$

(iv)



$$d_1 = d_2$$

$$\frac{|-12+4a-8|}{\sqrt{9+16}} = \frac{|6+12-8|}{\sqrt{9+16}}$$

$$\Rightarrow \quad |4a-20| = |10|$$

$$\Rightarrow \quad 4a-20 = \pm 10$$

$$\Rightarrow \quad a = \frac{32}{4} = 8$$

p)

Ans : p, r, s, q

⟹ THE END ⟸